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"The work of science is to substitute facts for appearances and demonstrations for impressions."—RUSKIN.

TRANSACTIONS

October 10th and 11th, 1907

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OCTOBER 10TH AND 11TH, 1907.

Valuation of Policies on the Select and Ultimate Basis.

BY H. N. SHEPPARD.

As it is necessary by Section 84 of Chapter 326, Laws of the State of New York, for every life insurance company doing business in that State to report the excess of the valuation of its policies over the legal minimum standard, it is necessary to consider the best method of calculating reserves on the Select and Ultimate basis. The writer believes the best method is to calculate the value of $-{}_0V_{[x]}$, which represents the discounted profit from mortality, by Mr. Henderson's formula given in the number of the *American Underwriter* for August, 1906, viz. : $-{}_0V_{[x]} = \frac{M_x - M_{[x]}}{D_{[x]}} - \pi^x \frac{N_x - N_{[x]}}{D_{[x]}}$ and then by means of Select "u" and "k" columns to calculate through to ${}_5V_{[x]}$, the agreement of the result so found with that of the standard table proving the accuracy of the work.

The following values of $U_{[x]+n}$ and $K_{[x]+n}$ at $3\frac{1}{2}\%$ were calculated from the formulas

$$U_{[x]+n} = \frac{D_{[x]+n}}{D_{[x]+n+1}} \text{ and } K_{[x]+n} = \frac{C_{[x]+n}}{D_{[x]+n+1}}$$

using the figures on pages 36 and 39 of Vol. II of Mr. Dawson's Practical Lessons in Actuarial Science, the values of $C_{[x]+n}$ being

calculated by differencing the values of $M_{[x]+n}$. The values so found were checked by the formulas :

$$U_{[x]+n} = \frac{1+i}{p_{[x]+n}} \text{ and } K_{[x]+n} = \frac{d_{[x]+n}}{l_{[x]+n+1}}$$

When this was done the following errors were found in the printed tables :

Page 39 age 32, the successive values of $M_{[x]+n}$ should be 9475.09, 9360.37, 9214.76, 9051.57 and 8871.87 ; age 43, 7313.52 and 7227.02, and Page 44 age 32, 12657.32, 12522.72, 12351.05, 12157.72 and 11943.80 ; age 43, 10045.91 and 9938.87 ; the remaining values at age 43 being correct.

The values by the two pairs of formulas did not agree exactly, as five figure logarithms were used in the calculation of the original tables ; so the values calculated from the commutation columns are the ones printed below, as all net premiums and reserves finally depend on these columns.

AMERICAN $3\frac{1}{2}\%$—SELECT AND ULTIMATE VALUES OF $U_{[x]+n}$.

$[x]$	$U_{[x]}$	$U_{[x]+1}$	$U_{[x]+2}$	$U_{[x]+3}$	$U_{[x]+4}$
20	1.039055	1.040325	1.041154	1.042044	1.042947
21	1.039073	1.040356	1.041204	1.042116	1.042985
22	1.039096	1.040394	1.041263	1.042143	1.043062
23	1.039125	1.040441	1.041307	1.042182	1.043158
24	1.039169	1.040447	1.041337	1.042270	1.043183
25	1.039170	1.040495	1.041417	1.042287	1.043297
26	1.039218	1.040567	1.041428	1.042395	1.043341
27	1.039259	1.040600	1.041495	1.042468	1.043419
28	1.039294	1.040661	1.041561	1.042541	1.043534
29	1.039328	1.040723	1.041629	1.042618	1.043660
30	1.039362	1.040750	1.041740	1.042703	1.043756
31	1.039435	1.040819	1.041782	1.042839	1.043867
32	1.039443	1.040899	1.041878	1.042953	1.043998
33	1.039535	1.040992	1.041944	1.043084	1.044150
34	1.039563	1.041060	1.042075	1.043196	1.044334
35	1.039652	1.041150	1.042188	1.043388	1.044508
36	1.039719	1.041265	1.042382	1.043463	1.044718
37	1.039768	1.041364	1.042513	1.043681	1.044923
38	1.039894	1.041499	1.042631	1.043892	1.045181
39	1.040012	1.041625	1.042853	1.044106	1.045444
40	1.040122	1.041741	1.043014	1.044377	1.045708
41	1.040223	1.041909	1.043235	1.044659	1.046052
42	1.040318	1.042202	1.043464	1.044884	1.046484
43	1.040536	1.042315	1.043698	1.045331	1.046874
44	1.040635	1.042565	1.044020	1.045666	1.047452
45	1.040804	1.042831	1.044443	1.046110	1.048004
46	1.041062	1.043125	1.044749	1.046677	1.048787
47	1.041268	1.043525	1.045254	1.047223	1.049563
48	1.041503	1.043888	1.045814	1.047922	1.050431
49	1.041771	1.044386	1.046355	1.048706	1.051302
50	1.042170	1.044861	1.047065	1.049595	1.052367
51	1.042534	1.045497	1.047768	1.050586	1.053589
52	1.042963	1.046125	1.048691	1.051593	1.054919
53	1.043569	1.046811	1.049612	1.052792	1.056399
54	1.044070	1.047635	1.050657	1.054113	1.058063
55	1.044701	1.048548	1.051817	1.055583	1.059891
56	1.045389	1.049572	1.053093	1.057207	1.061951
57	1.046160	1.050651	1.054564	1.059031	1.064192
58	1.047013	1.051906	1.056144	1.061038	1.066707
59	1.047949	1.053275	1.057913	1.063277	1.069510
60	1.049014	1.054799	1.059854	1.065752	1.072572
61	1.050184	1.056467	1.062033	1.068486	1.076023
62	1.051458	1.058361	1.064435	1.071546	1.079824
63	1.052858	1.060423	1.067099	1.074945	1.084033
64	1.054440	1.062723	1.070070	1.078669	1.088777
65	1.056178	1.065260	1.073355	1.082857	1.093987
66	1.05812	1.06807	1.07701	1.08748	1.09976
67	1.06025	1.07119	1.08101	1.09256	1.10613
68	1.06260	1.07467	1.08544	1.09819	1.11295
69	1.06522	1.07843	1.09035	1.10421	1.12034
70	1.06813	1.08262	1.09558	1.11071	1.12828

AMERICAN $3\frac{1}{2}\%$—SELECT AND ULTIMATE VALUES OF $K_{[x]+n}$.

$[x]$	$K_{[x]}$	$K_{[x]+1}$	$K_{[x]+2}$	$K_{[x]+3}$	$K_{[x]+4}$
20	.003931	.005117	.005956	.006821	.007682
21	.003940	.005169	.006004	.006865	.007722
22	.003960	.005200	.006052	.006910	.007774
23	.003992	.005242	.006090	.006955	.007862
24	.004025	.005262	.006140	.007024	.007915
25	.004046	.005305	.006191	.007071	.008006
26	.004091	.005350	.006243	.007142	.008062
27	.004102	.005406	.006308	.007203	.008143
28	.004148	.005451	.006349	.007288	.008262
29	.004184	.005510	.006428	.007364	.008346
30	.004219	.005569	.006496	.007465	.008458
31	.004268	.005606	.006589	.007556	.008583
32	.004318	.005704	.006661	.007650	.008687
33	.004380	.005780	.006745	.007782	.008856
34	.004432	.005845	.006857	.007904	.009028
35	.004498	.005936	.006984	.008068	.009167
36	.004577	.006030	.007115	.008211	.009387
37	.004646	.006139	.007235	.008396	.009599
38	.004729	.006263	.007398	.008572	.009844
39	.004827	.006417	.007565	.008780	.010082
40	.004915	.006535	.007736	.009020	.010408
41	.005032	.006696	.007952	.009281	.010717
42	.005151	.006876	.008189	.009577	.011090
43	.005288	.007101	.008432	.009923	.011545
44	.005443	.007306	.008738	.010310	.012017
45	.005617	.007559	.009084	.010738	.012607
46	.005824	.007864	.009455	.011282	.013248
47	.006025	.008208	.009928	.011862	.014004
48	.006290	.008594	.010446	.012509	.014851
49	.006580	.009054	.011016	.013259	.015766
50	.006927	.009533	.011671	.014076	.016805
51	.007318	.010093	.012386	.015008	.017960
52	.007743	.010742	.013200	.016051	.019242
53	.008220	.011438	.014120	.017199	.020666
54	.008751	.012220	.015122	.018465	.022294
55	.009378	.013095	.016234	.019895	.024052
56	.010056	.014074	.017484	.021456	.026042
57	.010794	.015115	.018908	.023221	.028205
58	.011599	.016353	.020417	.025158	.030642
59	.012516	.017647	.022139	.027333	.033320
60	.013541	.019141	.024026	.029688	.036318
61	.014668	.020745	.026126	.032354	.039631
62	.015899	.022559	.028434	.035322	.043320
63	.017271	.024558	.031031	.038582	.047387
64	.018786	.026775	.033883	.042195	.051969
65	.020472	.029254	.037045	.046238	.056988
66	.022319	.031954	.040579	.050726	.062552
67	.024408	.034950	.044466	.055620	.068691
68	.026690	.038306	.048758	.061018	.075313
69	.029208	.041990	.053452	.066861	.082468
70	.032017	.045984	.058548	.073110	.090140

NOTE.—The figures for the ages 66 to 70 inclusive in the U and K columns above were kindly furnished to the writer by Mr. J. D. Craig.

Recent Insurance Legislation.

E. E. RHODES.

It is my purpose to review in this paper the laws regulating the business of life insurance enacted subsequently to the Armstrong legislation in New York, which do not relate solely to companies of local origin. Such laws have been enacted in the following States:

Alabama, Arizona, Arkansas, California, Colorado, Connecticut, Delaware, Illinois, Kansas, Louisiana, Maine, Massachusetts, Michigan, Minnesota, Missouri, Montana, New Hampshire, New Jersey, North Carolina, North Dakota, Oregon, South Dakota, Tennessee, Texas, West Virginia, Wisconsin and Wyoming.

The Legislatures of Florida, Georgia, Idaho, Indiana, Nebraska, New Mexico, Pennsylvania, Rhode Island, South Carolina and Utah, have not enacted any new laws relating to life insurance.

In the States of Colorado, Massachusetts and Minnesota, the entire code was revised, and the references herein relate to sections of the new code.

Unless otherwise indicated, the reference in each case will be to the laws of 1907.

1. *Policy Forms to be filed with the Insurance Commissioner.*

In the States of Colorado,[1] Illinois,[2] Michigan,[3] Minnesota,[4] New Jersey,[5] North Dakota[6] and Tennessee,[7] no policy may be issued until the form thereof has been filed with the Insurance Commissioner, and it shall be unlawful for any company to issue a policy if the form thereof is disapproved by him. His action, however, is made subject to review by the courts.

[1] Section 40.
[2] Approved May 20, 1907.
[3] Bill No. 679.
[4] Section 88.
[5] Chapter 72.
[6] S. B. No. 172.
[7] Chapter 457.

In Massachusetts [8] the policy forms must be filed with the Commissioner at least thirty days before they can be issued, and the Commissioner's disapproval must be expressed within that time. His action is subject to review by the courts.

2. *Policy to Constitute Entire Contract.*

In Colorado,[9] Delaware,[10] Louisiana,[11] Massachusetts,[12] Michigan,[13] Minnesota,[14] Montana,[15] New Hampshire,[16] New Jersey,[17] North Dakota [18] and Tennessee,[19] the policy is to constitute the entire contract. In Louisiana and New Jersey nothing can be incorporated therein by reference to any constitution, by-laws, rules, application or other writings, unless the same are indorsed upon or attached to the policy when issued. If on its face the policy is the entire contract, the spirit, at least, of the law would be complied with if the policy did not expressly state that it was the entire contract.

In Illinois [20] the policy, together with the application therefor, a copy of which is to be indorsed upon or attached to the policy and made a part thereof, shall constitute the entire contract, but the copy of the application need not be attached to any policy containing a clause making the policy incontestable from date of issue.

3. *Payment of Premiums.*

Colorado,[21] Illinois,[22] Michigan,[23] Minnesota,[24] New Jersey,[25] North Dakota [26] and Tennessee,[27] require a provision in the policy that premiums shall be payable in advance either at the home office of the company or to an agent on delivery of a receipt signed by one or more of the officers who shall be named in the policy.

[8] Section 75.
[9] Section 36.
[10] Approved March 5, 1907.
[11] Act 52, Laws of 1906.
[12] Section 75.
[13] Bill No. 679.
[14] Sections 85 and 92.
[15] H. B. 213.
[16] Approved April 3, 1907.
[17] Chapter 72.
[18] S. B. 53 and S. B. 172.
[19] Chapters 441 and 457.
[20] Approved May 20, 1907.
[21] Section 36.
[22] Approved May 20, 1907.
[23] Bill No. 679.
[24] Section 85.
[25] Chapter 72.
[26] S. B. 172.
[27] Chapter 457.

Colorado, Minnesota and New Jersey provide that the policy may be a receipt for the first premium, but the laws of the other States could be likewise construed. In Minnesota and New Jersey the receipt is to be countersigned by the agent, but this, so far as I know, is a general practice, with which the laws of the other States would not conflict. In Illinois the first premium is not included in the provision.

4. *Statements of Insured to be deemed Representations.*

In Louisiana,[28] Massachusetts,[29] Michigan,[30] Minnesota,[31] New Jersey,[32] North Dakota [33] and Tennessee,[34] the policy must provide that statements made by the Insured shall, in the absence of fraud, be deemed representations and not warranties. The elimination of warranties from the application would seem to be, in effect, a compliance with this requirement.

5. *Avoidance of Policy.*

In Colorado,[35] Massachusetts,[36] Michigan,[37] Minnesota,[38] North Dakota [39] and Tennessee,[40] no statement made by the Insured shall avoid the policy unless it is contained in a written application and a copy of such application shall be indorsed upon or attached to the policy when issued. The omission of any provision contrary to the above would seem to be a practical compliance with the law. In Massachusetts [41] no oral or written misrepresentation or warranty shall be deemed material, or defeat or void the policy unless made with actual intent to deceive, or unless the matter misrepresented or made a warranty increased the risk.

[28] Act 52, Laws of 1906.
[29] Section 75.
[30] Bill No. 679.
[31] Section 85.
[32] Chapter 72.
[33] S. B. 172.
[34] Chapter 457.
[35] Section 36.
[36] Section 75.
[37] Bill No. 679.
[38] Section 85.
[39] S. B. 172.
[40] Chapter 457.
[41] Section 21.

6. *Incontestability.*

In Colorado [42] the policy is to be incontestable after "not more than" two years from its date, except for non-payment of premiums and violation of the conditions of the policy relating to naval and military service in time of war, or other prohibited risks. By the careless use of words, this law is in effect exactly the opposite of what its framers intended it to be.

In Michigan,[43] Minnesota,[44] North Dakota [45] and Tennessee,[46] there must be a provision in the policy that the policy shall be incontestable after two years from its date except for non-payment of premiums and except for violations of the conditions of the policy relating to naval and military service in time of war.

In Minnesota a special form of policy, however, may be issued on the life of a person employed in an occupation classed by the company as extra hazardous, or as leading to hazardous employment, which shall provide that service in certain designated occupations may reduce the company's liability under the policy to a certain designated amount not less than the full policy reserve.

In Illinois [47] the policy is to be incontestable after two years from its date of issue, except for non-payment of premiums and for violation of the conditions of the policy relating to naval or military service in time of war.

In Massachusetts [48] the policy is to be incontestable after two years from its date, except for non-payment of premiums and for engaging in military or naval service in time of war without the consent in writing of an executive officer of the company.

In New Jersey,[49] after a specified time, not later than two years from its date, the policy is to be incontestable except for non-payment of premiums and violations of its expressed conditions, if any, relating to hazardous travel, residence or occupation, in which case the liability of the company may be limited to a definitely determinable reduced amount, not less than the full reserve for the policy and any existing dividend additions.

[42] Section 36.
[43] Bill No. 679.
[44] Section 85.
[45] S. B. 172.
[46] Chapter 457.
[47] Approved May 20, 1907.
[48] Section 75.
[49] Chapter 72.

7. *Misstatement of Age.*

In Colorado,[50] Massachusetts,[51] Michigan,[52] Minnesota,[53] New Jersey,[54] North Dakota [55] and Tennessee,[56] the policy must provide that if the age of the insured has been understated, the amount payable shall be such as the premium would have purchased at the correct age.

In Illinois [57] the policy must provide that if the age of the insured has been *mis*-stated, the amount payable shall be such as the premium would have purchased at the correct age, or the premium may be adjusted and credit given to the insured or to the company, according to the company's published rates at date of issue.

8. *Distribution of Surplus.*

In Colorado [58] the policyholder is to be permitted at the time the first dividend is declared, to select from among the options set forth in the policy the manner and method of payment of the surplus to be annually *apportioned* to his policy. The distribution may be deferred, but on all policies under which the distribution is deferred to a fixed or specified time and made contingent upon the policy being in force and the insured alive at that time, the company must annually ascertain, beginning at the end of the fifth policy year, the amount of the surplus to which all such policies as a separate class are entitled, and shall annually apportion to such policies as a class the amount of surplus so ascertained, "subject to the mortality experience of the company to the end of the dividend period." Upon application the company must furnish to the policyholder an annual statement showing the contingent surplus accumulation to the credit of the policy at the beginning of the preceding year, the rate of interest earned upon the accumulation, the amount of interest, the amount of saving and profit contingently accredited to the policy during the pre-

[50] Section 36.
[51] Section 75.
[52] Bill No. 679.
[53] Section 85.
[54] Chapter 72.
[55] S. B. 172.
[56] Chapter 457.
[57] Approved May 20, 1907.
[58] Sections 42 and 44.

ceding year, and the total amount of the surplus accumulation then contingently credited to the policy. The form of statement is prescribed.

Section 43 of the Colorado law states that the right to cash dividends provided by Section 42 may be specifically waived in the policy, but Section 42 does not give any right to cash dividends.

In Illinois [59] there is no restriction upon either the time or the manner of distributing surplus, except that participation shall begin not later than the end of the twentieth policy year. The policy may provide that each dividend shall be paid subject to the payment of the premium for the next ensuing year. The insured shall have the right to have any dividend to which he may be entitled paid in cash. If any company shall issue policies under the terms of which the payment of dividends is deferred beyond the third year, it shall furnish the Superintendent of Insurance with a statement showing the number and amount of all policies with deferred dividends, in force at the beginning of the year for which the statement is made, of all such policies issued and revived, or terminated during the year, with the mode of termination, and the number and amount of all such policies in force at the end of said year; also a statement showing any and all amounts provisionally set apart, ascertained or calculated or held awaiting apportionment upon such policies at the beginning of said year, the additions made to the said fund during the year, with the sources from which such additions arose, the deductions made from the fund during the year, with the reasons therefor, and the amount of said fund at the end of the year, which shall be carried as a separate liability to such classes of policies on and for which the fund was accumulated. Upon written request of the insured under any deferred dividend policy, after the policy shall have been in force more than three years, the company shall furnish the policyholder with a statement of the amount of surplus provisionally declared, ascertained or set aside on such policy and held awaiting apportionment at the expiration of the deferred dividend period.

In Louisiana [60] a foreign company which shall not provide in every participating policy that the proportion of the surplus accruing upon said policy shall be ascertained and distributed annually

[59] Approved May 20 and 23, 1907. [60] Act 88, Laws of 1906.

and not otherwise, and which shall not ascertain and distribute the surplus accruing upon said policies annually either by providing for their payment in cash, or their application to the payment of premiums, or to the purchase of paid-up additions, or for their accumulation as provided in case of domestic companies, shall not be permitted to do business in the state. It would seem that if any one of the options above named be given, the law would be complied with.

In Michigan [61] and Tennessee,[62] beginning not later than the end of the fifth policy year, the company must determine and account for the portion of the divisible surplus accruing on the policy. The owner of the policy shall have the right to have the current dividend arising from such participation paid in cash. At the option of the policyholder such accounting and payment shall be made at periods of not more than five years. Moreover, in Tennessee [63] whenever such apportionment has been made and the dividend left with the company, the amount thereof shall be reported as a separate liability, and the term "Divisible Surplus" is defined to mean the excess of admitted assets over and above the total liabilities "including legal reserve, paid-up or guaranteed capital stock, and any surplus set aside for the specific purpose of retiring said guaranteed capital stock, or $100,000 of required surplus if there be no capital stock, *and the contingency reserve.*"

In Massachusetts [64] the policy is to participate in surplus annually, beginning not later than the end of the third policy year. The amount apportioned is to be either paid in cash or applied in payment of premiums or to the purchase of paid-up additions, or accumulated by the company subject to withdrawal by the policyholder. The granting of any one of these options would seem to be all that is required.

In Minnesota the provisions seem to be conflicting. Section 72 provides that the company shall make an annual apportionment and accounting of divisible surplus to each policyholder, beginning not later than the end of the *third* policy year, and that each such policyholder will be entitled to and be credited with such a portion of the entire divisible surplus as has been contributed thereto by his

[61] Bill No. 679.
[62] Chapter 457.
[63] Chapter 454.
[64] Sections 75 and 76.

policy. Section 74 provides that every policyholder shall be permitted after his policy has been in force *five* years, annually to select the manner of the application of the dividend from among those set forth in the policy, and that all apportioned surplus not actually paid over to the insured or applied in the reduction of current or future premiums or in the purchase of paid-up insurance or pure endowment additions, shall be credited to the insured and carried as an actual liability to be paid at the maturity of the policy. Section 85 requires that the policy shall contain a provision for participation in surplus and that beginning not later than the end of the *third* policy year the company will annually determine and account for the portion of the divisible surplus accruing on the policy, and that the owner of the policy shall have the right each year after the *fifth,* to have the current dividend arising from such participation, paid in cash. If the policy provides other dividend options and the insured shall not elect any such other options, the dividend shall be paid in cash. The provision may stipulate that any dividends payable during the first *five* years shall be conditioned upon the payment of the next ensuing premium.

In Section 77 of the Minnesota law it is provided that on all deferred-dividend policies heretofore issued, the company shall annually ascertain the amount of surplus to which such policies as a separate class are entitled, and shall annually apportion to such policies as a class the amount of surplus so ascertained, and carry the same, plus the actual interest earnings and accretions of the fund, as a distinct and separate liability to such class of policies on and for which the same was accumulated. A company is not permitted to use any part of such apportioned surplus fund for any purpose whatsoever, other than the express purpose for which the same was accumulated.

In Montana,[65] North Dakota[66] and West Virginia,[67] the requirement is similar to Sections 72 and 74 of the Minnesota Law, "except that the five year provision found in Section 74 of the Minnesota law is omitted."

In North Dakota[68] and South Dakota[69] in case of deferred-dividend policies heretofore issued the company must annually

[65] H. B. 212.
[66] S. B. 61 and S. B. 172.
[67] Chapter 67.
[68] S. B. 59.
[69] Chapter 168.

ascertain the amount of surplus to which such policies as a separate class are entitled, apportion the same thereto, and carry the amount of such apportioned surplus, plus interest earnings and accretions to such fund, as a liability to the class of policies for which the same are accumulated. A company is not permitted to use any part of the fund for any purpose other than for the express purpose for which it was accumulated, and it must at the time of mailing the premium notice, show the contingent surplus accumulated to the credit of the "policy" at the beginning of the preceding year, the rate of interest earned upon the accumulation, the amount of the interest, and the amount of savings and profit contingently accredited to the policy during the preceding year, with a showing of the total amount of the accumulation then contingently accredited to the policy. The form of statement prescribed in South Dakota differs somewhat from that in North Dakota.

It will be observed that in Section 44 of the Colorado Law above referred to, and in the North Dakota and South Dakota laws, the required apportionment relates only to a particular class or classes of policies. The requirement regarding individual policies would seem to be nugatory.

In New Jersey [70] the policy must contain a provision that it shall participate in the surplus of the company, and that beginning not later than the end of the fifth policy year, the company at uniform intervals, not less than one nor more than five years, to be specified in the policy, will determine and account for the portion of the divisible surplus accruing thereon. The owner of the policy shall have the right to have the dividend paid in cash at the end of the then current policy year, provided no other dividend option given in the policy or in the dividend notice shall have been duly elected, and provided no part of any yearly premium for the ensuing policy year remains unpaid. The last proviso is optional with the company. The provision is not required in insurances issued or granted in exchange for lapsed or surrendered policies.

9. *Non-Forfeiture.*

In Arizona [71] the stipulations and requirements of Paragraph

[70] Chapter 72.

[71] Approved March 21, 1907.

809 of the Civil code are to be read into and be a part of the policy. Paragraph 809 defines the non-forfeiture provisions which every policy issued upon the life of a resident of the Territory and delivered within the Territory shall contain "unless specifically contracted between the insurer and the insured for tontine insurance or for other term or paid-up insurance."

In Colorado [72] the policy must provide that in event of default in premium payments after three years' premiums have been paid, there shall be secured to the insured without action on his part, as specified in the policy, either paid-up insurance or extended insurance, the net value of which shall be at least equal to the entire net reserve held by the company on the policy, including dividend additions (if any) less two and one-half per centum of the amount insured by the policy and dividend additions, if any, or one-fifth of such reserve, and less any outstanding indebtedness to the company on the policy at time of default. In lieu of the above, the policy may provide that unpaid premiums shall be charged as a lien against the policy, provided the amount that would otherwise be applicable to the purchase of paid-up or extended insurance is sufficient to secure such loan with interest added at a rate not exceeding six per centum. There shall also be secured to the insured the right to surrender the policy to the company at its home office within one month from date of default, for a cash value equal to the sum available for the purchase of paid-up or extended insurance. This latter requirement may be specifically waived in the policy.

In Louisiana [73] every policy other than a term policy for twenty years or less, must provide that in event of non-payment of premium, or of any loan or interest, after being in force three full years, not less than four-fifths of the entire reserve, less any indebtedness, shall, upon surrender of the policy be applied as a surrender value as agreed upon in the policy, provided that if no other option expressed in the policy be availed of by the owner thereof, the value shall be automatically applied to continue the insurance in force at its full amount, including dividend additions, less any indebtedness, so long as the surrender value will purchase non-participating term insurance at net single-premium rates by the standard adopted by the company. In case of Endowment

[72] Section 43.

[73] Act 193, Laws of 1906.

policies, if the value is more than sufficient to continue the insurance to the end of the endowment term, the excess shall be used to purchase pure endowment insurance payable at the end of the term. Any value allowed in lieu of the extended insurance must be at least equal to the net value of the extended insurance.

Massachusetts [74] requires in the policy a provision specifying the options to which the policyholder is entitled in event of default in premium payments after three years' premiums have been paid. It requires nothing more than a provision.

In Illinois,[75] Michigan,[76] Minnesota,[77] North Dakota [78] and Tennessee, [79] there must be in the policy a provision which in event of default in premium payments after premiums have been paid for three years, shall secure to the owner of the policy a stipulated form of insurance, the net value of which shall be at least equal to the entire reserve at date of default, specifying the mortality table and rate of interest adopted for computing the reserve, less a sum not more than two and one-half per centum, of the amount insured by the policy and any existing dividend additions, and less any existing indebtedness to the company on the policy. The provisions shall also stipulate that the policy may be surrendered to the company at its home office within one month from date of default, for a specified cash value at least equal to the sum which would otherwise be available for the purchase of insurance as aforesaid, and may stipulate that the company may defer payment not more than six months after the application is made.

The above provision is not required in term insurances of twenty years or less.

In Minnesota, Section 75, there is a provision regarding nonforfeiture, differing from Section 85 referred to above, in that it requires the granting of paid-up or extended insurance or cash value, of the same minimum value as that required under Section 85, provided the policy shall not be continued in force by virtue of an automatic loan provision therein, and in that Section 75 need not be incorporated in the policy. The two sections appear to be another evidence of careless legislation.

In Montana [80] and West Virginia [81] in event of default in pre-

[74] Section 75.
[75] Approved May 20, 1907.
[76] Bill No. 679.
[77] Section 85.
[78] S. B. 172.
[79] Chapter 457.
[80] H. B. 212.
[81] Chapter 67.

mium payments after three years' premiums have been paid, there shall be secured to the Insured without action on his part either paid-up insurance or extended insurance, as specified in the policy, the net value of which shall be at least equal to the entire net reserve held by the company, less two and one-half per centum of the amount insured by the policy and dividend additions, if any, and less any outstanding indebtedness to the company. There shall also be secured to the insured the right to surrender the policy to the company at its home office within one month from date of default, for the cash surrender value otherwise available for the purchase of paid-up or extended insurance.

The respects in which S. B. 61 in North Dakota differs from S. B. 172 will be noted by the reader.

Chapter 454 of the Tennessee laws also differs from Chapter 457 in that it follows Section 75 of the Minnesota Law, the only respect in which it differs from the latter being that instead of specifying paid-up or extended insurance, it specifies a stipulated form of insurance. It differs from Chapter 457 of the Tennessee law in that the latter does not refer to an automatic loan provision.

In New Jersey [82] there must be a provision in the policy which in event of default in premium payments after premiums have been paid for three years, secures to the owner of the policy a stipulated form of insurance the net value of which shall be at least equal to the entire specified reserve held by the company, less a specified percentage (not more than three) of the amount insured by the policy, including dividend additions, and less any outstanding indebtedness on the policy. The specified percentage need not be stated for the policy years included in the required table of surrender values, and the provision is not required in term policies of twenty years or less.

10. *Policy Loans.*

In Illinois [83] there must be a provision in the policy that after three full years' premiums have been paid, the company at any time while the policy is in force will loan on the execution of a proper note or loan agreement by the insured and on proper assignment and delivery of the policy and on the sole security thereof,

[82] Chapter 72.

[83] Approved May 20, 1907.

at a specified rate of interest, a sum equal to or at the option of the insured, less than the reserve at the end of the current policy year on the policy and dividend additions thereto, (the reserve basis to be specified) less a specified percentage, not more than two and one-half, of the amount insured by the policy and any dividend additions thereto, and that the company will deduct from such loan value any existing policy indebtedness and any unpaid balance of the premium for the current policy year, and may collect interest in advance on the loan to the end of the current policy year. The loan may be deferred for not exceeding six months after application therefor is made. No condition other than that provided by law shall be exacted as a prerequisite to any such loan. The provision is not required in term insurances, nor does it apply to temporary insurance or pure endowment insurance issued or granted in exchange for lapsed or surrendered policies. In ascertaining the indebtedness due upon policy loans, the interest, if not paid when due, shall be added to the principal of such loans, and shall bear interest at the rate specified in the note or loan agreement.

In Michigan,[84] Minnesota,[85] North Dakota[86] and Tennessee,[87] the law differs from the Illinois law, in that it must be stipulated that failure to repay any loan or interest shall not avoid the policy unless the total indebtedness shall equal or exceed the loan value at time of such failure, nor until one month after notice shall have been mailed by the company to the last known address of the insured and assignee, if any. In these four States the provision is not required in term insurances.

11. *Grace.*

In Massachusetts[88] the policy must contain a provision that the insured is entitled to a grace of thirty days, within which the payment of any premium after the first year may be made, subject, at the option of the company, to an interest charge not in excess of six per cent. per annum. During the period of grace the policy shall continue in full force, but in case the policy

[84] Bill No. 679.
[85] Section 85.
[86] S. B. 172.
[87] Chapter 457.
[88] Section 75.

becomes a claim during the period before the overdue premium or deferred premiums of the current policy year, if any, are paid, the amount of such premiums with interest on any overdue premium, may be taken from the face of the policy in settlement.

In Illinois,[89] Michigan,[90] Minnesota,[91] North Dakota[92] and Tennessee,[93] the policy must contain a provision for a grace of one month for the payment of every premium after the first year (in Minnesota and North Dakota the word "year" is omitted) which may be subject to an interest charge, during which month the insurance shall continue in force, which provision may contain a stipulation that if the insured shall die during the month of grace, the overdue premium will be deducted in any settlement of the policy.

In New Jersey[94] there must be a provision in the policy for one month's grace for the payment of each premium after the first, subject to an interest charge, during which month the insurance shall continue in force, provided the overdue premium shall be paid within the month.

It would seem that in all the above cases the company could not be required to accept the premium, unless it was paid within the month of grace.

12. *Reinstatement.*

In Massachusetts[95] the policy must contain a provision that the holder shall be entitled to reinstate it at any time within three years from date of default, unless the cash value has been duly paid or the extension period expired, upon evidence of insurability satisfactory to the company and the payment of all overdue premiums and any other indebtedness to the company on the policy, with interest at a rate not exceeding six per centum per annum.

In Illinois,[96] Michigan,[97] Minnesota,[98] North Dakota[99] and Tennessee,[100] the policy must contain a provision that if in event

[89] Approved May 20, 1907.
[90] Bill No. 679.
[91] Section 85.
[92] S. B. 172.
[93] Chapter 457.
[94] Chapter 72.
[95] Section 75.
[96] Approved May 20, 1907.
[97] Bill No. 679.
[98] Section 85.
[99] S. B. 172.
[100] Chapter 457.

of default in premium payments the value of the policy shall be applied to the purchase of other insurance, and if such other insurance shall be in force and the original policy shall not have been surrendered to the company and canceled, the policy may be reinstated within three years from default, upon evidence of insurability satisfactory to the company and payment of all arrears of premiums with interest.

In New Jersey [101] the policy must contain a like provision, except that the insured may be required to pay compound interest on arrears of premium and on all loans, if any.

13. *Settlement of Claims.*

In Illinois,[102] Michigan,[103] Minnesota,[104] New Jersey,[105] North Dakota [106] and Tennessee,[107] the policy must contain a provision that when it becomes a claim by the death of the insured, settlement shall be made immediately upon receipt of due proof, or not later than two months thereafter.

14. *Table of Surrender Values.*

Colorado,[108] Illinois,[109] Massachusetts,[110] Michigan,[111] Minnesota,[112] New Jersey,[113] North Dakota [114] and Tennessee,[115] require a table in the policy showing the loan values, if any, and the options available each year upon default in premium payments during at least the first twenty years of the policy, beginning with the year in which such values become available. In Massachusetts the table must cover twenty years.

15. *Table of Instalment Values.*

In Colorado,[116] Illinois,[117] Massachusetts,[118] Michigan,[119] Min-

[101] Chapter 72.
[102] Approved May 20, 1907.
[103] Bill No. 679.
[104] Section 85.
[105] Chapter 72.
[106] S. B. 172.
[107] Chapter 457.
[108] Section 36.
[109] Approved May 20, 1907.
[110] Section 75.
[111] Bill No. 679.
[112] Section 85.
[113] Chapter 72.
[114] S. B. 172.
[115] Chapter 457.
[116] Section 36.
[117] Approved May 20, 1907.
[118] Section 75.
[119] Bill No. 679.

nesota,[120] New Jersey,[121] North Dakota [122] and Tennessee,[123] the policy must contain a table showing the amounts of instalments in which the proceeds of the policy may be payable. In Massachusetts the table must include Annuity Payments as well as instalments.

16. *Policy Title.*

In Illinois,[124] Michigan,[125] Minnesota,[126] North Dakota [127] and Tennessee,[128] there must appear on the face and back of the policy a title correctly describing the same. In New Jersey [129] the title need appear only on the first page.

17. *Forfeiture for Failure to Repay Loan or Interest.*

Colorado,[130] Illinois,[131] Michigan,[132] Minnesota,[133] New Jersey,[134] North Dakota [135] and Tennessee,[136] prohibit any provision in the policy for forfeiture for failure to repay any loan, or interest thereon, while the total indebtedness on the policy is less that the loan value thereof, or (except in Illinois) any provision for such forfeiture unless it contain a stipulation that no such forfeiture shall occur until at least one month after notice shall have been mailed to the last known address of the insured. In Colorado, Michigan, Minnesota, North Dakota and Tennessee, notice must also be mailed to the assignee, if any, and in New Jersey notice must be mailed to the person to whom the loan was made.

18. *Dating Back.*

Colorado [137] prohibits any provision by which the policy shall purport to be issued or dated back more than one year before the

[120] Section 85.
[121] Chapter 72.
[122] S. B. 172.
[123] Chapter 457.
[124] Approved May 20, 1907.
[125] Bill No. 679.
[126] Section 85.
[127] S. B. 172.
[128] Chapter 457.
[129] Chapter 72.
[130] Section 37.
[131] Approved May 20, 1907.
[132] Bill No. 679.
[133] Section 86.
[134] Chapter 72.
[135] S. B. 172.
[136] Chapter 457.
[137] Section 37.

original application was made "if thereby the assured would rate at an age younger than his age not more than one year at date when application was made, according to his age nearest birthday." What is probably meant is that the insured cannot be rated at an age more than one year younger than his age (nearest birthday) at the time of the application.

Illinois [138] prohibits a provision by which the policy shall purport to be issued or dated back for more than six months before the original application was made.

Michigan [139] and Tennessee,[140] prohibit a provision by which the policy shall purport to be issued or to take effect before the original application was made, if thereby the insured would rate at an age younger than his age immediately preceding the date when the application was made. The Minnesota [141] and North Dakota [142] laws in this respect are the same, except that the insured must rate at his age nearest birthday.

19. *Limiting Time for Commencement of Action.*

Colorado,[143] Minnesota,[144] New Jersey,[145] North Dakota [146] and Tennessee,[147] prohibit any provision limiting the time at which any action at law or in equity may be commenced to less than five years after the cause of action shall accrue. In Illinois [148] the time is three years, in Michigan [149] six years, in Massachusetts [150] two years, and in West Virginia [151] one year.

20. *Mode of Settlement.*

Illinois [152] prohibits a provision for any mode of settlement at maturity of less value according to the company's published rates therefor then in use, than the amount insured by the face of the policy, plus dividend additions, if any, less any indebtedness

[138] Approved May 20, 1907.
[139] Bill No. 679.
[140] Chapter 457.
[141] Section 86.
[142] S. B. 172.
[143] Section 37.
[144] Section 86.
[145] Chapter 72.
[146] S. B. 172.
[147] Chapter 457.
[148] Approved May 20, 1907.
[149] Bill No. 679.
[150] Section 29.
[151] Chapter 77.
[152] Approved May 20, 1907.

on the policy and less any premium that may by the terms of the policy be deducted.

Michigan,[153] Minnesota,[154] North Dakota[155] and Tennessee,[156] have a similar law, which does not, however, refer to the company's published rates. Michigan exempts sub-standard policies.

21. *General Prohibition.*

Michigan,[157] North Dakota[158] and Tennessee,[159] prohibit any provision in the policy contemplating any proposed benefit not essentially a part of the insurance contract and any connection of the insured with the company other than that of policyholder.

22. *Reciprocal Provisions.*

In Illinois,[160] Michigan,[161] Minnesota,[162] North Dakota[163] and Tennessee,[164] the policies of foreign companies may contain any provision which the law of the state, territory, district or country under which the company is organized prescribes shall be in such policies when issued therein.

In New Jersey[165] no provision of the act regarding contents of policies shall apply to policies of life insurance issued or delivered in that state by a company operating therein but organized outside thereof, when such provision is inconsistent with any law of the state in which the company is organized.

23. *Agents.*

In Delaware,[166] Louisiana,[167] Minnesota,[168] Missouri,[169] Montana,[170] New Hampshire,[171] New Jersey,[172] North Carolina,[173]

[153] Bill No. 679.
[154] Section 86.
[155] S. B. 172.
[156] Chapter 457.
[157] Bill No. 679.
[158] S. B. 172.
[159] Chapter 457.
[160] Approved May 20, 1907.
[161] Bill No. 679.
[162] Section 89.
[163] S. B. 172.
[164] Chapter 457.
[165] Chapter 72.
[166] Approved March 21, 1907.
[167] Act 94, Laws of 1906.
[168] Section 63.
[169] H. B. 86.
[170] H. B. 123.
[171] Approved April 3, 1907.
[172] Chapter 72.
[173] Approved March 11, 1907.

North Dakota,[174] Tennessee[175] and West Virginia,[176] any person who shall solicit an application for insurance upon the life of another, shall, in any controversy relating thereto between the insured or his beneficiary and the company, be regarded as the agent of the company, and not as the agent of the insured. In Delaware, Minnesota, Missouri, Montana, New Hampshire, North Dakota and Tennessee, the words "relating thereto" are omitted from the law. In Delaware[177] and Tennessee,[178] no corporation or stock company shall be licensed as the agent of a life insurance company.

24. *Valuations.*

In California,[179] Colorado[180] and Illinois,[181] policies issued after January 1st, 1908, are to be valued on the basis of net premiums by the American Experience Table of Mortality and interest at 3½ per cent. yearly.

In Michigan[182] and Tennessee,[183] policies may be issued providing for not more than one year preliminary term insurance, by the incorporation therein of a clause on the face of the policy distinctly specifying that the first year's insurance is term insurance. If the premium charged for the term insurance under a limited payment life or endowment preliminary term policy providing for the payment of all premiums thereon in less than twenty years from the date of the policy, exceeds that charged for like insurance under 20 payment life preliminary term policies of the same company at the same age, the reserve thereon at the end of any year, including the first, shall not be less than the reserve on a 20 payment life preliminary term policy issued in the same year and at the same age, together with an amount which shall be equivalent to the accumulation of a net level premium sufficient to provide for a pure endowment at the end of the premium payment period equal to the difference between the value at the end of such period of such 20 payment life preliminary term policy and the full reserve at such time of such a limited payment life

[174] S. B. 175.
[175] Chapter 442.
[176] Chapter 53.
[177] Approved March 21, 1907.
[178] Chapter 444.
[179] Section 629 Code.
[180] Section 35.
[181] Approved May 23, 1907.
[182] Bill No. 679.
[183] Chapter 457.

or endowment policy. Illinois [184] has a similar provision affecting only limited payment life policies.

In North Dakota [185] and West Virginia,[186] the provision is the same as in Michigan and Tennessee, except that if the premium charged for term insurance on *any* endowment preliminary term policy exceeds that charged for insurance under whole life preliminary term policies of the same company, or if such is the case under a limited payment life preliminary term policy providing for the payment of all premiums thereon in less than twenty years, the reserve under such policies at the end of any year, including the first, shall not be less than the reserve on a whole life preliminary term policy issued in the same year and at the same age, together with an amount which shall be equivalent to the accumulation of a net level premium sufficient to provide for a pure endowment at the end of the premium payment period of such a whole life preliminary term policy and the full reserve at such time of such a limited payment or endowment policy. In West Virginia policies issued after January 1, 1907, are to be valued by the American Experience Table and 3½ per cent. interest.

In Minnesota [187] policies may provide for not more than one year preliminary term insurance, and shall be valued accordingly if the following clause is incorporated therein: "The first year's insurance under this policy is term insurance."

In New Jersey [188] in case of policies issued after 1907 a modified net reserve plan is to be used under certain conditions. Under this plan, in case of a policy having a net premium equal to or greater than the net premium at the same age for a continuous premium whole life policy, the reserve for the first year is found by reducing the usual net reserve (American Experience 3½ per cent.) by the excess of the reserve for the ordinary life policy over that for a one-year term policy at the same age. For the second year the usual reserve is diminished by five-sixths of the aforesaid excess; for the third year by four-sixths; for the fourth year by three-sixths; for the fifth year by two-sixths; for the sixth year by one-sixth. In case of any policy having a net premium less than that of the ordinary life policy, the usual reserve for the first

[184] Approved May 23, 1907.
[185] S. B. 172.
[186] Chapter 77.
[187] Section 84.
[188] Chapter 73.

year is reduced by the excess of the usual reserve over that for a one-year term policy; for the second year the usual reserve is reduced by three-fourths of the above mentioned excess; for the third year by one-half; and for the fourth year by one-fourth. Subject to the above, no policies issued after December 31st, 1907, shall be valued as term insurance if the premiums charged for term insurance exceed those charged by the company for like term insurance under any other form of policy. The Commissioner of Insurance may accept the valuation of the department of any other state when made upon a specified basis according to which the reserves shall be at least as large as if they had been computed upon the basis prescribed by the New Jersey law, if the insurance officer of the other state or country accepts the certificate of valuation of the New Jersey Commissioner, or if the insurance officer of any other state by express requirement of law shall have made yearly, for not less than thirty years past, valuations of policies of companies transacting business in such state. The modified net reserve plan is not to be employed in valuing the policies of any company which has premium reserves amounting to four hundred millions of dollars or more, and as to other companies, its application after December 31st, 1910, shall be subject to the following limitations: If the reserves amount to twenty-five millions on December 31st, 1910, or at the close of any year thereafter, the plan shall not be employed in the valuation of the endowment policies of such company issued in any succeeding year; and if the reserves of any company are found to amount to fifty millions of dollars on December 31st, 1910, or at the close of any year thereafter, the plan shall not be applied to the valuation of any policies thereafter issued. Request for the application of the plan must be filed with the Commissioner of Insurance not later than March 31st, and such request when made shall be considered as continued for succeeding years, unless the company shall withdraw such request. The request shall be accompanied by an agreement on the part of the company that the sum of the expenses incurred and payable wholly in connection with the first year's premiums on policies issued in the year, including commissions, agents' salaries, or other compensation based on new business, agents' advances, and medical fees and a proper share of other expenses, together with the modified mean net reserves to be held for such policies, shall not exceed the total amount of said first year's premiums including premiums

deferred or in course of transmission, and also that the aggregate expenses in each calendar year shall be so regulated that the percentage of such expenses upon premiums received in such year shall be less than when similarly computed for the calendar year next previous. In reckoning such aggregate expenses, investment expenses not exceeding one-fourth of one per cent. of the mean invested assets at book value, and taxes on real estate and other necessary outlays, exclusively connected with real estate, shall not be included. Companies shall, not later than the first day of February, file with the commissioner an exhibit showing how far it has fulfilled its agreement as to the calendar year next previous, and if the commissioner after due notice and hearing shall consider that the company has failed to make a reasonable and proper compliance with its agreement, he shall notify it that the modified net reserve plan will not be applied to the valuation of policies of said company issued during the previous year and in succeeding years.

In Oregon [189] the valuation shall be made on the American Experience Table with 4½ per cent. interest, or upon the Actuaries' Table with interest at 4 per cent., or according to the select and ultimate method on the basis that the rate of mortality during the first five years shall be calculated according to the following percentages of the rates shown by the American Experience Table, viz.: first insurance year, 50 per cent.; second, 65 per cent.; third, 75 per cent.; fourth, 85 per cent.; and fifth, 95 per cent.; or according to the preliminary term plan, or modified preliminary term plan.

25. *Contingency Reserve.*

In Minnesota [190] a company may accumulate and maintain in addition to the capital and surplus contributed by stockholders, and in addition to the net values of its policies, a contingency reserve not exceeding the following percentages of said net values: If the net values are less than $100,000, 20 per centum thereof, or the sum of $10,000, whichever is greater; when said net values are greater than $100,000, the percentage thereof measuring the contingency reserve shall decrease half of 1 per centum for each $100,000 of such net values up to $1,000,000; when said net

[189] Section 3722.

[190] Section 73.

values are greater than $1,000,000, but do not exceed $25,000,000, the contingency reserve shall not exceed 15 per centum thereof; when said net values are greater than $25,000,000, but do not exceed $50,000,000, the contingency reserve shall not exceed 12½ per centum thereof; when said net values are greater than $50,000,000, but do not exceed $100,000,000, the contingency reserve shall not exceed 10 per centum thereof; when greater than $100,000,000 but not in excess of $150,000,000, 7½ per centum; when greater than $150,000,000, the contingency reserve shall not exceed 5 per centum thereof. A company may maintain any accumulated contingency reserve although for the time being it may exceed the maximum percentage, but may not add to the contingency reserve when the addition will bring it beyond the maximum percentage. The commissioner may permit a company to accumulate and maintain a contingency reserve in excess of the limitation for a prescribed period, not exceeding one year under any one permission. This law does not apply to a company doing exclusively a non-participating business.

In Montana,[191] North Dakota,[192] Tennessee[193] and West Virginia,[194] a company may accumulate and maintain in addition to the amount of capital and surplus contributed by stockholders, or if there be no capital stock, in addition to the $100,000 of required surplus and in addition to the amount of the net values of its policies and the amount of temporary capital stock, if any, a contingency reserve not exceeding the following respective percentages of the amount of the net values: when the amount of the net values is less than $100,000, 20 per cent. thereof; or the sum of $10,000, whichever is the greater; when the amount of said net values is more than $100,000, the percentage thereof measuring the contingency reserve shall decrease ½ of 1 per centum for each $100,000 of the amount of said net values up to $1,000,000; ½ of 1 per cent. for each additional $1,000,000 up to $10,000,000; ½ of 1 per cent. for each additional $2,500,000 up to $20,000,000; ½ of 1 per cent. for each additional $5,000,000 up to $50,000,000; ½ of 1 per cent. for each additional $25,000,000 up to $75,000,000. If the amount of said net values equals or exceeds the last mentioned amount, the contingency reserve shall not exceed 5 per

[191] H. B. 212.
[192] S. B. 61.
[193] Chapter 454.
[194] Chapter 67.

centum thereof. The provisions for continuing to hold the contingency reserve accumulated and authorizing the commissioner to permit a company to accumulate and maintain a contingency reserve in excess of the limit, are the same as in Minnesota; and as in Minnesota, the law does not apply to a company doing exclusively a non-participating business.

In North Dakota the law does not exclude $100,000 of surplus of a mutual company in ascertaining whether the contingency reserve is within the limitation.

However much or little can be said in justification of other laws referred to in this article, these particular laws are utterly without justification.

26. *Premium Notices.*

In Louisiana[195] no company shall, within one year after default in payment of any premium or interest, declare forfeited or lapsed any policy except a term policy for one year or less, unless written or printed notice stating the amount of the premium or interest due, the place where it shall be paid, and the person to whom the same is payable, shall have been duly mailed to the insured or assignee at least fifteen and not more than forty-five days prior to the date when the same is payable. The notice shall also state that unless the premium or interest shall be paid by or before the day it falls due, the policy and all payments thereon will become forfeited and void except as to the right to a surrender value. No policy shall be forfeited until the expiration of thirty days after the mailing of such notice. The affidavit of anyone authorized to mail the notice that the same was duly mailed, shall be presumptive evidence that the notice was given. No action shall be maintained to recover under a forfeited policy unless the same is instituted within two years fom the date of default.

27. *Misrepresentation by Officers or Agents.*

In Colorado,[196] Delaware,[197] Illinois,[198] Louisiana,[199] Massachu-

[195] Act 68, Laws of 1906.
[196] Section 48.
[197] Approved March 5, 1907.
[198] Approved May 20, 1907.
[199] Act 107, Laws of 1906.

setts,[200] Minnesota,[201] Missouri,[202] Montana,[203] New Jersey,[204] North Dakota,[205] Tennessee [206] and West Virginia,[207] there is a prohibition against issuing or circulating or causing or permitting to be issued or circulated, any estimate, illustration, circular or statement, of any kind, misrepresenting the terms of any policy or the benefits or advantages promised thereby, or the dividends or share of surplus to be received thereon, or the use of any name or title of any policy, misrepresenting the true nature thereof.

28. *Discrimination.*

In Colorado,[208] Maine,[209] Massachusetts,[210] Minnesota,[211] Missouri,[212] Montana,[213] New Hampshire,[214] New Jersey,[215] North Dakota,[216] Oregon,[217] West Virginia [218] and Wyoming,[219] stringent laws have been passed regarding rebating and discrimination. Colorado and Maine include the receiver, as well as the giver, of the rebate in the prohibition. In West Virginia any policy sold in violation of the law is null and void. All except Massachusetts and Oregon prohibit "any paid employment or contract for services of any kind." Tennessee [220] and the above named states except Minnesota, New Jersey and Oregon, prohibit giving, selling or purchasing, or offering to give, sell or purchase, as inducement to insurance or in connection therewith any stocks, bonds or other securities of any insurance company or other corporation, association, or partnership, or any dividends or profits to accrue thereon, or anything of value whatsoever, not specified in the policy.

In Louisiana [221] the contract is to have the benefit of all legislative or legal enactments of the state in which the company is domiciled.

[200] Section 74.
[201] Section 65.
[202] H. B. 86.
[203] H. B. 132.
[204] Chapter 72.
[205] S. B. 176.
[206] Chapter 455.
[207] Chapter 77.
[208] Sections 49, 50 and 51.
[209] Chapter 121.
[210] Section 69.
[211] Section 61.
[212] H. B. 85.
[213] H. B. 146.
[214] Approved April 3, 1907.
[215] Chapter 74.
[216] S. B. 179.
[217] Section 3722.
[218] Chapter 77.
[219] Chapter 38.
[220] Chapter 456.
[221] Act 96, Laws of 1906.

29. *Agents' Commissions.*

New Jersey[222] prohibits companies issuing policies upon both the participating and non-participating plan, from making any distinction in the rate of commission or in compensation paid to an agent based upon the participating or non-participating character of any policy issued through said agent.

30. *Misrepresentation by Insured.*

In Arizona[223] and Kansas,[224] no representation made in obtaining a policy of insurance shall be deemed material or render the policy void, unless the matter misrepresented shall have actually contributed to the contingency or event on which the policy is to become due and payable. No defense based upon misrepresentation shall be valid unless the defendant shall at or before the trial deposit in court for the benefit of the plaintiff the premiums received on such policy.

31. *Political Contributions.*

In Colorado,[225] Delaware,[226] Massachusetts,[227] Minnesota,[228] New Hampshire,[229] New Jersey,[230] North Carolina,[231] North Dakota,[232] South Dakota,[233] Tennessee[234] and West Virginia,[235] political contributions are prohibited.

32. *Vouchers.*

In Tennessee[236] no company shall make any payment out of its funds amounting to $100.00 or more, unless the same shall be evidenced by a voucher properly signed and correctly describing the consideration for the payment. If the payment be for both

[222] Chapter 74.
[223] Approved March 18, 1907.
[224] Chapter 226.
[225] Section 47.
[226] Approved March 5, 1907.
[227] Section 22.
[228] Section 64.
[229] Approved April 3, 1907.
[230] Chapter 34.
[231] Approved February 5, 1907.
[232] S. B. 52.
[233] Chapter 142.
[234] Chapter 443.
[235] Chapter 52.
[236] Chapter 439.

services and other purposes, the voucher shall set forth an itemized statement of the specific services rendered, and of all other expenditures. If the expenditure be in connection with any matter pending before any legislative or public body, or before any department or officer of any state or government, the voucher shall correctly describe in addition the nature of the matter and the interest of the company therein. When such voucher cannot be obtained, the expenditure shall be evidenced by an affidavit describing the character and object of the expenditure and stating the reason for not obtaining the voucher.

33. *Salaries.*

In Tennessee [237] no company may pay any salary, compensation or emolument to any officer, trustee or director thereof or any salary, compensation or emolument amounting in any one year to more than five thousand dollars, to any one person, firm or corporation, unless such payment be first authorized by a vote of the Board of Directors. Any agreement with officers, trustees or salaried employees providing for any salary, compensation or emolument extending beyond a period of twelve months is prohibited. Any officer, director, or trustee who is paid a salary for his services of more than one hundred dollars per month, shall not receive any other compensation or emolument. The above does not apply to General Agents. All pensions are prohibited.

In Missouri [238] and North Dakota,[239] no company which pays as a salary or as compensation for services more than fifty thousand dollars per annum to any one person, shall be licensed to transact business.

34. *Misuse of Funds.*

In Missouri [240] and Oregon,[241] any person connected with any insurance company using or employing or permitting others to use or employ any of the funds or securities of the company for private profit or gain, shall be subject to imprisonment.

[237] Chapter 440.
[238] S. B. 118.
[239] S. B. 173.
[240] S. B. 95.
[241] Section 3722.

35. *Removal of Suits to Federal Courts.*

The laws of Alabama [242] and Missouri,[243] now provide that the removal of a suit from the State to the Federal courts will result in the cancellation of the company's license. In Missouri the same is true if a company institutes a suit in the Federal courts. In Minnesota [244] the company must agree not to remove a case from the State to the Federal courts. In Colorado [245] a company cannot remove a case without the consent of the other party.

36. *Annual Reports.*

Section 103 of the New York Insurance Law has been enacted in Colorado,[246] Delaware,[247] Louisiana,[248] Minnesota,[249] New Jersey [250] and North Dakota,[251] with the following modifications:

Colorado does not require a statement of any certificate issued by the Insurance Commissioner extending the time for the disposition of real estate.

In connection with the third item Minnesota and New Jersey do not call for a statement of the substitutions of securities held for loans, other than substitutions made during the current year, and New Jersey does not call for the names of the borrowers. In Item 11 there is no reference to the Select and Ultimate method of valuation.

In New Jersey Item 12 of the New York Law is omitted.

In Colorado a company issuing or "having issued" both participating and non-participating policies shall make a separate statement of profits and losses, etc. The words "having issued" may, however, be construed as referring to the year for which the statement is made.

Section 101 of the Massachusetts law relating to Annual Statements, has been amended, but the amendments cannot well be summarized here.

[242] Act 245.
[243] H. B. 36.
[244] Section 196.
[245] Section 34.
[246] Section 53.
[247] Approved April 9, 1907.
[248] Act 87, Laws of 1906.
[249] Section 95.
[250] Chapter 73.
[251] S. B. 177.

In Massachusetts the Gain and Loss Exhibit need not be filed before May 1st, and in New Jersey before June 30th.

37. *Power of Attorney.*

In Minnesota,[252] Montana,[253] and West Virginia,[254] the commissioner is to be appointed attorney for service of process.

38. *Taxes.*

In Alabama[255] the license to do business must be procured from the Secretary of State, the fee for which is $10.00 per annum. The tax has been fixed at two per cent. of the premiums collected. In Delaware[256] a tax of two per cent. of the premiums is imposed.

In Arkansas[257] companies must pay a franchise tax ranging from $10.00 to $100.00, based upon their authorized capital stock.

In West Virginia[258] the tax appears to have been increased to two per cent. of the premiums.

39. *Insurance of Minors.*

In New Jersey,[259] in respect of insurance heretofore or hereafter issued upon the life of any person not of the full age of twenty-one years, but of the age of fifteen years or upwards, for the benefit of such minor, or for the benefit of the father, mother, husband, wife, child, brother or sister of such minor, the assured shall not by reason only of such minority be deemed incompetent to contract for such insurance, or for the surrender of such insurance, or to give a valid discharge for any benefit accruing, or for money payable under the contract.

The person liable for the support of a minor may secure insurance for a limited amount on the life of the minor.

[252] Section 197.
[253] H. B. 226.
[254] Senate Bill 1, Extraordinary session.
[255] Acts 245 and 259.
[256] Approved April 9, 1907.
[257] Act 443.
[258] Chapter 77.
[259] Chapter 72.

40. *Concluding Remarks.*

In Connecticut the only laws passed regarding life insurance related to rebating and companies' advertisements.

In California[260] companies must furnish a bond for $20,000, the conditions of such bond to be as follows: 1. That the company and its agents will pay all state, county and municipal property and license taxes in the manner and at the time prescribed by law; 2. That the company will conform to all the provisions of the revenue and other laws made to govern them: and 3. That the company will promptly pay all fees, assessments, taxes, penalties and fines that may be laid upon or against it.

From the above summary it will be seen that the much-heralded conference of Governors, Attorneys-General and Insurance Commissioners, which was held ostensibly to secure uniform legislation for the benefit of policyholders, met with failure. Instead of uniformity there is confusion, and instead of legislation which will benefit policyholders, there is a mass of laws which are not understood by their sponsors, and an arbitrary enforcement of which must result in great harm.

It has not been deemed necessary to refer to the legislation in Texas which resulted in the enforced withdrawal of practically all the established companies doing business in that State. Neither have I referred to the legislation in Wisconsin. In the latter State there are new laws prohibiting misrepresentation by companies or agents; requiring companies to furnish a copy of the application; requiring them to report to the Commissioner of Insurance moneys disbursed in opposing and promoting legislation; prohibiting the writing of participating and non-participating business by the same company; requiring companies to report to the Commissioner of Insurance contributions made for political purposes; requiring foreign stock life insurance companies to determine and report the respective rights of policyholders and stockholders; prohibiting discrimination; requiring separate gain and loss exhibits for participating and non-participating business; the publicity requirements of the New York Law, with much additional information; prohibiting corporations from acting as agents; providing for an annual license fee of three hundred dol-

[260] Section 623, Code.

lars, and repealing Chapter 455 of the laws of 1905; and a law providing that where no other penalty is prescribed, any violation of the insurance laws of the state shall be punished by a fine of not more than five thousand dollars in case of a company, and by a fine of not more than one thousand dollars and by imprisonment in the county jail for not exceeding one year, or by both fine and imprisonment in case of a person. In addition to these laws there are others regarding annual apportionment of dividends and the apportionment of deferred dividend surplus, defining certain words and phrases, and limiting expenses and premium loadings, in the preparation of which the framers displayed such crass ignorance that they themselves are unable to place any consistent interpretation upon them. In order to secure the passage of these laws it was necessary to employ the arts of the demagogue, to prostitute political power, and to engage in specious sophistry. Such laws can only become a lasting reproach to the men primarily responsible for them.

It should be borne in mind that this article is intended to cover only regular forms of insurance. While care has been exercised to make it complete and correct, errors doubtless exist, and the original laws should be consulted in every case.

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Valuation and Distribution.

Some Thoughts aroused by recent Discussion and Legislation.

by henry moir.

Part I. Valuation.

Net Premium Valuation.[1]

Until last year net-premium valuation was strongly entrenched in the legislation of the various States of the United States; but since April, 1906, the laws of several States have recognized its weakness and danger, and have authorized other valuation methods.

The net-premium method became popular about 50 years ago as a result of reaction from the abuse of the gross-premium method. Originally the term "net-premium" seems to have indicated nothing more than that in the valuation formula an allowance had been made for future expenses and future surplus, in contradistinction to the gross-premium method which made no such allowances. While Elizur Wright was a staunch advocate of this method, he took a very sane view of the subject, which is well expressed by him thus: "Regarding it only as the prophecy of common sense that future expenses and dividends should be about the same as the past, our insurance laws since 1854 have

[1] I wish to call particular attention to the following important papers from which I have quoted but little, because anyone interested in this subject should read each one through, and that carefully:—

H. W. Manly, on "*A Comparison of the Values of Policies, etc.*" (*J. I. A.*, Vol. xiv, p. 249).

S. G. Warner, on "*Some Notes on the Net Premium Method of Valuation, as affected by recent tendencies and developments.*" (*J. I. A.*, Vol. xxxvii, p. 57).

W. S. Nichols, on "*The Limitations of the System of Net Valuations.*" (*Transactions Second International Congress*, p. 161).

required a net valuation, etc." The distribution of the expenses has changed completely since he wrote this, and it is therefore natural that this change should be made effectual in valuation also.

Authoritative definitions as to what constitutes a net-premium method of valuation are very hard to find. The method has been described as a valuation by a true table of mortality at a true rate of interest, the net-premium used in the valuation being based on that true table. If we add the stipulation that the office premium be computed from the same net-premium with provision for a true rate of future expense, this form of net-premium valuation would be unassailable except in minor details. Another definition, and the one which has received the greatest favor in the minds of Actuaries, was propounded by the late Mr. James Meikle, who described the method as one by which the value of the sum insured should be exactly equal to the value of the net-premiums at the date of issue of the policy; in other words, that the net value at the date of issue should be zero. (*J. I. A.*, XI, p. 241). From personal knowledge of Mr. Meikle, I do not believe that he was an advocate of this method of valuation for practical purposes, although he advanced the definition on theoretical grounds.

This endeavor to bring the net-premium method down to a scientific and mathematical basis has resulted in serious blunder and misunderstanding. It has led away from the true net-premium system and has encouraged an arbitrary standard which ignores the fundamental principles of valuation, the greatest of which is that *the thing to be valued must govern the method of valuation.* It would be considered the height of absurdity to value an Endowment Assurance Policy as a Whole Life Policy, or to value a $10,000 policy as if it were for $9,000 only. Yet it is not considered out of place to ignore entirely the terms of the contract so far as the premium payable is concerned. It is conceivable that a company might issue three non-participating whole life policies in the same year at age 35, one for $10,000 at a premium of $200, another for $9,500 also at a premium of $200, and a third for $10,000 at a premium of $210.50; and I cannot suppose that any Actuary would hold that the *true values* of the first two contracts differ only because of the difference in their sums assured. It would just be about as logical to go to the opposite extreme and say that because the premiums are the same, therefore the values are the same. Under the so-called net-premium

method the first two policies would always have different values from one another, while the first and third would have identical values, although either the expense provisions or the true net-premiums under the policies are different.

The net-premium method assumes that expenses will be evenly distributed throughout the premium-paying period and makes no allowance whatever after any policy has become paid up. Here we have two important objections to the method. It does not take account of the initial cost of placing the business on the books; it eliminates from observation the actual premium received. An office valuing an ordinary policy by the net-premium method cannot hope to meet first year's expenses and mortality out of the first year's office premium, and have sufficient remaining with which to provide the first year's net-premium reserve on the basis of Mr. Meikle's definition. As a result, long-established companies draw upon the surplus contributed by older members to set up the reserves on the policies of new members, while young companies must resort to some modification of the contract as in preliminary term valuation..

When the premium-paying period is less than the term of the assurance, the so-called net-premium method fails to make any provision for expenses after the premium-paying period is completed.

This will appear to be of little consequence to American Actuaries; but in Britain the common practice is to give the same share of surplus to a paid-up Limited Payment Policy as accrues to a Whole Life Policy of the same age. The question is there a very important one. This objection is usually met by the reserve value being computed at a rate of interest sufficiently low to allow a good margin of surplus interest for the future of the business. Sometimes the reserve on Limited Payment Policies is for this reason computed at a lower rate of interest than the reserve on Whole Life Policies, and very properly so. But this condition is scarcely logical; nor is it net-premium valuation, although it is an excellent way of making a practical approximation to a true valuation.

Recently a good many young companies have been started with a capital stock; and, in addition to the paid-up capital, the promoters have very frequently arranged to have a large surplus paid in before commencing operations. The stock is usually sold at a premium, and the premium goes to the credit of the company as surplus.

This surplus is for the purpose of meeting preliminary expenses over and above the usual cost of new business which is generally met by use of the preliminary term contract form.

The net-premium method was applied in Massachusetts, the District of Columbia, and in Canada by ignoring all preliminary term devices. The principle was followed that, *irrespective of the terms of the contract and of the basis of the premium calculations, a level gross premium necessarily implied a level net premium.* A policy contract written clearly as a term insurance for one year, and with a premium in subsequent years based upon the rate for the next higher age, with proper renewal expense loadings for that age, was treated in those States not as a one-year term policy, followed by a life policy taken at the next age, but as a life policy taken from the original date. In all other States of the Union, however, such a policy was valued according to the terms of the contract, and two decisions were rendered by the Supreme Court of Vermont (in 1901 and 1904 respectively) upholding the preliminary term method of valuation as applied to such policies when so expressed. The second decision was rendered after a change had been made in the wording of the Vermont law so as to follow literally the Massachusetts Statute.

This condition of affairs has been completely changed during the last year. In New York State the Select and Ultimate method of valuation is permitted by law, and is prescribed as the method for measuring the limitation of first year's expense. It has already been fully discussed and is the subject of much consideration. In other States the recommendations of the Committee of Fifteen have been adopted with more or less modification. These recommendations imply that life policies and limited payment policies with premiums limited to 20 payments or more can be valued by the preliminary term method, if the policies so provide, but that endowment policies and policies with premiums limited to less than 20 years, can only be valued by the "modified" preliminary term method. The allowance for initial expense under the modified method is the same amount as would be available under a Whole Life Policy.

Canada Valuation Law.

In Canada a third method has been proposed, the discussion

of which is already scheduled as part of our business (*Transactions*, Vol. X, p. 30). In the briefest possible way I may describe the method as being for the first year equivalent to modified preliminary-term. The additional expense allowance for all higher priced policies is the same as for ordinary life policies. Under ordinary life policies no reserve is required at the end of the first policy year; under limited payment and endowment policies the reserve at the end of the first year consists of the investment element in these policies in excess of the same element under ordinary life policies. One main feature of the method is that this initial expense allowance has to be made good out of the four subsequent premiums, so that at the end of five years the reserve is the same as it would be under the full net-premium, or the Select and Ultimate methods.

New Jersey Valuation Law.

In New Jersey yet another method has become a law, of which the following is a brief résumé:

> All valuations shall be made upon a net premium basis, or such modification thereof as hereinafter expressly provided.
>
> The modification applies to policies issued after January 1st, 1907. If the net premium for the policy to be valued is equal to, or greater than, that of an Ordinary Whole Life Policy, the strict net premium reserve may be reduced as follows:—
>
> 1st year, by the excess of the reserve of an Ordinary Life policy over a one-year-term policy;
> 2nd year, five-sixths of the aforesaid excess;
> 3rd year, four-sixths of the aforesaid excess,
> 4th year, three-sixths of the aforesaid excess;
> 5th year, two-sixths of the aforesaid excess;
> 6th year, one-sixth of the aforesaid excess;
>
> After the sixth year, the regular reserves will apply.
>
> In the case of policies with net premiums *less* than Ordinary Life Policies, reduce the reserves the first year by the difference between (1) the reserves for the policy applied for, and (2) a one-year-term reserve, and adjust this to the regular reserve by equal parts over the subsequent four years, thereby reaching the regular reserve in five years.

In the case of large companies with assets exceeding $25,000,000 the modification in the reserve is subject to further restrictions; when the reserves reach $50,000,000 the modified plan cannot be applied to any policies thereafter issued.

The modification is made only "upon the request of any Company," and it is combined with a restriction of the expenses of such Company. On adopting the modified reserve plan the Company must agree that the first year's expenses and modified reserves taken together shall not exceed the first year's premiums; and, if the Commissioner shall consider that a Company has failed to comply with this restriction, he shall withdraw the privilege of using the modified reserve plan.

The New Jersey method is subject to two particular objections:—*The first* is that it purports to be a concession to young and weak companies; the larger, stronger and older companies cannot and must not use it. The law seems to uphold the theory that a large company is justified in borrowing from the surplus of old members the expense which it incurs in writing new business; also that a vigorous young company is necessarily weak and needs to have concessions made to it in order to make suitable reserves for its policies. The system of "borrowing" from older policyholders the money with which to pay the expense of introducing new members is a questionable practice. It looks like the misuse of trust funds, and the laws of New Jersey require that trust funds should not be misapplied. *The second* objection is that the larger and stronger companies are under no restriction as to the expenses they incur in writing new business, while the smaller growing companies are under such restriction. This is grossly unfair and illogical, because it means that if the large companies pay high commissions for their first year's business, the smaller companies will be driven out of business altogether, since they cannot meet the competition for business and comply with the requirements of the modified reserve law.

Objects of Valuation Modifications.

All of these methods of modifying the reserves have the same ostensible object:—to make a moderate allowance for first year's expenses, and afford relief to young companies, or to companies

doing a rapidly expanding new business. It is worthy of note that in but few instances has the thought of equity been introduced; the reasons for changes have been those of expediency and safety. Yet in comparing the net-premium and the gross premium methods of valuation, there are two conditions which should not be forgotten; they are both objectionable:—

1st. In the gross premium method, the anticipation of future loadings giving negative values or incorrectly small reserves.

2nd. In the net premium method, the taking of surplus from policyholders to whom it now belongs in order to set up reserves which the Company cannot under present-day conditions have accumulated from the premiums themselves, to hold this surplus (whether under the title of reserves or otherwise) for many years, and to pay it to future policyholders who may or may not have contributed to its accumulation.

In the first case, the true condition of a company is lost in the calculation; the future is discounted in favor of the present under any such gross premium method of valuation, and the equities of policyholders are disturbed, the present policyholders getting a much larger share of surplus than they are justly entitled to. The future of such valuation usually spells disaster and ruin. In the second case, exactly the opposite takes place; the existing surplus is taken from the present policyholders and carried to reserve. One most important question therefore is that of the equities between different classes of policyholders.

Valuation should, properly speaking, be conducted in two different ways according to the object in view; *first,* to test the solvency of the company; and *second,* to ascertain the surplus for distribution. In this country the two methods have been confused. To test the solvency of a company involves a different problem from that of valuation for distribution, and to this subject I shall revert later.

The more important valuation is that of a "going concern," with reference to the surplus distribution. If the full reserve be carried for the first year's insurance under annual dividend policies, it is not possible to distribute to older policyholders all the surplus they have earned; a certain part must be withheld to supply the deficiency in reserve on new entrants in addition to special reserves for possible mortality fluctuation or investment losses.

True Net Premium Valuation.

It has always seemed to me that in premium computations (see *Transactions Faculty of Actuaries,* Vol. II, p. 208, *et seq.*) the net premiums should be computed from a true table of mortality at the rates of interest and expense which a company can reasonably expect to experience; also that a true net-premium system of valuation must of necessity follow the computation of the premiums. Otherwise the net premium used in valuation is merely an arbitrary factor which has nothing whatever to do with the facts on which the contract between the policyholder and the company had been based. It is usually contended that as the net value at 3½ per cent. is greater than the net value at 4 per cent., therefore it is better for a company to value at the lower rate rather than to continue at 4 per cent. with a falling rate of interest. Of course it is better to do so! But this does not seem to make it net-premium valuation at the lower rate, but only a strengthening of the reserves beyond those required on the net 4 per cent basis to an extent which is not altogether apparent in the mere reduction in the rate of interest.

Let us consider briefly Sprague's well known formula for non-participating office premiums, the figures used being taken from the $O^{[M]}$, Select Table at 4 per cent.:—

$$P_x = 1.075\left(\pi_{[x]} + \frac{.01}{a_{[x]}} + .00125\right)$$

$$P_{30} = 1.075(.01567 + .00054 + .00125)$$
$$= 1.075(.01746)$$
$$= .01877\ ;\ \text{or } \$18.77 \text{ per } \$1000.$$

If the same calculation were made at 3½ per cent. interest the resulting premium rate would be $19.85 per $1,000; and if made according to the mortality experience on non-participating policies, the $O^{[NM]}$ Table 3½ per cent. interest, the resulting rate would be $21.12 per $1,000.

In this formula the first factor 1.075 gives a *percentage* loading to the first and renewal premiums of 7½ per cent. to cover renewal commissions, collection charges, etc. The last factor .00125 furnishes a constant loading each year based on the sum assured for executive and other fixed charges, while the addition of $\frac{.01}{a_{[x]}}$

gives an extra loading for the special expenditures of the first year, estimated at $10. per $1,000 of face value.

A true valuation following the above formula would require the use of a valuation premium of $16.21 per $1,000. In symbols, the formula would be

$$ {}_nV'_{[x]} = A_{[x]+n} - \left(\pi_{[x]} + \frac{.01}{a_{[x]}}\right) a_{[x]+n} $$

which, in the example given,

$$ = A_{[30]+n} - (.01567 + .00054) a_{[30]+n} $$
$$ {}_nV_{[30]} = A_{[30]+n} - .01621 a_{[30]+n} $$

The provision for initial expenses of $10. per $1,000 would be thereby spread over the duration of the policy and treated as an addition to the net premium. If this were not done and a mutual company were started to write business on the basis of this formula, the company would be technically insolvent, although the conditions might coincide exactly with those provided for in the original calculations.

If in a few years the rate of interest were to fall and the company should desire to strengthen its valuation to a 3½ per cent. basis, the usual process is to use a reduced rate of interest in all three factors (1) the sum assured, (2) the premium, and (3) the annuity value. This is the only process which meets the requirements of the definition that the valuation formula should give zero at date of issue. But is this net-premium valuation under the conditions laid down? The net premium was first computed at 4 per cent., namely $15.67 per $1,000, and now it is proposed to value a net premium entirely different, namely, $16.71 per $1,000, the net 3½ per cent. rate. This increased net premium is actually greater than the 4 per cent. premium plus the initial expense provision. Such a change assumes that the net premium has been increased. But no such change has actually taken place; the contract between the company and the policyholder remains as it was. It would be more clearly net-premium valuation if changes were made only in the reversion value and the annuity value, leaving the net premium on the original basis, 4 per cent., with the expense allowance which the original premium formula provided.

If the foregoing proposition holds, presumably the opposite

should also hold, namely, that if a company had based its premium rates on 3 per cent. interest, *with sufficient loadings to meet the necessary expenses,* and afterwards found that by Legislative enactment or otherwise the business could properly be valued at 4 per cent., then the net premium to be valued should not be reduced below the old 3 per cent. rate, but should remain at the fixed rate on which the contract was based, the valuation factors, namely, the reversion and the annuity, being the only factors changed to the increased rate of 4 per cent. As a test of solvency this condition is more than a mere theory.

The common method of treating this net-valuation question by which all three factors are changed (and it is the only method which permits of the reduction in the valuation rate of interest and yet shows a value of zero at the date of issue of the policy) is reduced to an absurdity if, as sometimes happens, the net premium should exceed the gross premium.

Mr. S. C. Warner says (*J. I. A.*, XXXVII, p. 71), that when a net-premium valuation is made "a lowering of the valuation rate means of course an automatic increase in the net-premium valued," and he adds that any company adopting this course, even although the loading all but disappear "might state with perfect truth that a net-premium valuation was made." With this conclusion I cannot agree. I think that this is not in any true sense net-premium valuation. Such a statement would deceive the public, deceive even the Directors and Officers of the companies interested, and would be scientifically untrue. Indeed Mr. Warner himself seems to agree with me in this regard, notwithstanding the statement I have quoted above, when he afterwards says that the method would "throw a cloud over the results." But the statement I criticize evidently arises from the acceptance of the definition of net-premium valuation.

Perhaps I should point out also that the preliminary term method of valuation as commonly applied in the United States, fulfils this definition of net-premium valuation. The policy is usually written as a double contract, first, a one-year-term insurance, and afterwards a Life, a 19 Payment Life, or such other form as may be desired. The net valuation of the Term Policy at the date of issue is zero and the net valuation of the Life Policy taken the following year is in like manner zero, thus fulfilling the conditions of the so-called definition.

Misconception of British Valuations.

British companies value at very low rates of interest, and in America it is often thought that this course is adopted because of the lower interest rates prevalent in Britain and the natural conservatism of the Managers of the Companies in discounting the future. This is a common misapprehension, because there is one much more important reason than either of these which makes it *necessary, not optional,* for British companies to have a large margin between the rates of interest earned and the valuation rates.

The Royal Commission in Canada seems to have been misled by this condition when they say in their Report: "In Great Britain, no rate is fixed by law;[1] but the companies value at from 2½ to 3½ per cent."

I think the true condition can best be explained if I first give a mathematical illustration of a common situation. Consider the case of participating policies under which the premium has been computed to provide a reversionary addition of $10 per $1,000 each year the policy is in force. In this particular case, so as to simplify the investigation, let us assume that special first year's expenses do not exist, but that they are the same as renewal expenses, and that the premium at age 30 is therefore:—

$$\begin{aligned} P_{30} &= 1.10\left(\pi_{[30]} + .00125 + .01\frac{R_{[30]}}{N_{[30]}}\right) \\ &= 1.10(.01567 + .00125 + .00423) \\ &= .02327\ ;\ \text{or } \$23.27 \text{ per } \$1000. \end{aligned}$$

Again, suppose that the expense conditions are exactly fulfilled. The surplus at the end of the first year would be the amount provided in the formula, namely, $.01\frac{R_{[x]}}{N_{[x]}}$, improved at interest and mortality, that is,

$$\begin{aligned} \text{Cash Surplus} &= .01\frac{R_{[x]}}{N_{[x]}} \times \frac{D_{[30]}}{D_{[30]+1}} \\ &= .00441 \\ &= \$4.41 \text{ per } \$1000. \end{aligned}$$

[1] It is not correct to say "no rate is fixed by law." 4% is the rate fixed by an 1872 Amendment of the Life Assurance Company's Acts of 1870 and 1871. Moreover some British companies still use 4%.

Now if this sum were applied to purchase a reversionary addition to the policy, it would, on the theory of Select Tables, be divided by $A_{[30]+1}$. The result would be a reversionary addition of \$14.76 per \$1,000, all these calculations being made at O[M] 4 %.

Apparently then the surplus at the end of the year would buy a reversionary addition of \$14.76, or 1.476 times the amount which the premium calculations were designed to furnish. The cause of this is easily seen; the cash surplus each year would remain very nearly the same, increasing only as the ratio $\frac{D_{[x]+n-1}}{D_{[x]+n}}$ increases, and as the surplus is divided by $A_{[x]+n}$, which increases much more rapidly with the increase in the value of n, then the reversionary additions would diminish each year. They would therefore be much more than 1 per cent. in the early years, and much less in the later years, averaging 1 per cent in all. The condition which the formula was designed to produce would not be fulfilled. But such conditions *can* be fulfilled by having the valuation follow the premium computations, and valuing the sum assured as an increasing quantity, with the net premium as

$$\pi_{[x]} + \frac{.01 R_{[x]}}{N_{[x]}}$$

This would be a true net-premium valuation, and the formula, following the formula for calculation of the premium, would be

$${}_nV_x = A_{[x]+n} + .01\frac{R_{[x]+n}}{D_{[x]+n}} - \left(\pi_{[x]} + .01\frac{R_{[x]}}{N_{[x]}}\right) a_{[x]+n}$$

Additions which have been declared immediately become liabilities, and would be valued separately. They are ignored in above formula because they can be surrendered for cash and treated as independent factors.

This method of valuation is a troublesome process, and British actuaries effect a compromise on the question by reducing the rate of interest they use for valuation purposes so as to realize nearly the same practical effect. It is easily seen how this comes about. By using a lower rate of interest, the practical effect is to increase both sides of the valuation equation and also to increase the net value. The surplus is diminished in the early years, and increased in the later years from additional surplus interest. It has been found by many tests that a reduction in the interest rate of about

½ per cent. below that realized is sufficient to equalize and maintain a uniform reversionary addition of 1 per cent. annually to the sum assured. If a larger addition than 1 per cent is to be maintained, then of course the interest margin must be greater.

If 4 per cent. is realized by any company, then in order to provide a proper margin for future reversionary additions, it may use 3½ per cent. interest in its valuation. But this surely cannot be called net-premium valuation at 3½ per cent., when it is conducted in this way merely to approximate to a net valuation at 4 per cent. interest with an allowance for the future additions which have been provided in the premiums and promised.

As British companies generally earn less than 4 per cent. interest net, and as they desire to maintain higher bonus rates than 1 per cent. per annum, 3 per cent. valuations are necessary to meet the conditions. Many American Actuaries are not aware that British companies use strict interest rates not only because of the lower interest earnings from good investments in Britain; but also, and probably to a greater extent, because of their method of surplus distribution. They do not think the low rate of interest a requirement for safety or solvency; but they use it merely as a means of approximating to a more exact method of valuation for surplus distribution with an allowance for future bonus additions.

Compound Reversionary Additions.

Perhaps I should point out further that when compound reversionary additions are promised, it is necessary to make provision for a still greater increase in the surplus in the later years when the compounding feature becomes pronounced. Accordingly, while a margin of surplus interest of from ½ per cent. to ¾ per cent. is necessary to maintain the usual rates of simple reversionary addition, to maintain the rates of compound reversionary addition commonly paid necessitates a margin of surplus interest of about 1 per cent.

Several companies in Great Britain use as low a rate of interest as 2½ per cent. for their valuations, but these companies are nearly all advocates of the increasing reversionary addition system, and their average interest rates earned only approximate to 3¾ per cent., so that they have not much more than the proper margin

for the maintenance of the increasing reversionary additions which they desire, and have virtually promised, to pay. If they were to make a true net-premium valuation at the rate of interest they earn, 3¾ per cent., with suitable provision for the future surplus distribution on the increasing reversionary plan, the result would not differ so very widely from the results which are published and *misunderstood* as 2½ per cent. net-premium valuation.

Please remember that I am not condemning the system; on the contrary, the attitude of those companies is to be highly commended in the care and foresight which they exercise in making such excellent provision for future results. But I dislike exceedingly to see such an approximate system misunderstood and called net-premium valuation at 2½ per cent., especially when the misunderstanding includes such men as the Royal Commission in Canada, charged with recommending new legislation.

It looks as if the plan of reducing the rate of interest in all functions must have resulted from a desire on the part of companies to appear a little better than they really are, so that they might say that they valued at a very low rate of interest, and claim unusual conservatism. If the net premium were left at its original figure and a modification were made only in the reversion and in the annuity values (see *J. I. A.*, Vol. XXXVI, p. 25), we should have a much smaller number of companies valuing their business at the low rates. A true valuation, with a specific provision for further surplus would more frequently be made.

There is one difference between the practice of American and British companies to which I wish to call attention. It is the attitude towards a change of premium rates. Most American companies have no hesitation in changing their premium rates from time to time as conditions change. Thus when the basis of valuation is changed most companies at the same time effect a change in their premiums. The surplus is usually ascertained by the contribution plan, and a change in the basis of valuation is frequently applied only to the future business as it is written. The past is not disturbed, but the business is allowed to work itself out on the old basis. In Britain the system of distribution is often incorporated in the company's charter; and a change in valuation basis is generally applied to all the business in force. A change in the premium rates, where

the system of distribution remains fixed by some simple rule, may, indeed generally will, disturb the equities of policyholders, the same surplus rule being applied to those paying different premiums. Accordingly, it is sometimes found desirable to change the charter provisions before changing the premium rates.

The two foregoing illustrations have been taken from common British practices, because this subject of net valuation has been more fully discussed there than here. I have therefore dealt first with those two points, which, so far as I have observed, British writers have mostly ignored. But there are several questions peculiar to American conditions which are overlooked in the rigid adherence to net-premium valuation, and these also should not be forgotten. Briefly they relate to

1. Companies charging inadequate premiums;
2. Agency company plans;
3. Board schemes; and
4. General agency contracts.

I cannot enter at proper length into these subjects, partly because I have not had time to get exact information regarding individual cases and partly because this paper is going to be quite long enough without such details. Perhaps, however, before the question comes up for discussion next Spring other members of the Society may be able to quote authoritatively instances within their knowledge of the conditions to which I direct only general attention.

Inadequate Premiums.

Amongst companies organized recently I have observed one or two with premiums which are exceedingly small, close to the net 3 per cent. American rates. There is not a sufficient expense provision for the probable future outlays, especially when the agency contracts of the company, the renewal commissions, taxes, salaries, and general administration expenses are computed. Yet such a company would under the system of net-premium valuation be permitted to value at 3½ per cent. or 3 per cent. and publish the fact, although only a very slight provision for future expense would be made. Indeed sometimes even participating contracts are issued by companies whose premiums do not furnish

loadings sufficient to pay the regular expenses. Yet I do not say that such a company is necessarily *insolvent,* it may be quite solvent, able to continue business, to meet all its obligations or to reinsure on favorable terms if reinsurance be thought desirable. The present system of valuation does not, however, apply properly to such companies, nor does it make any discrimination (as a proper system of valuation should) between the real strength and real weakness of many companies.

Agency Company Plans.

The mathematical interpretation of net-premium valuation as a test of solvency has been directly responsible for another outgrowth of American Life Insurance, namely, the agency company. Several life insurance companies are held in the clutches of these noxious parasites. I cannot speak very definitely as to the extent to which the agency agreements have forestalled the future loadings on the premium, but I have a very distinct impression that under one such plan at least the entire expenses of the business are to be met by the agency company, *and the entire loadings on the premiums are payable to the agency company, although policies promise participation in surplus.* Accordingly, participating premiums are charged, with loadings added for surplus requirements, and yet all of these loadings are paid to an agency company. Not only so, but the insurance company is under contract with the agency company to pay the latter all the loadings on all business for 20 or 30 years to come. The amounts paid by innocent policyholders in expectation of receiving dividends are paid away in expenses and in profits to stockholders of the agency company. Perhaps I should explain that the agency company compensates the writing of business, pays commissions, officers' salaries, printing, stationery, advertising, and all other expenses. There is something wrong with the system of valuation, whether it be called net-premium valuation or otherwise, which permits a plan of this kind to be in operation and does not take into consideration as a liability of the insurance company the contract between that company and the agency company.

There is an interesting phase of this situation which I think the designers of this scheme may have overlooked—the human element. The agency company may prove a great success and earn

colossal dividends. If so will the company be permitted to distribute these dividends to stockholders while the policyholders get a mere pittance? On the other hand, the agency company may be a failure in which case the insurance company may be enabled to regain control of its own affairs through some process of reorganization. There seems a possibility that agency companies may find the old saying reversed "Heads I lose, and tails I don't win."

Doubtless the claim will be made that a life insurance company is perfectly solvent under such conditions. Certainly! But the promises made to policyholders in the issue of participating contracts are not being fulfilled, and adequate steps are not being taken to see that there is at least a chance that such promises may be made good.

Board Plans.

During recent years a good many companies have revived and have pushed vigorously the Board plan of obtaining applications for life insurance. This plan consists in approaching the more prominent men in any community and asking them to become associated with the company in an advisory capacity; in consideration of their assistance in procuring applications and in furnishing information, a fund will be formed from the business on all the premiums written in the district. This fund will be distributed annually amongst the members of the Board, who thus in a sense are agents of the company. If this business were legitimately conducted, and if these men really became interested in the company and gave their services freely, there would seem to be little theoretical objection to such a plan. But in most instances the system has been used in an objectionable manner. Men with no real influence, and with no intention of rendering help, have been induced to take policies on the basis of estimated future income which they may derive from their membership of the Board. The most usual compensation fixed for the members of such a Board is $1.00 per $1,000 of sums insured remaining in force.

Some of the excesses of this plan may properly be mentioned, as the issue of contracts covering the entire business which a company may write, combined with the promise that such contracts will be limited to 100 in all; when this limit is reached, a second

series is commenced, or even a third or fourth series of contracts, each series calling for $1.00 per $1,000 on the business of the company. It is scarcely necessary to point out to a body of Actuaries what the effect of an accumulating charge like this must be on the future business. It is made under a strictly drawn contract and is frequently payable in addition to agency commissions and collection charges. Furthermore, this Board contract may endure for a period of as much as twenty years. Needless to say the usual method of valuation does not take into consideration these things.

In themselves, and if properly, honestly and conservatively used, I can see very little which is objectionable in either the agency company plan or the Board plan of operation, but neither of these plans have been used in the manner which I have outlined. On the contrary, they have been made merely a means of deferring the evil day when an obligation must be paid. They have not been used in an economical method but rather on the same principle which induces a man to sign a note for $100 due a year hence if he can thereby get the use of $75 now. Moreover, deceptive estimates and illustrations are submitted to prospective applicants, prepared without regard to either reason or truthfulness.

General Agency Contracts.

When I enter on this field I realize that I am treading upon dangerous ground, and that I may provoke the wrath of some of the most conservative Actuaries in the country whose companies issue, or have issued, general agency contracts, promising renewal commissions for an extended period. In some cases a company gives supervision of a State or large territory to an agent for his entire lifetime, sometimes also during the lifetime of a successor, and contracts to pay to him *and to his heirs* renewal commissions, usually 7½ per cent., for a long period of years after the issue of each policy, in some instances indeed during the entire continuance of the policy contract. When a contract of this kind is granted it becomes at once an obligation of the company just as binding as a policy of insurance. Properly, the value of this contract should be taken into consideration in a true valuation of the company's liabilities. I know that this sounds like rank heresy from the standpoint of net-premium valuation, but I fail

to see wherein a company's true position is disclosed in valuation if obligations of this nature are ignored, as is the common practice.

Definition.

It will be gathered from the foregoing that I consider that we should endeavor to get back more nearly to the old definition of net-premium valuation, and that in my opinion the proper definition would be that the sums assured be valued by a true table of mortality at a true rate of interest approximating reasonably to the probable experience of the company, that the annuities used in valuing the net premiums should be on the same basis as the reversion values for valuing the sums assured, and that *the net premiums should be those on which the office premiums were based.* It is assumed, of course, that the office premiums were scientifically computed with appropriate and adequate expense allowance, and that no obligations have been incurred by the company which would increase the claims against the future premiums beyond the expense loadings which have been provided.

Valuation for Purposes of Distribution.

The system of net valuation is in many instances a good compromise on practical grounds when distribution of surplus is the object in view; but it does not serve as a good basis of comparison between different companies. It is not a good guide to their strength or their weakness, and much misconception has arisen through the popular belief that it is the correct scientific system. This is one of the most widespread actuarial errors. In many circumstances, the net system fails utterly and it can scarcely be said to give a true picture of the condition of a company.

On the other hand, it gives the *relative* position of the same company from time to time with satisfactory regularity. For the particular purpose of surplus distribution, therefore, the method may be found reasonably satisfactory. The future charges against the business for commissions, etc. may properly be met as they accrue, and may therefore be ignored at the time of valuation. As a matter of equity the first year's expenses should not be ignored unless the business being written by the company is of a very stationary character. If the company's business has reached a

stationary stage and if the expenses incurred for writing new business are nearly the same from year to year, then this also could be ignored in making a valuation for surplus distribution.

A valuation of this nature was explained clearly and forcibly so long ago as 1851, and the views of the writer seem to me so much more nearly correct than those now frequently advanced that I desire to quote as follows:—

"*Valuation of Life Policies for Bonus or Profit.* . . . In the estimation of the present value of the premiums the net amount and not the gross amount is taken, that is, all marginal additions or surplusage laid on the original premium, for commission, expenses or fluctuation of mortality, are rejected.

"Hence all additions made to the net premiums (which premiums represent the true value of the life contingencies at a given rate of interest), and which additions, in the case of assurances taken out on a non-participating scale, are wholly absorbed by the agency and other charges incidental to the working of a life business, must be altogether disregarded; otherwise, the present value of the income of the office really available for future claims, will be greatly exaggerated, and a wrong foundation laid, whereupon to form a report of the condition, favorable or unfavorable, of its affairs."

James on "*Life and Fire Assurance,*" London, 1851.

Valuation as a Test of Solvency.

The object here is entirely different. It is not a question of the amount of the surplus and of the equities of different policyholders. It is a question as to whether the company can meet its obligations at all. For that purpose all legal obligations should be charged in the liabilities and every available asset should be credited. In judging the solvency of an office we have to consider the actual premiums paid while the rates of interest, mortality and expenditure to be experienced in the future should be carefully computed. Forfeiture and surrender must also be borne in mind, especially the fact that surrender values may be guaranteed in the policies in excess of the natural share which these policies would have in the assets.

Amongst the assets there are several items which in my opinion should be allowed and which at present are treated as not admitted by the usual practice of Insurance Departments. For example, I can see no reason why office furniture, bills payable and even agents' balances should not be conservatively valued and admitted. On the other hand, in charging the liabilities and testing the solvency of a company I consider that the only fair and reasonable method is a modified gross premium valuation, proper consideration being given to the legal obligations for compensation under general agency contracts, agency agreements, Board plans, and all other claims which can be sustained against the company.

Considering the case of two companies in embarrassed circumstances, and assuming conditions to be identical excepting that one charges low and the other high premiums, the net-premium method would show the same reserve liability in both cases, yet it cannot be denied that the recuperative power of the company charging the higher rates is the greater, and it need not therefore have so large a fund on hand. This condition should be recognized by the State in fixing the standard of solvency.

One of the only cases in recent years in which I can remember that a company was declared insolvent arose through a technical ruling as to net valuation. The company was treated as insolvent but yet was reinsured in another company for a very large sum, all its policy obligations were guaranteed and the shareholders received more than the par value of their shares as the result of the reinsurance. As a matter of fact the company was perfectly solvent all the time, able to meet all its obligations, and the ruling of insolvency arose on technical grounds only.

In New York State there is a wise provision in what may be called a double valuation standard, the first for a company in good standing, and the second for a weaker concern. If the assets do not measure up to the first standard, the company must cease writing new business; but it is not insolvent until its assets fall short of the second standard, namely, the American Experience 4½ per cent. Table. The trouble is that the lower standard also calls for net-premium valuation, which is a very poor basis for testing the solvency of any company.

The questions thus brought up for discussion have been in my mind for many years, but I refrained from dealing with the subject because the time did not seem ripe, and more harm might be

done than good. Now, a change in sentiment can be observed. Questions have been forced upon our attention, and Actuaries should take the lead in discussing such matters. If we do not, our profession is belittled. We shall be looked upon as mere mathematicians, able to make calculations but unable to discuss principles or to help frame the laws which must govern the business of life insurance. Actuaries should stand on a much higher plane, and we should educate the public to know that no insurance laws should be enacted without consultation with a committee of the more eminent actuaries in the country. There are many changes in the insurance laws which could with great advantage be adopted, and none more necessary than those which arise out of this subject of valuation. Greater uniformity is desirable; principles of equity should strictly govern surplus distribution; and valuation as a test of solvency should be treated as a separate question, under which a weak company should have every chance to recuperate.

Part II. Surplus Distribution.

This question of surplus distribution has always been one of difficulty. So far back as 1823 a committee of scientific experts in discussing the mode adopted in the old Equitable Society reported that "the mode adopted in the Equitable office is not the correct mode." But upon the three following points, namely, (1) What is the correct mode of distributing profit? (2) Whether the correct mode be the practical mode? And (3) what would be the expedient mode for the Society to adopt?—*they all differ in opinion.* This difference of opinion seems to have existed from that day to this. I do not suppose there is much likelihood that uniformity will be brought about; but by treating some of the methods now in use analytically, we may be able to obtain clearer ideas on the subject, and open discussion may bring us all a little more into harmony than we now are.

One of the reasons for the difference in opinion is that each Actuary has a different problem to consider. He naturally takes up the subject from the viewpoint of the Company with which he

is connected, and each Company brings to the problem varying conditions which must affect the result. It is only necessary to deal with these very briefly by making reference to (1) promises which Companies may have made in the sale of their policies; (2) charter provisions which sometimes govern such matters; and (3) the traditions of the Company, combining with all these the nature of the education which the Actuary himself may have received.

Legal Requirements.

This question of distribution has been made a very important one through the change in the New York Law which became effective on January 1st of this year. The Law now reads:—

> "Upon the 31st day of December of each year, or as soon thereafter as may be practicable, every such corporation shall well and truly ascertain the surplus earned by such corporation during the said year . . . . . such corporation shall apportion the . . . . . surplus equitably to all other policies entitled to share therein."

> "The dividend declared as aforesaid in the case of a policy issued on or after the first day of January, 1907, shall be payable respectively either upon the anniversary of the policy next after said 31st day of December or upon a day certain in the year following said date, according to the rules of the corporation or the terms of the policy, and upon the sole condition that the premium payments for the policy year current upon said 31st day of December shall have been completed."

It is evident from these extracts that the surplus which the law contemplates is that which has actually been earned during the preceding year. The amount distributed will immediately be charged as a liability; no condition can be imposed whereby the surplus would be forfeited for failure to pay the next premium or for any other reason, because a lapsing policyholder can claim his share of the surplus in cash and presumably he can claim it at any time after the amount has been ascertained.

After a policy has been three years in force the dividends are, with the reserve, applied automatically to maintain the contract in the form of extended insurance, unless some other option expressed in the policy be availed of. The law is silent as to what should be done with surplus which policyholders do not claim in cash under policies lapsing during the first three years. Apparently dividends will be forfeited as well as reserve values, although there seems but little justice in this provision.

From the foregoing it will be seen that the first step is to ascertain the total amount of surplus available for distribution, and after this has been done the amount has to be apportioned "equitably." In ascertaining this amount I have already shown that the process should not be the same as that of testing the solvency of the company; in other words, the surplus for distribution is not necessarily the same as the surplus admitted by State Insurance Departments. There are certain forms of investments and assets which are disallowed by State Departments, such as bills receivable, agents' balances, office furniture, etc., but which nevertheless have a defined value; they may properly be treated for company purposes as surplus, if dealt with on a conservative basis. If a company carries the full ordinary reserve for its business in the first year in which it is written, there is an investment in this business, part of which can reasonably be viewed as a good asset. But if the company uses the select-and-ultimate, or modified-preliminary-term method of valuation or any of the other plans which have been proposed, this asset would not be available. Then on the other hand it is desirable, if not necessary, for a company distributing annually to carry forward a contingency reserve for fluctuations in mortality and investments. The limitation of this contingency reserve under the New York Law must be observed. All these reasons show that the surplus for distribution may differ radically from the surplus under State Department rules.

Contribution Plan of Distribution.

The general theory of the contribution plan of distribution has been accepted by nearly all Actuaries, namely, that the surplus should be apportioned to those persons who as a class have earned it. It is, however, impossible to deal exact justice to all, because there

are many sources of surplus, and each source introduces its own particular problem as to distribution. Dealing first with the question on the basis of pure theory, let us examine the various sources of surplus and come to some sort of agreement regarding the method in which each part of the total surplus should be disposed of. Some of the theories must be cast aside and merged in broad principles, but it is useful to discuss the theory first so long as Principles are not forgotten in a mass of detail.

Loadings.

The profit from loadings should surely be distributed with reference to the loadings contributed by each individual policy remaining in force; but not always *in proportion* to such loadings. It is easy to ascertain the amount by bookkeeping methods and as easy to distribute it. If the total loadings paid in any year on policies entitled to participate, amount to L, and the available loadings on an individual policy amount to l, then such individual policy may receive that share of the total surplus from loading which the ratio of l to L would produce. There are two difficulties however. *The first* is to determine what the true net premium, or the true loading, may be: whether it should be on the valuation basis, presumably the American Experience Table, or by some other table of mortality such as a Select Table, whether at the valuation rate of interest or at some other rate. *The second* is to apportion the expenses equitably against the loadings. It is not theoretically accurate to treat the expenses as a ratio of the total premiums. These expenses should be apportioned more accurately, partly as a percentage, and partly as a constant depending upon the amount insured. Moreover, the first year's expenses should be separately charged, renewal expenses being put in the form of a percentage of the premium and a constant for each thousand dollars of sums assured. A distribution of surplus by using only a ratio of the loadings paid might be far from just as between different ages and different classes of policies. (*See Transactions Faculty of Actuaries,* Vol. II, p. 257.)

Interest.

The interest earnings in excess of the reserve requirements should be distributed in accordance with the amount of funds

held on behalf of each policyholder, making allowance for the varying amount during the year. This is usually a large item, but the principles of its distribution are well defined; it is therefore the simplest factor of all.

Mortality.

Properly speaking, the mortality profit, which is usually a large item also, should be distributed according to the mortality risk incurred under each policy. The principal objection to treating this profit by itself is that the mortality differs greatly from year to year and this would cause extreme irregularity in annual dividends. Even in a fairly large company the mortality fluctuation as between one year and the next may be as much as 15 per cent. or 20 per cent.; and, under certain forms of policy, this would mean a reduction in dividend of possibly as much as 10 per cent. or even 15 per cent. of the premium. In theory a true system of apportionment would require us to allocate the mortality risk under each policy. A mortality fluctuation fund should be formed in those years when light mortality is experienced, in order to meet the heavy years which are likely to follow and preserve a reasonable harmony in the dividends from year to year.

Annuities.

This item more frequently results in a loss than in a profit, there being two conflicting elements (1) a loss from light mortality, and (2) a profit from surplus interest. The loss under the former usually exceeds the profit from the latter and there is a very small provision for expenses. As annuities are usually written on a non-participating basis, the profit or loss, as the case may be, has to be borne by the general body of policyholders. Strictly speaking, the amount should be analyzed according as there is profit or loss from loadings, from interest, or from mortality, and the distribution should follow the methods above mentioned.

Surrenders and Lapses.

Profit from this source, according to first principles of the contribution method, should be distributed in proportion to the

amount which each policyholder would lose on surrender or lapse of his policy, that is, in proportion to the difference between the reserve value and the cash surrender value. In this particular case, the rule scarcely seems to apply, because we can go still further back to the reason for making any surrender charge. The reason is that continuing policyholders may be compensated for the loss caused by those who withdraw, whether that loss be caused by adverse selection, or by those remaining being burdened by a larger share of the expenses. An initial expense must be incurred to get a new policyholder to replace the old. The distribution of this surplus therefore raises a more intricate question. If the amount were distributed according to the risk each policyholder runs of losing the difference between the surender value and the reserve, then policies in the first three years of their existence would get a larger share of this surplus than older members. The lapse rate is higher in the early policy years; and, as no surrender values are payable, the loss of each is greater. On the other hand, this ignores the question of substituting a new policyholder for him who discontinues. On this latter basis the gain from surrenders and lapses should be applied to reduce the initial expenses. If liberal surrender values are paid, and a fair volume of new business is being written, I consider this the best practical solution.

Special Funds, and Profit and Loss.

The interest on special funds should either be credited to these funds or, if distributed as surplus, should be carried to the section for which the special fund is held. For example, if a special mortality fluctuation fund be carried, then the interest on this fund should be credited to mortality account from which the fund was doubtless originally drawn. In like manner the interest on any investment contingency fund would be carried to surplus interest.

Profit or Loss on Investments.

This is one of the troublesome and debatable questions. Dr. Sprague holds that it should be treated in exactly the same way as surplus interest, on the basis that the funds from which the profit or loss arise were at risk in proportion to the amount belonging to each policyholder. But it is objected that this makes the rate

of interest fluctuate severely from year to year, so much so that in some years the company through investment losses would earn less than the rate required to maintain the reserve; in other years the rate of interest would be exceedingly large. To overcome this objection it seems to be desirable for a company to hold an "investment fluctuation account" to which unusual profits could be carried from year to year, and from which losses and depreciation of securities could be met in times of stress. A large part of the contingency reserve permitted by the New York Law should be treated in this manner. The reserve for fluctuation in investments should be gradually built up to about 5 per cent. of the marketable securities held by the company in times of prosperity. In those times when values become unusually high it is not desirable to take credit in the accounts for the largest value which the market will show; it is better to be somewhat conservative in estimating market values. The practice of carrying all bonds at their par value, like mortgages, is one worthy of consideration.

British Contribution Plans.

As there is now a tendency to introduce modifications of the contribution plan along lines which have been applied in Great Britain, perhaps it will be interesting to members if I give briefly the history of the contribution plan in that country. The general principles of the system propounded by Sheppard Homans met with the hearty approval of Dr. Sprague and other British Actuaries. Sheppard Homans' plan was published in the *Journal of the Institute of Actuaries* (Vol. XI, p. 121) in 1863, and in 1868 Dr. Sprague wrote a brief editorial (*J. I. A.*, XIV, p. 396) explaining the modified method which he had *previous to that date* introduced in the Company with which he was connected. Conditions in Great Britain do not favor the application of the contribution plan on the lines proposed by Sheppard Homans: the prevailing custom is to make a valuation of the business of a company for Government returns each five years and bonus distributions at the same periods. The Actuary ascertains how much surplus has been accumulated, and most companies distribute it by fixed rules prescribed either in their Charter or By-Laws. Some companies give a simple percentage of the premiums paid during the bonus period; others a

reversionary bonus addition to the sum assured for each year; others a compound reversionary addition; while various other more or less complex plans are also in use.

Of recent years, the unfairness of fixed proportionate methods of distribution combined with premiums which do not harmonize with the particular method, has impressed itself strongly upon the Actuaries, and to overcome this unfairness they have in many cases, following Dr. Sprague's lead, adopted modifications of the contribution plan. Scales of premium rates in Britain have often been in use for 40 or 50 years without change. The Actuaries now in charge of the companies sometimes do not know on what basis the premiums were originally computed; yet it is felt that to change the basis would disturb the equities of the policyholders. Accordingly methods of distribution which sometimes are unjust have been continued because of hesitation on the part of the management of the companies to introduce innovations for which they might be criticized. The hesitation which is felt in changing premium rates is a matter which will appear strange to American Actuaries. In this country the same company may have three or four scales of premiums for the same kinds of policies in a period of ten or twenty years.

When the premiums are unfair as between entrants at different ages or as between different classes of policies, a frequent change has been to leave the premiums on the old scale, but adopt a modified contribution plan. The favorite modification divides the surplus in two portions as recommended by Mr. Weeks and Mr. Dawson last year. For example, Mr. T. G. C. Browne divides the surplus into three groups as follows:—

Group 1. Loading (less expenses and commissions), also profit from surrenders and lapses;
Group 2. Interest earned above valuation rate.
Group 3. Mortality profit, profit from investments, interest on special funds, and other surplus.

Mr. Browne next apportions group 3 ratably between groups 1 and 2 on the principle that this group 3 contains the fluctuating elements, while groups 1 and 2 are relatively constant and more distinctly within the control of the management.

Mr. Browne then having only two groups derives two surplus factors, a loading factor and an interest factor. The loading

factor multiplied by the loading on the policy gives the share of surplus derived from group 1, and the interest factor multiplied by the reserve value of the policy gives the surplus derived from group 2. The system has been in operation for a period commencing in 1875, and has been found to give excellent results. It may be considered typical of the variations of the contribution plan used in Great Britain.

Simplicity essential.

It must be apparent to all from what has been already said as to the sources of the surplus and the different ways in which each portion should theoretically be divided, that the surplus could be analyzed into all its component parts and each part treated as an independent factor in distribution. This would be exceedingly complex and would lead to confusion of the worst kind; and we must find a system which would be reasonably just and completely practical. One of the essentials of a good method of distribution is simplicity. The method adopted by any company may have to be explained at any time in Court, and must be such a method as can be understood by a person of ordinary intelligence who is without training in life insurance. It is better in practice to have a simple method (so long as it is roughly equitable) than it is to have a very accurate and fair but complex method of distribution. If an arrangement can be made whereby a certain amount of surplus is set apart by resolution of the Directors for distribution, and this amount is distributed amongst all the policyholders by simple rule, or according to factors which may be readily calculated for each policy, the arrangement can be explained.

Contribution Plan. One Factor.

The simplest method of all is that of having *one factor* for each policy, and one sum for distribution. This condition may be reached in several ways. Sometimes the division takes place simply in proportion to the premiums paid and as a percentage of such premiums. If the premiums are loaded with a straight percentage and if the expenses are charged also as a percentage, the system has the advantage of simplicity and consistency. In certain circumstances it might even be reasonably fair; but for general use the rule is not one to be commended.

The best of the "one-factor" plans which I have seen is that advanced by Mr. Dow at the last meeting of this Society; but it does not seem to be particularly simple. This method, if carefully applied, would give rough justice to all policyholders, and some analysis of it will show forth both its advantages and its weaknesses. The method consists in distributing the surplus according to the loadings in the premiums. But this cannot be defended unless the net premium used for computing the loadings be a true net premium by the experience of the company itself. It would be unjust to use the American Experience Table and the valuation rate of interest for fixing such net premiums. From motives of prudence premiums are frequently calculated and the valuation is made at as low a rate of interest as 3 per cent. But if the company earns 4 per cent. net or more (as most companies do) Endowment Assurance Policies would contribute far more to the surplus from excess interest than Ordinary Life or Term Policies. On the principle that those who contribute the surplus should have it refunded to them, a larger proportion of this excess should go to Endowment Policies than to Whole Life Policies. An illustration will best show this. Suppose that the premiums charged by a company at age 35 are as shown in the following schedule:—

Kind of Policy.	Gross Premium.	Net Amer. 3% Premium.	Loading.
(1) Whole Life Policy.....	$26.50	$21.08	$5.42
(2) 20 Payment Life Policy.....	36.50	29.85	6.65
(3) Endowment Assurance maturing in 20 years.....	50.00	41.97	8.03
(4) 20 Year Term Policy.....	17.50	10.91	6.59

The distribution as regards these policies would therefore be made in the proportion which 5.42, 6.65, 8.03 and 6.59 bear to each other. But if the Company were to earn 4 per cent. interest and to experience the mortality of the $O^{[M]}$ Select Table,[1] the condition would be as follows:—

Kind of Policy.	Gross Premium.	Net $O^{[M]}$ 4% Premium.	Loading.
(1) Ordinary Life.....	$26.50	$18.49	$ 8.01
(2) 20 Payment Life.....	36.50	24.64	11.86
(3) 20 Year Endowment.....	50.00	37.44	12.56
(4) 20 Year Term.....	17.50	10.07	7.43

[1] The $O^{[M]}$ Select Table is used merely for convenience, not that this table is applicable to American conditions.

From this schedule it will be seen that the ratio of distribution would be entirely different under the second plan from what it would be under the first where the valuation rate of interest is used. If $20 of surplus were distributed amongst four policies in the proportions above shown, the results would be as given in the following table according as (1) the valuation basis is used, and (2) the assumed experienced basis:—

	Proportionate Distribution of $20.	
Kind of Policy.	Valuation Basis, 3%.	Assumed Basis, 4%.
(1) Ordinary Life.....	$ 4.06	$ 4.02
(2) 20 Payment Life.....	4.98	5.95
(3) 20 Year Endowment.....	6.02	6.30
(4) 20 Year Term.....	4.94	3.73
	$20.00	$20.00

It will be seen that the two methods give different results, and the difference would be still greater if a more favorable mortality and higher rate of interest were experienced. Of the two the second is more equitable; it would be fairer for entrants at different ages, as well as for Life and Endowment Policies. But the use of the actual interest rate and actual mortality is open to one very serious criticism in the case of Limited Payment Policies. With this "one factor" plan of distribution, the surplus earned from year to year is being distributed as it is earned, and no provision is made for surplus after the premiums cease to be payable. In the case of Limited Payment Policies this involves an anomalous and objectionable condition. It pays them too large a proportion of the surplus from interest as well as from loadings during the premium-paying term. The assumption is made that the rate of interest used in determining the net premium will be earned in the future until all the policies become claims, an unwise assumption.

If the valuation basis be used for computing the loadings, the result is equally objectionable in the case of Limited Payment policyholders. In this case it is unfair to the policyholder, because, the loadings being payable for a period of years only, the surplus distribution will cease after that time, and no more surplus will be received by these policyholders although they are undoubtedly earning surplus if the Company is valuing at 3 per cent. and earning 4 per cent. interest. In the one case the Limited Payment

policyholder may get the proper amount of surplus for the entire duration of life, but discounted into the premium-paying period. In the other case he would only get the surplus earned during the premium-paying period and would lose the surplus interest, also mortality profit, for the balance of his lifetime. This seems to me the greatest objection to the "one-factor" method of distribution.

The best solution of this difficulty in regard to Limited Payment Policies appears to lie in the calculation of the premium rates. If the policies are exactly what they purport to be, namely, Ordinary Life Policies with premiums payable over a shorter period, then properly speaking the Limited Payment policyholders should get exactly the same surplus from year to year as the ordinary life policyholders get. Accordingly, for surplus distribution the loading factor under Limited Payment Policies would be the same as under Life policies taken at the same age. The participation would be continued at the same rate as Ordinary Life Policies after the premiums cease. When this is done provision for surplus after the premium-paying period expires should be made and carried as a special reserve by the company. This is the method commonly employed in Great Britain, but so far as I have heard it is seldom, if ever, used in America.

Another complication in this "one-factor" method arises on account of the fluctuations in the rates of interest and mortality which might necessitate the recalculation of the loading factors periodically; when this recalculation takes place a disturbing influence would be introduced in the distribution. For example, if the company were to use 4 per cent. for 5 years, and then were to find that the rate had decreased to 3½ per cent. it would be necessary to recalculate the loadings in the above schedules, and the following would then represent the relative positions of the policies above mentioned:—

Kind of Policy.	Gross Premium.	$O^{[M]}$ Select 3½% net Premium.	Loading.	Proportionate share of $20.
(1) Ordinary Life.....	$26.50	$19.59	$ 6.91	$ 3.99
(2) 20 Pay Life.....	36.50	26.80	9.70	5.60
(3) 20 Year Endowment	50.00	39.23	10.77	6.21
(4) 20 Year Term.....	17.50	10.22	7.28	4.20
				$20.00

This "one-factor" method unduly favors the new entrant; it does not take into account the fact that a large part of the surplus namely, the surplus interest, is earned on past contributions. It assumes that each year's surplus is earned from payments made by the policyholders during that year. If the conditions regarding any company remain uniform from year to year, the method will produce the same cash surplus to any policy each year, while it is usually considered preferable and more popular to have the dividends increase as the policy grows older. Unless a modification were made for the initial expenses, this method would give the same share of surplus at the end of the first year as would be given at the end of any other year. Of course there are two opinions as to the justice and equity of this arrangement. Personally, I think that Mr. J. M. Craig made a wise statement when he said last May:—"It is a dangerous policy to undertake the payment of dividends before they are earned."

I am inclined to think that a more just apportionment can be made by loading the net premium for expenses in accordance with the probable experience of the company, analyzing the expense into its component elements of percentage and constant. Then by using the difference between this theoretical gross premium and the premium actually charged, which includes the loading for surplus, a fairer basis for distribution would be arrived at.

By introducing variations on the method as above suggested and also changes from time to time as the mortality experience and interest earnings may change, we would have under this method the practical advantages of the "two-factor" or the "three-factor" methods afterwards described. But these complications reduce the value of the method and make it as elaborate as others which can be just as easily explained. Indeed I consider that the justice of the "two-factor" method can be explained more easily to a layman than this method which calls for a theoretical net premium and explanations relative thereto.

Class Mortality.

One interesting question is raised in considering different classes of policies, viz:—whether a company can properly and fairly divide its mortality experience into two or three sections, giving credit to the policyholders of each section for the mortality profit or loss

by the experience of that particular class. It is well known that Life policies have different rates of mortality from Endowment policies, while Term policies are subject to heavier rates than either, the difference being generally considerable. Some companies have had a favorable experience under Term insurances, but these seem to be the exception, and if *renewable* term insurance is issued the difference between the mortality affecting Term policies, after the first term expires and they have been renewed, and that affecting Life policies is likely to be 15 per cent. or 20 per cent. or more. It seems unjust to charge this extra mortality against the dividends of Life policyholders and give large dividends to Term policyholders which they themselves as a class do not earn. But then it would be difficult for any company of ordinary size to keep the sections distinct in mortality investigations. It would seem better to write certain policy forms only on a nonparticipating basis, or to cut them out altogether.

Two Factors.

The "two-factor" method has already been discussed so fully that I need not enter further into the general question. In addition to the contribution by Mr. Weeks in the last volume (Vol. IX, p. 310) of *The Transactions,* Mr. Sanderson gave (Vol. X, pp. 102-6) an excellent outline of various modifications of this plan. He also gave one or two original suggestions which should be carefully considered. I have already referred to Mr. T. G. C. Browne's method and would suggest that students should also read Mr. G. J. Lidstone's valuable paper (*J. I. A.*, Vol. XXXII, p. 73).

Three Factors.

Under a "three-factor" method the surplus is usually divided into three groups derived from (1) loadings, (2) surplus interest, and (3) mortality. The first is divided according to the loading under each policy, the second according to the Reserve Value, and the third according to the Cost of Insurance or Death Strain. From experience I find it almost impossible to get a layman to understand what the Cost of Insurance is. I have argued with an intelligent man on this subject for hours and he always came back with the statement "But I have lived; it has cost nothing to

insure me." Perhaps a judge could be convinced of the fairness of using this factor, but it would be hopeless before a jury; at the best we should have to get the jury to accept the statement on faith. Theoretically the method is perhaps a little more equitable towards policyholders, and the additional factor does not add seriously to the amount of trouble involved in the calculation; the principal difficulty lies in its explanation.

We should consider whether mortality profit ought to be a regular factor from year to year. Should not the mortality table in use be changed if this condition regularly holds? Although the legal standard be the American Experience Table there is no reason why for purposes of distribution a company may not use such mortality table as may follow closely the experience of the company itself. Then again if we use mortality as a third factor we have either to classify the experience according to the kind of policy under consideration, or give to Term policyholders much more mortality surplus than they earn, and to Endowment policyholders less than they earn. Is it not better to merge this mortality element in the other two?

Four or more Factors.

There are five important factors from which the surplus is derived, so it would be possible to take into consideration separately each one of these factors. The two main factors in addition to the three already dealt with are: (1) profit from surrender and lapse, and (2) profit from investment with profit and loss generally. But I have already shown that the profit from surrenders and lapses is probably more fairly distributed by setting it against the initial expenses of acquiring business than it would be if distributed according to the risk which each policy runs of losing a proportion of that policy's reserve. The last important factor is Profit and Loss, mainly from investments. This is a fluctuating element, as will be more clearly shown in the tables afterwards submitted, and a large part of such profit should be carried to a contingency reserve fund. Any balance which may properly be distributed can better be added to the interest earned than distributed as a separate factor. The main arguments therefore seem to lie in favor of either the two-factor or the three-factor methods.

Assessment of Expenses.

If premiums are loaded with a percentage only, then, if the surplus from loading is distributed according to the loadings received, the result is equivalent to charging the expenses of the company as a percentage of the office premiums. This can be demonstrated mathematically, but it is unnecessary, as the statement is obvious. On the other hand if the premiums are differently loaded, the distribution of surplus must follow, in a more or less definite manner, the method in which the loadings have been added. Loadings are introduced for two purposes: *Firstly,* to meet the expenses, which usually consist partly of percentage expenses and partly of constant charges depending on the sums assured; and *secondly,* to furnish surplus which may be refunded to the policyholders in one form or another. It is conceivable that at some ages the loading may be barely sufficient to meet the expenses, while at other ages there may be a considerable contribution to surplus from this source. Or a like condition may exist as between different classes of policies. To overcome any such unfairness the expenses should be carefully assessed in a manner hereinafter discussed, and the amount of such expense should be deducted from the gross premium before determining the surplus loading to be used as the factor for distribution.

There is no reason why old policyholders should be charged much more for the administrative care of their business than young policyholders. The premium rate at age 20 is under $20 per thousand, and five per cent. of this would mean a charge of $1 per thousand. On the other hand at age 60 the premium rate usually exceeds $70 per thousand and five per cent. of this amount represents no less than $3.50 per thousand. The amount of trouble involved in taking care of the business in the case of young lives is practically the same as it is in the case of old lives, that is as regards administrative expense. Some other expenses are a percentage of the premiums; in particular agency commissions and state taxes. Sometimes the two together will average nearly ten per cent., in other cases they will average less than five per cent., as the renewal commissions depend so much upon the number of general agency contracts where long continued renewals are payable.

To illustrate the difference between charging the expenses sim-

ply as a percentage, or as a percentage and constant, I submit the following table:

Age at issue.	Premium charged.	Loading.	Distribution of Surplus of $20. Distribution according to loading.	Distribution if percentage charged 5%, and constant expense $1.67.
20	17.97	4.49	2.87	1.93
40	31.33	7.83	5.01	4.59
60	75.77	18.94	12.12	13.48
			$20.00	$20.00

The premiums charged are loaded with twenty-five per cent. of themselves, that is one-third added to the net American 3% premiums.

If the premiums were loaded differently a different result as regards the distribution would be obtained when allotted according to loadings. Therefore, it is necessary in equity to have the premiums loaded in a scientific manner to meet the expenses likely to be incurred even with the contribution method of distribution.

It is a comparatively easy matter to assess expenses according to their natural incidence. They consist of three elements: *Firstly,* there are initial expenses to be charged against first premiums. These consist principally of first year commissions, advertising, and a large proportion of the agency expenses. From this aggregate amount should be deducted the profit from surrenders and lapses. The balance can then be compared with the premiums received. *Secondly,* there are the percentage expenses chargeable against renewal premiums for commissions and taxes, with a small share of miscellaneous outlays. This percentage can be ascertained very nearly accurately over a period of years; it does not fluctuate much. *Thirdly,* the balance of the expenditure of the company, caused mainly by the necessities of administration, should then be apportioned as a constant on the outstanding sums assured.

In symbols the percentage expense, k, should first be charged on the renewal premiums, R, and the amount, kR, subtracted from the total expenses, E, after deducting initial expenses, I. This balance would therefore be $E - I - kR$; it should be distributed as a constant for each $1,000 of outstanding sums assured, the total sums assured being represented by $S.\ A.$ The formula for the constant expenditure per $1,000 would therefore be

$$C = \frac{1000(E - I - kR)}{S.A.},$$

C is the unknown, and variable quantity to be determined by calculation each year. To some this method may look complicated, but it is easier in practice than its appearance would lead one to suppose. Its justice and fairness to all are what make it appeal to me. It is necessary to keep first year expenses separate if we wish to follow the very reasonable condition laid down by Mr. Weeks: "Dividends ought not to rise and fall according to the volume of new business." We thus have a scientific and just apportionment of the expenses not only as between old and young entrants but also as between different classes of policies. It is thus unnecessary to have factors for Twenty Payment Life or Twenty Year Endowment policies differing from those for Whole Life policies.

In view of the recent New York laws, which make it necessary to pay smaller renewal commissions on Ten and Fifteen Year Endowments, it might be desirable to make an adjustment for these particular classes. The amounts are small and the adjustment could very easily be made by keeping the business separate and adding to the total expenses the additional percentage for such premiums before calculating the expense factors. This amount would afterwards be deducted when making the allocation to Ten and Fifteen Year Endowment policies.

Surplus Earnings—Practical Results.

As theories are valueless unless they are borne out by facts and tested by practice, I have had prepared the following tables showing the sources of surplus in two groups of representative companies during the last eleven years. All the companies in the first group have adopted 3 per cent. as their valuation basis for recent business. Those in the second group have adopted 3½ per cent. In each of the two groups the old business is valued at 4 per cent., so that in the meantime the effect of this difference must be slight; but the difference between the two groups of companies will become greater year by year. The profit from loading will thereby be diminished and the profit from surplus interest increased in the case of those companies using the 3 per cent. valuation rate.

TABLE I.

STATEMENT SHOWING THE SOURCES OF SURPLUS EARNINGS IN FOUR REPRESENTATIVE COMPANIES, WHICH VALUE THEIR NEW BUSINESS ON A 3% BASIS.

Compiled from the Gain and Loss Exhibits as furnished by the Companies.

Year ending 31st. Dec.	Profit from loading.	Surplus Interest.	Profit from Mortality.	Gain from Surrenders and Lapses.	Profit or Loss Investments.	Miscellaneous.	Total.
1896	$ 3,041,361	$ 5,064,424	$ 4,437,294	$ 3,705,344	$ —295,174	$ 39,802	$ 15,993,051
1897	$ 3,238,259	6,226,386	5,792,103	4,966,155	1,440,251	—7,000	21,656,154
1898	2,920,679	6,913,753	5,751,309	3,344,936	5,669,278	—7,000	24,592,955
1899	2,002,833	6,571,871	4,978,283	3,161,158	2,542,905	—7,000	19,250,050
1900	3,977,166	6,154,842	7,120,972	3,194,220	3,118,569	748,971	24,569,868
1901	3,407,828	6,409,970	6,726,529	3,138,071	2,958,503	5,114	22,390,887
1897-1901	15,546,765	32,276,822	30,369,196	17,804,540	15,729,506	733,085	$112,459,914
1902	2,633,734	5,189,414	8,429,351	2,997,118	$ —257,230	$ —33,974	18,958,413
1903	2,004,295	7,599,231	9,323,785	4,611,485	—10,230,149	—7,000	13,301,647
1904	2,980,757	8,446,437	9,996,696	3,720,636	7,011,047	—7,000	32,148,573
1905	3,247,288	10,276,076	11,475,180	4,801,450	—20,395,464*	—7,000	9,397,530
1906	11,111,005	12,270,131	13,650,598	6,968,046	—13,526,681	2,104,408	32,577,507
1902-1906	21,977,079	43,781,289	52,875,610	23,098,735	—37,398,477	2,049,434	$106,383,670
1896-1906 11 years.	40,565,205	81,122,535	87,682,100	44,608,619	—21,964,145	2,822,321	
						GRAND TOTAL.....	$234,836,635

* This item is to a great extent caused by a different treatment of assets in one large Company.

TABLE II.

STATEMENT SHOWING THE SOURCES OF SURPLUS EARNINGS IN FOUR REPRESENTATIVE COMPANIES, WHICH VALUE THEIR NEW BUSINESS ON A 3½% BASIS.

Compiled from the Gain and Loss Exhibits as furnished by the Companies.

Year ending 31st Dec.	Profit from loading.	Surplus Interest.	Profit from Mortality.	Gain from Surrenders and Lapses.	Profit or Loss on Investments.	Miscellaneous.	Total.
1896	$ —826,744	$ 1,879,919	$ 2,838,107	$ 2,502,656	$ 200,284	$ —46,885	6,547,337
1897	46,964	2,048,049	3,456,179	2,354,652	2,025,776	—25,000	9,906,620
1898	—67,724	2,275,819	3,117,051	2,327,174	4,964,268	—25,000	12,591,588
1899	—1,763,823	2,544,777	1,807,802	1,634,033	5,179,585	—25,000	9,377,374
1900	—1,387,480	2,150,163	4,554,870	1,331,947	4,028,131	—25,759	10,651,872
1901	—1,655,450	2,168,300	4,966,149	1,296,236	4,208,612	—16,735	10,957,112
1897-1901	—4,837,513	11,187,108	17,902,051	8,944,042	20,406,372	—117,494	$53,484,566
1902	—1,608,741	2,573,185	6,078,834	1,815,504	1,085,635	— 1,753	9,942,664
1903	—1,366,193	3,665,716	4,876,090	2,103,938	—9,921,042	—13,972	—655,463
1904	—1,223,716	4,411,877	4,352,898	2,607,408	9,337,798	—24,134	19,462,131
1905	359,831	4,914,292	4,722,480	2,931,271	—2,949,632	12,387	9,990,629
1906	3,175,019	6,874,651	6,065,268	3,598,827	—6,409,507	101,131	13,405,389
1902-1906	—663,800	22,439,721	26,095,570	13,056,948	—8,856,748	73,659	$52,145,350
1896-1906 11 years.	—6,328,057	35,506,748	46,835,728	24,503,646	11,749,908	—90,720	
						GRAND TOTAL.....	$112,177,253

TABLE III.—(Combining Tables I and II.)

STATEMENT SHOWING THE SOURCES OF SURPLUS EARNINGS IN EIGHT REPRESENTATIVE COMPANIES.

Compiled from the Gain and Loss Exhibits as furnished by the Companies.

Year ending 31st Dec.	Profit from Loading.	Surplus Interest.	Profit from Mortality.	Gain from Surrenders and Lapses.	Profit or Loss on Investments.	Miscellaneous.	Total
1896	$ 2,214,617	$ 6,944,343	$ 7,275,401	$ 6,208,000	$ —94,890	$ —7,083	22,540,388
1897	3,285,223	8,274,435	9,248,282	7,320,807	3,466,027	—32,000	31,562,774
1898	2,852,955	9,189,572	8,868,360	5,672,110	10,633,546	—32,000	37,184,543
1899	239,010	9,116,648	6,786,085	4,795,191	7,722,490	—32,000	28,627,424
1900	2,589,686	8,560,133	11,675,842	4,526,167	7,146,700	723,212	35,221,740
1901	1,742,378	8,323,142	11,692,678	4,434,307	7,167,115	—11,621	33,347,999
1897-1901	10,709,252	43,463,930	48,271,247	26,748,582	36,135,878	615,591	165,944,480
1902	1,024,993	7,762,599	14,508,185	4,812,622	828,405	—35,727	28,901,077
1903	638,102	11,264,947	14,199,875	6,715,423	—20,151,191	—20,972	12,646,184
1904	1,757,041	12,858,314	14,349,594	6,328,044	16,348,845	—31,134	51,610,704
1905	3,607,119	15,190,368	16,197,660	7,732 721	—23,345,096	5,387	19,388,159
1906	14,286,024	19,144,782	19,715,866	10,566,873	—19,936,188	2,205,539	45,982,896
1902-1906	21,313,279	66,221,010	78,971,180	36,155,683	—46,255,225	2,123,093	158,529,020
1896-1906 11 years.	34,237,148	116,629,283	134,517,828	69,112,265	—10,214,237	2,731,601	
						GRAND TOTAL.....	$347,013,888

These tables are both interesting and instructive. They were kept in separate groups for several reasons, one of which was to see whether the same features would appear in each group and thus to a certain extent confirm the tendencies. They were then brought together in the third table, from which general conclusions may be drawn. It will be observed that the loading profit, over a period of ten years to 1905 inclusive, scarcely increased at all notwithstanding the tremendous expansion of business during that time. The interest profit in 1906 is nearly three times the amount in 1896, and the mortality figures show results approximately the same; the profit from surrenders and lapses, like the profit from loading, shows a very slight increase. An interesting feature is the abnormal growth in profit from loading for the year 1906, caused (1) by reduction in new business written that year by the New York Companies, and (2) by economies in management introduced after the Insurance Investigation.

In order to have a better idea of the tendency towards changes in the sources of surplus from period to period, the following statements were prepared, giving the percentages which the four *regular* items of profit [namely, (1) loading, (2) interest, (3) mortality, and (4) surrenders and lapses] bear to the total earnings from these four sources.

Table from Group 1.	Proportion of Profit from			
	Loading.	Interest.	Mortality.	Surrenders and Lapses.
1897–1906	15.8%	32.0%	35.0%	17.2%
1897–1901	16.2%	33.6%	31.6%	18.6%
1902–1906	15.5%	30.9%	37.3%	16.3%
Table from Group 2.				
1897–1906	5.8%	35.7%	46.7%	23.4%
1897–1901	14.6%	33.7%	53.9%	27.0%
1902–1906	1.1%	36.8%	42.8%	21.5%
Table from Combined Groups.				
1897–1906	9.6%	33.1%	38.3%	19.0%
1897–1901	8.3%	33.6%	37.4%	20.7%
1902–1906	10.5%	32.7%	39.0%	17.8%

General conclusions from tables may be summarized as follows:

First. The tables separately and combined show that there are wide fluctuations from year to year; some are fat years and some lean years.

Second. The mortality, even in large companies and groups of companies, shows much variation year by year; and in smaller companies this feature is necessarily more pronounced.

Third. If the actual surplus earned in each year were distributed in the year following (as some of the laws seem to expect) there would be no regularity at all in the dividends; it is far from practical to make such distributions.

Fourth. The profit and loss items do not as a rule yield much surplus. Probably much of the loss caused by charging out agents' balances appears under this head; and a separation has not been made from the investment profits and losses.

Fifth. The increase in the surrender and lapse profit is exceedingly small when viewed in relation to the increased number of policies discontinued, showing the effect of more liberal surrender values now paid.

Sixth. The swing of investment profits and losses ranges over a period of several years; for three or four years large profits are derived, then for several years large losses follow. This item therefore should not be distributed when earned, but should be held against the probable losses to follow in later years.

The section of the foregoing paper dealing with surplus distribution is analytical and has not been prepared with a view of advancing new theories or new plans of distribution. It is intended rather to clear up our ideas on the subject so far as the plans now in use are concerned. Perhaps by the time the paper comes up for discussion next spring I may have brought some nebulous ideas into more definite shape; but meantime I am inclined to the belief that, as I stated in the early part of the paper, the problems confronting actuaries in different companies are so varied, and the conditions surrounding the companies so different in each case, that no fixed and arbitrary rules for surplus distribution can be evolved. We have each to work out our own problems.

There are one or two general principles which have been made more clear in my own mind by the writing of this paper. The first is that the premium rates charged for different ages and different kinds of policies have a close relation not only to the method of valuation to be adopted, but also to the method of surplus distribution. *To attain equity these three important factors, Premiums, Valuation, and Distribution, must all be in har-*

mony. The second point which has been emphasized is that valuation as a test of solvency should be something entirely different from valuation for the purpose of distributing the surplus. A certain amount of leniency is desirable in the former, while the latter should be conducted on conservative principles, reserving amply for the future, and taking credit for nothing but what has been already earned. As a test of solvency different conditions can be introduced, because a company deserves every possible chance to pull itself out of a difficult situation either by reinsurance or otherwise, without the tremendous loss to policyholders which must inevitably follow a forced realization.

NOTE.—I desire to record my appreciation of the help I received in preparing the foregoing paper from Mr. George Graham, Jr. (then my assistant and now Actuary of the Capitol Life) who gave valuable suggestions as the work progressed, and who compiled most of the figures and tables.

On Surplus Distribution.

BY D. E. KILGOUR.

In eliciting the judgment of the members of the Actuarial Society on this vital, though hackneyed subject, I wish to confess at the outset my inability to arrive at any very fixed opinion as to the merits of the method outlined in this paper. It is hoped that in the discussion evoked, it will be shown whether the method possesses much practical value or will lend itself to material economy of labour.

The method, while adhering somewhat to the Contribution Principles, can scarcely be termed a Contribution Method of surplus distribution. It ignores a precise division according to the usual component elements, but it recognizes the principle that injustice is done, where no attention is paid to the fact that the standard reserve determining the surplus is not based on rates of interest and mortality, obtaining in actual business operations. It differs from the exact contribution method in that, while in the latter the margins of surplus from mortality and interest vary from year to year, in the former they are determined in advance and remain constant.

The following notation and equations will assist to an understanding of the method :

Notation for Single Age, Plan, and Year of Issue.

Let λ_n = Number living at end of n years.

V_n = Reserve at end of n years based on a fixed table of mortality and a rate of interest i.

P = Net premium based on fixed table of mortality and a rate of interest i.

L = Loading, where $P + L$ = Gross Premium.

$P - A$ = Net premium based on a select hypothetical table of mortality deduced from the experience of the company, with such refinements as may be considered wise to employ, and a rate of interest $i + \rho$.

ρ = Rate of interest in excess of i, which it is thought will be earned for a term of years.

$\sigma =$ A positive or negative quantity determined from the relation, $i + \rho + \sigma =$ actual rate of interest for year (less investment charge).

$\frac{d_n}{\lambda_n} =$ Rate of mortality in the $(n+1)^{th}$ year according to the fixed table of mortality.

$\frac{d_n - \delta_n}{\lambda_n} =$ Rate of mortality in the $(n+1)^{th}$ year according to the hypothetical table of mortality or assumed actual experience.

$E =$ Expense charge.

$V'_n =$ Reserve at end of n years based on the hypothetical table of mortality and rate of interest $i + \rho$.

$S_n =$ Surplus earned in n^{th} year $=$ Total Funds $- V_n$.

$S'_n =$ Surplus earned in n^{th} year $=$ Total Funds $- V'_n$.

The fixed table of mortality corresponds to the standard table for calculating the company's reserves. The data upon which it is based are usually much more extreme than a continuance of actual existing conditions would necessitate selecting. The hypothetical table should approximately reflect the company's experience, exhibiting if anything a slightly higher rate of mortality than actually experienced.

Equations showing relations among functions, as disclosed by the growth of the company's funds.

$$\lambda_n(V_n + P + L - E)(1 + i + \rho + \sigma) - (d_n - \delta_n) = \lambda_{n+1}(V_{n+1} + S_{n+1}) \quad (1)$$

$$\lambda_n(V_n + P)(1 + i) - d_n = (\lambda_{n+1} - \delta_n) V_{n+1} \quad (2)$$

$$\lambda_n(V'_n + P - A + L + A - E)(1 + i + \rho + \sigma) - (d_n - \delta_n) = \lambda_{n+1}(V'_{n+1} + S'_{n+1}) \quad (3)$$

$$\lambda_n(V'_n + P - A)(1 + i + \rho) - (d_n - \delta_n) = \lambda_{n+1} V'_{n+1} \quad (4)$$

From (1)–(2) :

$$\lambda_n(V_n + P + L - E)(\rho + \sigma) + \lambda_n(L - E)(1 + i) + \delta_n = \lambda_{n+1} S_{n+1} + \delta_n V_{n+1} \quad (5)$$

From (3)–(4) :

$$\lambda_n(V'_n + P + L - E)\sigma + \lambda_n(L + A - E)(1 + i + \rho) = \lambda_{n+1} S'_{n+1} \quad (6)$$

From (6) :

$$S'_{n+1} = \frac{\lambda_n}{\lambda_{n+1}} \left\{ (L + A - E)(1 + i + \rho) + (V'_n + P + L - E)\sigma \right\} \quad (7)$$

From (5) :

$$S_{n+1} = \frac{\lambda_n}{\lambda_{n+1}} \left\{ (L-E)(1+i) + (V_n + P + L - E)(\rho+\sigma) + \frac{\delta_n}{\lambda_n}(1 - V_{n+1}) \right\} \quad (8)$$

From (7)-(8) :

$$S'_{n+1} - S_{n+1} = \frac{\lambda_n}{\lambda_{n+1}} \left\{ A(1+i+\rho) - \rho(V_n + P) - \frac{\delta_n}{\lambda^n}(1 - V_{n+1}) - \sigma(V_n - V'_n) \right\} \quad (9)$$

From (3)-(1) :

$$\lambda_n(V_n - V'_n)(1+i+\rho+\sigma) = \lambda_{n+1}(V_{n+1} + S_{n+1} - V'_{n+1} - S'_{n+1}) \quad (10)$$

From (10) :

$$S'_{n+1} - S_{n+1} = (V_{n+1} - V'_{n+1}) - \frac{\lambda_n}{\lambda_{n+1}}(V_n - V'_n)(1+i+\rho+\sigma) \quad (11)$$

$$S_{n+1} = S'_{n+1} - (S'_{n+1} - S_{n+1})$$

From (7) and (9) :

$$\begin{aligned} S_{n+1} &= S'_{n+1} - \frac{\lambda_n}{\lambda_{n+1}} \left\{ A(1+i+\rho) - \rho(V_n + P) - \frac{\delta_n}{\lambda_n}(1 - V_{n+1}) \right\} + \frac{\lambda_n}{\lambda_{n+1}}(V_n - V'_n)\sigma \\ &= \frac{\lambda_n}{\lambda_{n+1}} \left\{ (L + A - E)(1+i+\rho+\sigma) \right\} \\ &\quad - \frac{\lambda_n}{\lambda_{n+1}} \left\{ A(1+i+\rho) - \rho(V_n + P) - \frac{\delta_n}{\lambda_n}(1 - V_{n+1}) \right\} \\ &\quad + \frac{\lambda_n}{\lambda_{n+1}} \sigma(V_n + P - A). \end{aligned} \quad (12)$$

The expression

$$\frac{\lambda_n}{\lambda_{n+1}} \left\{ A(1+i+\rho) - \rho(V_n + P) - \frac{\delta_n}{\lambda_n}(1 - V_{n+1}) \right\}$$

may be positive or negative according to the duration and kind of policy. It is the quantity which must be deducted from the true surplus or the surplus which would have been available for division, if the hypothetical reserve had been used, in order to arrive at the divisible surplus disclosed by the standard reserves adopted.

Illustration of Function

$$\left\{ A(1+i+\rho) - \rho(V_n + P) - \frac{\delta_n}{\lambda_n}(1 - V_{n+1}) \right\}$$

tabulated for 20 years on 20 year Endowment Plan, age 30.

The Assumed or Hypothetical Mortality Table is the Canada Life Select and the assumed rate of interest is $4\frac{1}{2}\%$.

The Standard Basis for Premiums and Reserves is H^M $3\frac{1}{2}\%$,

$P = 39.47.$ $P - A = 34.27.$

Year.	$A(1+i+\rho)$	$-P(V_n+P)$	$-\frac{\delta_n}{\lambda_n}(1-V_{n+1})$	Total.
1	5.434	— .3947	—4.7461	+ .293
2		.7286	3.3924	1.313
3		1.0749	2.4103	1.949
4		1.4346	1.7860	2.213
5		1.8084	1.5084	2.117
6		2.1967	1.4576	1.780
7		2.5998	1.5490	1.285
8		3.0183	1.6035	.812
9		3.4530	1.6030	.378
10		3.9050	1.5589	— .030
11		4.3757	1.4428	.385
12		4.8666	1.2790	.712
13		5.3790	1.1337	1.079
14		5.9138	1.0356	1.515
15		6.4718	.9152	1.953
16		7.0544	.8196	2.440
17		7.6627	.7001	2.929
18		8.2985	.5208	3.385
19		8.9640	.2843	3.814
20		9.6619	0	4.228

I have supposed here that a company makes a very careful review of its whole experience and gives due weight to all relevant circumstances which are likely to affect it in the future. It is supposed that the rate of interest upon which the fixed standard of reserve is based is (i), that the company anticipates that it will earn in excess of this rate of interest (ρ); further, that the company actually earns this rate of interest increased or decreased by (σ), which quantity should be very small, some years positive and some years negative. It has been introduced here so as to make the formula exact, but there is a question in my mind whether it might not be neglected entirely in the practical distribution formula. The various equations showing the progress of a fund and the relations amongst the functions are, I assume, self-explanatory. They lead to function

$$\frac{\lambda_n}{\lambda_{n+1}}\left\{ A(1+i+\rho) - \rho(V_n + P) - \frac{\delta_n}{\lambda_n}(1 - V_{n+1} \right\}$$

which I have defined as the contribution from the true surplus earned in a given year and retained or disbursed by the company in order to maintain a fixed reserve.

This function will be positive in the early years and negative in the later years of the policy's duration, and will vary according to the plan and age of the insured.

The suggestion is to evaluate it for all plans, ages and durations.

The method of actual apportionment would be as follows: When making an annual valuation of the policies, which is effected by multiplying the amount in force at the end of the year (exhibited by most American companies on summation sheets or cards) by the appropriate reserve, multiply the same outstanding amount by the function referred to for the same corresponding plan, age and duration. This would not necessarily entail an entire duplication of the work, as the reserve sheets and sheets giving this function might be kept together and the operation completed at the same time. It would, of course, be only necessary to do this work in the case of the participating policies.

Having determined the total valuations, which will be the sum of both positive and negative quantities, but which in the aggregate we may assume to be positive, we would add the amount so found to the total distributable surplus of the year, ascertained by deducting from the assets of the company the liabilities as based upon the fixed standard of reserve. This amount might be termed the true distributable surplus of the year, and would, except for the incidental interest savings which if thought necessary might first be apportioned according to the reserve, be distributed in proportion to the true premium or true loading, or in some other determined way.

Having found, now, for each individual policy what might be termed the true apportionable surplus, it remains to deduct from this the amount already added, the sum of which amounts has been added to the distributable surplus in order to determine the true aggregate surplus of the year.

There appears to be no reason why a set of such tables as suggested when once prepared, would not serve a company for a considerable term of years. The actual mortality rates, while varying from year to year, would not likely exhibit material fluctuations on the average; nor is there reasons to suppose that with fairly permanent conditions obtaining in the company, the rates of interest earnings should radically change.

While a lower rate of interest is thought necessary to use in the calculation of rates and reserves, the actual rate earned at the present time and likely to be earned for some years to come, can be anticipated with fair accuracy. At any rate, no great loss of equity would occur by assuming that the general tendency of a long period is towards lower interest rates and to select at the outset, a rate $(i + \rho)$ which is lower than the actual margin of interest earnings at the present time warrant. Incidental savings in interest being the sum of the functions $\frac{\lambda_n}{\lambda_{n+1}} \cdot \sigma \cdot (V_n + P - A)$ might properly constitute a contribution to a contingency fund and therefore be deducted from the actual distributable surplus of the year.

The use of true net premiums in apportioning surplus, is not, of course, new. If I am not mistaken, one of the largest American companies divided surplus in simple proportion to the true net premiums long before the introduction of the contribution method. The idea, however, of using a simple corrective function, thereby virtually arriving at a contribution method, so far as I am aware, has not been suggested or proposed. In merely outlining the method, no attention has been paid to the incidence of expense, which has been supposed for purposes of simplicity to fall as heavily on renewal as on first year premiums. Nor has the method been elaborated to provide for any but an annual distribution.

NOTE:—The steps taken to arrive at the final relation in equation (12) are, as will be observed, redundant. They are given in the present form, so as to present important relations amongst the functions.

Mortality Experience of Yale Graduates, 1792 to 1901 Inclusive.

BY EDWARD B. MORRIS.

In preparing this Mortality Experience on the Lives of Yale Graduates the writer has been hopeful of presenting new data of sufficient volume to be both of interest and of value and based upon conditions differing from those found in any previously completed experience. The limitations and inaccessibility of proper data have heretofore barred experiment, with the possible exception of the Mortality of Yale Graduates of the eighteenth century and the Mortality Experience on Graduates of the Yale Divinity School, papers on which have been presented to the Society by Messrs. Hunter and Gaines and Mr. Strong respectively. While at the outset the mere tabulation of the death rate of college graduates cannot be of more than historical value, the facts obtainable in such a research will be of interest and of value in comparison with the relative findings of other experiences.

The Mortality Experience of college graduates brings out conditions which do not confront any of the regular Mortality Experiences. It includes a group of selected men necessarily of higher culture and circumstances than the average, at the commencement of their business or professional careers, and views them until death. Such an experience eliminates all question of lapse. It differs from the experience of population tables in that it views the same risks throughout life. Thus relatively it is an experience on a small group of men for a long period and not an experience on a large group for a short period, as is the case of most mortality tables based upon population.

The Experience is not a select table (in the insurance sense) as no medical examination is obtainable. Neither is it an ultimate or an aggregate table, although perhaps approaching in many ways the

former. An ultimate insurance table, while removing theoretically the question of selection, however, bases its experience on new entrances up to about age sixty-five and being usually founded upon the experience of insurance companies necessarily is adjusted by withdrawals of lapse and surrender. As already pointed out, this question is entirely removed from this college experience as the entrants are all observed from the age at graduation until death or the termination of the experience.

Possibly in many ways such a table from the conditions which surround it would approach the Peerage Tables published April, 1861, in the *Journal of the Institute of Actuaries*, Vol. XX, page 305.

The records of death of graduates of Yale University are perhaps kept as complete as will be found in any similar institution, as considerable care has been expended in compiling them. The volume of the experience and representative place held by the college and university during the past two centuries have led me to adopt the experience of her graduates as the subject of this paper.

Unfortunately, the records of birth are not given in the Yale records, neither are they given in the matriculation records of the early classes. The gathering of these facts necessarily involved some additional labor. The more recent classes have published their class records and thus such data is available. This statement is practically true, however, of the Academic Classes only, as it is comparatively recently that class secretaries have been appointed for the other departments of Yale. It can be imagined therefore that the task of biographical research necessary in order to obtain authentic records of the births of Yale graduates was considerable. Assuming an average age at graduation did not appear as being of sufficient exactness for an experience of such a size. That fortune favored in this research work will be seen from the fact that in summing up the entire number of graduates from 1792 the dates of death were missing in only thirty-three instances and the dates of birth in two hundred and fifteen, the missing data being mainly in the Law and Medical Schools where class records have only been kept in very recent years.[1]

[1] Where the death records have been incomplete the risks have been eliminated from the experience. Where the date of birth was not found an average age of twenty-one for the College and Sheffield Classes has been used and twenty-five for the professional schools. This assumption on account of the limited number of such cases could not affect the experience adversely.

In this paper the experience has been divided into seven sections :

(1) Graduates of Yale College (Academic), 1850 to 1900 inclusive ;

(2) Graduates of the Sheffield Scientific School, 1852 to 1901 inclusive ;

(3) Graduates of Yale Law School, 1843 to 1901 inclusive ;

(4) Graduates of the Yale Medical School, 1814 to 1901 inclusive ;

(5) Graduates of Yale College (Academic), 1792 to 1849 inclusive ;

(6) Combination of Sections (1), (2) and (3) ;

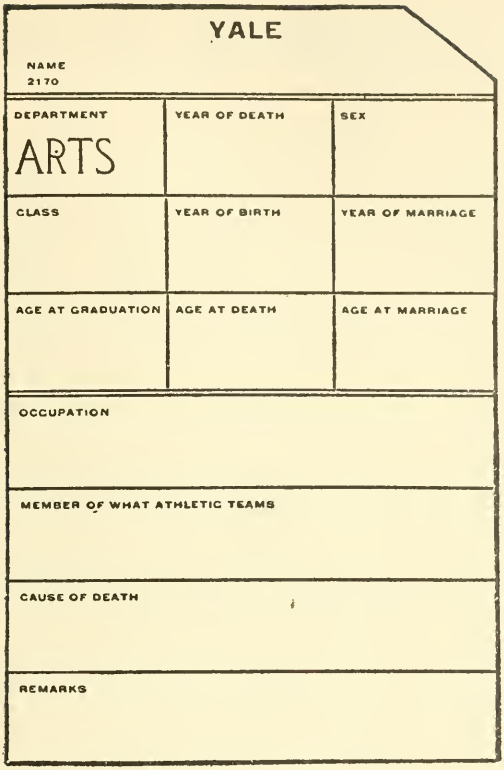
(7) All graduates from 1792 to 1901 inclusive (combination of the Sections (1) to (5) inclusive.

The exposures have been continued to June, 1905.

In working out the tables of exposure the following method has been used.

It was decided at the outset that the tabulation of the exact dates of birth and death was not necessary but that the exact year should be taken in each case. Inasmuch as the Yale classes graduate near the middle of the year (about the last of June) a very convenient assumption can be made, as the year of graduation less the year of birth gives the exact age at graduation. This is approximately correct in that we can assume that the birth rate is constant throughout any calendar year and averages in the middle of the year. This assumption cannot be carried out for the age at death, however. For example: date of graduation 1860, date of birth 1840, date of death 1861. The age at graduation is apparently twenty. The age at death, however, substracting the date of birth from the date of death, would be twenty-one. As a matter of fact, however, the age at death would be either twenty-one or twenty-two according as to whether the graduate died prior to or after July 1st. Hence in such a case the death is treated as $^5/_{10}$ at age twenty-one and $^5/_{10}$ at age twenty-two, which assumption it is clear would average correctly and at the same time would save considerable labor involved in getting the exact age at death. The experience shows a full year of exposure for the year of death.

Cards have been written for each graduate. A copy of card is here shown.


YALE

NAME
2170

DEPARTMENT	YEAR OF DEATH	SEX
ARTS		
CLASS	YEAR OF BIRTH	YEAR OF MARRIAGE
AGE AT GRADUATION	AGE AT DEATH	AGE AT MARRIAGE

OCCUPATION

MEMBER OF WHAT ATHLETIC TEAMS

CAUSE OF DEATH

REMARKS

The cards for the different schools were printed on different colors but also have the distinguishing abbreviation of the school. The card was made somewhat complete in the hope of obtaining interesting information for statistical purposes but in this paper it has been impossible to touch upon the points given further than as they are connected with the Mortality Experience in question. Also additional information was not generally obtained.

The cards after completion were assorted according to age at graduation and then according to age at death, and finally for the age existing at the termination of the experience in June, 1905, for those risks surviving. From this data the exposure is readily determined and the actual deaths tabulated.

The exposure at any age (x) equals exposures age ($x-1$) plus entrants age (x) minus (deaths age ($x-1$) plus existing age (x)). or, in the usual notation,[1]

$$E_x = E_{x-1} + N_x - (d_{x-1} + e_x).$$

In computing the expected deaths according to the exposure two tables have been used : first, the standard American Experience, and, second, the Modified Healthy English, which was used by the Actuarial Society in their Specialized Experience. This latter table I believe comes as near to fulfilling conditions similar to the Yale Experience as any available table, being based largely on the population and being without conditions involving medical selection.

The first division of the Experience given is the summary of the Experience on Yale Graduates in the Academic Department, usually called "Yale College,"—graduates from the Class of 1850 to 1900. The very favorable comparison with the American Table and also with the Modified Healthy English Table brought out by this section is noteworthy. Further, that it compares very closely with the experience that a healthy life company of some years standing might hope to experience after the effect of medical selection had worn off, although the higher ages are perhaps more favorable than any company would expect. It is remarkable also that the experience after the first age group should show almost a constant relation to that of the American Table, being throughout approximately 80 % of that table. The number of lives involved in this experience is 7,203, of which 1,484 are dead.

Comment is perhaps necessary from the fact that the Yale Academic Classes have been gradually increasing, especially since about 1875, the ratio of increase in the last ten years being most marked. This necessarily brings a relatively large number of the younger ages into this division of the experience but in no way affects the mortality percentages which are based upon the number of exposed lives.

[1] It should be noted that on account of the assumptions made in the experience (graduation being July 1st of each year), the formula is more easily worked than in the case of exposures under insurance tables, whether by the calendar year or by the policy year method, as all question of mean ages and withdrawals other than by death is eliminated.

Yale Graduates Academic Department (The College)[1] 1850 to 1900 inclusive.

Experience terminated June, 1905.

Age Attained.	Years of Exposure.	Actual Deaths.	By the American Table. Expected Deaths.	Ratio of Actual.	By the Modified Healthy English Table. Expected Deaths.	Ratio of Actual.
15–25	25,551	113.5	203.89	55.7%	192.43	59.0%
26–35	57,749	388	488.88	79.4	473.92	81.9
36–45	36,907	289	365.61	79.2	351.18	82.4
46–55	22,720.5	246.5	321.85	76.6	304.59	80.9
56–65	11,610.5	264	314.79	83.9	288.97	91.4
66–75	3,329	168	191.52	87.7	172.71	97.3
76–End	143	15	16.56	90.6	14.63	102.5
Total,	158,010	1,484	1,903.10	78.0	1,798.43	82.6

The second division of the Experience is for the graduates of the Sheffield Scientific School since its first class graduated in 1852, to 1901 inclusive and involved 2,674 lives, 277 of which have died. This table verifies in a remarkable degree the conditions found in the table of graduates from the college since 1850. This section covers practically the same period as that of the College Division and is composed of men of about the same general class, although it contains more men who have taken up engineering in its different branches. This last experience, however, although showing slightly more favorable results than that of the College Division, is not as comprehensive as the former in volume and consequently cannot be viewed with as much reliance.

[1] In the above table, and also in the table which follows of the Graduates of the Sheffield Scientific School, an interesting comparison may be made with the paper written by Mr. Gaines and Mr. Hunter entitled "Mortality among Athletes and Other Graduates of Yale University"—see Vol. IX of the *Transactions*, page 50. These gentlemen there made an approximation of the mortality of graduates from 1863 to 1902 of practically 70% of the Actuarial Society's Table (the Modified Healthy English Table), comparing same with the mortality shown in the experience which they had worked up amongst Yale athletes. It would seem, therefore, that their assumption as to a mortality of 70% is practically verified by the findings in these two sections of the Yale Experience, since we may consider that a large number of such graduates would not at the time of the estimate have been over forty-five years of age.

Yale Graduates Sheffield Scientific School, 1852–1901 inclusive.

Experience Terminates June, 1905.

Age Attained.	Years of Exposure.	Actual Deaths.	By American Table. Expected Deaths.	Ratio of Actual.	By the Modified Healthy English Table. Expected Deaths.	Ratio of Actual.
15–25	11,263.	39.5	89.75	44.0%	84.45	46.8 %
26–35	17,856	105.	150.70	69.7	146.06	71.9
36–45	8,030	62.5	79.07	79.0	75.94	82.3
46–55	3,309.	41.	45.83	89.5	43.44	94.4
56–65	987.	19.	26.08	72.9	23.95	79.3
66–75	250.	10.	13.95	71.7	12.60	79.4
76–End	4.	0.	.43	0	.39	.0
Total,	41,699.	277.	405.81	68.3	386.83	71.6

The very favorable comparison shown by these two groups naturally leads to the inquiry as to whether the data contained therein is correct and if a considerable number of risks involved have not been considered as existing when in reality they should have been treated as dead ; that is, cases of death which have not been reported to the college authorities. Careful work has been done in this connection, however, for not only have the college records been consulted but almost every case has been verified for this experience by the secretary of the graduate's class, or by a signed statement given by the graduate himself if he was still living. The percentages of such errors are undoubted extremely small.

Another fact should be constantly borne in mind in connection with this Experience and that is the character of its composition. It necessarily includes men of much higher culture and average circumstances than those in a population or life insurance experience. Taken as a whole, it would probably be impossible to find a class of men as large as that of the college graduates that will show as favorable conditions of environment. There can be no doubt but that these facts have their effect upon the experience in question.

The following two tables, showing the Mortality Experience of the graduates of two of Yale's professional schools, have been worked up with the hope of ascertaining if possible whether the Mortality Experience is effected by the profession as indicated by the graduate's degree.

It was hardly to be expected that any adverse symptoms would be shown by the graduates of the Yale Law School. This fact is found to be true, as is indicated in the annexed table. The high ratio of the actual to the expected deaths, age 46 to 55, is undoubtedly an accidental fluctuation and cannot be viewed as significant. It will be noticed, however, that the mortality shown by the table as a whole is favorable. This table is based on a total number of entrants of 1,413 of whom 221 were dead at the termination of the experience.

GRADUATES YALE LAW SCHOOL,
1843–1901 INCLUSIVE.

Experience terminated June, 1905.

			By American Table.		By the Modified Healthy English Table.	
Age Attained.	Years of Exposure.	Actual Deaths.	Expected Deaths.	Ratio of Actual.	Expected Deaths.	Ratio of Actual.
15–25	2,895	10.	23.15	43.2%	21.93	45.6%
26–35	10,748.5	60.	91.10	65.9	88.30	68.0
36–45	6,214.5	42.5	61.38	69.2	58.97	72.1
46–55	2,953.	43.0	41.18	104.4	39.03	110.2
56–65	1,271.5	26.0	35.00	80.9	32.13	80.9
66–75	555.	27.5	33.95	81.0	30.54	90.0
76–End	97.5	12.	11.93	100.6	10.45	114.8
Total,	24,735.0	221.	297.69	74.2	281.35	78.5

The next table, showing the summary of the mortality of the graduates of the Yale Medical School, is extremely interesting and indicates an extremely heavy mortality up to about age sixty-five, after which age the ratios are as favorable as any of those shown in the previous tables. This table is based on 1,239 entrants, 748 of whom are dead, and has a much larger ratio of the dead to the living on account of the longer existence of the school than is the case with the other tables heretofore shown and also from the fact that the ratio of increase in recent graduates is not as great as in the other divisions. The classes of the Yale Medical School until more recent years have been fairly constant in size.

This is the only table of recent years in the Yale Experience which shows an excessive mortality and is the only one which an insurance company could not safely carry as a whole. The mortality for the total ages is relatively high, being a trifle over 100% of the expected by the American Table.

GRADUATES YALE MEDICAL SCHOOL,
1814–1901 INCLUSIVE.

Experience terminated June 30, 1905:

Age Attained.	Years of Exposure.	Actual Deaths.	By American Table. Expected Deaths.	Ratio of Actual.	By the Modified Healthy English Table. Expected Deaths.	Ratio of Actual.
15–25	2371.	22.	18.99	115.9%	17.97	112.4%
26–35	9636.5	106.5	81.87	130.1	79.35	134.2
36–45	8136.	95.	80.96	117.3	77.76	122.2
46–55	6502.5	100.	93.36	107.1	88.28	113.3
56–65	4700.5	144.	130.82	110.1	120.03	120.0
66–75	2697.5	140.5	170.51	82.4	153.15	91.7
76–End	1053.	140.	168.97	82.9	136.14	102.8
Total,	35,097.	748.0	745.49	100.3	672.68	111.2

Research failed to show any peculiar conditions affecting this table, such as heavy war mortality, etc.

The results, however, as a whole verify the mortality found by the Actuarial Society in their Specialized Mortality Investigation for Class 12, composed of physicians.

The following table is taken from that experience :

PHYSICIANS ALL POLICY YEARS.

Ages at Entry.	Actual Deaths.	Expected Deaths.	Ratio Actual to Expected.
15–28	739	738.6	100.1%
29–42	2,859	3,028.6	94.4
43–56	1,702	1,493.8	113.9
57–70	343	277.7	123.5
Total,	5,643	5,538.7	101.9

The mortality on the above table of 101.9% of the expected compares with 111.2% of that found in the Medical School. It should be noted, however, that although the above table is based upon ages at entrance a comparison with the Medical School Table will indicate that the Society's Experience shows a more favorable mortality in the earlier years (where there are results from medical selection) and less favorable in the later years.

The final division of the experience is that of the mortality of graduates of the Academic Department from 1792 to 1849 inclusive, and is composed of 3,877 entrants over 3,660 of whom are dead. It will be noted that the mortality of this division is very heavy up to

about age fifty-five and is more favorable above that age, also that the mortality as a whole compares favorably with that of the American Table, being 86.2 % of the expected deaths.

GRADUATES ACADEMIC DEPARTMENT (YALE COLLEGE), 1792–1849 INCLUSIVE.

Experience terminated June 30, 1905.

Age Attained.	Years of Exposure.	Actual Deaths.	By the American Table. Expected Deaths.	By the American Table. Ratio of Actual.	By the Modified Healthy English Table. Expected Deaths.	By the Modified Healthy English Table. Ratio of Actual.
15–25	17,148	125.5	136.26	92.1%	127.86	98.2%
26–35	34,632.5	413.5	294.15	140.6	285.09	145.0
36–45	31,133	382.0	310.22	123.1	297.94	128.2
46–55	27,272.5	392.0	393.42	99.6	371.91	105.4
56–65	22,913.5	523.0	647.76	80.7	594.17	88.0
66–75	16,459.0	840.0	1055.53	79.6	946.55	88.7
76–End	8,002.5	990.0	1414.22	70.0	1085.28	91.2
Total,	157,561	3,666.0	4,251.56	86.2	3,708.80	98.8

The above table coincides in general with the results found by Mr. Gaines in a summary by him for Vol. IV of the *Yale Annals* based on the mortality of the graduates of Yale from 1701 to 1791, this being a continuation of the subject of his and Mr. Hunter's paper already read before the society.[1]

The summary published in the *Yale Annals* is as follows :

YALE GRADUATES 1701 TO 1791.

Age Attained.	Actual Deaths.	Expected Deaths by American Table.	Ratio Actual to Expected.
15–25	109	73.80	148%
26–35	179	149.80	119
36–45	200	158.67	126
46–55	247	199.82	124
56–65	332	308.42	108
66–75	403	468.11	86
76–End	510	746.09	68
Total,	1,980	2,104.71	94

The heavy mortality shown during the earlier ages under both the above tables is also found in Mr. Strong's adaption of Professor Newton's experience on the mortality of the graduates of the Yale Divinity School from 1825 to 1872.[2]

[1] See Society *Transactions*, Vol. IX, page 47.

[2] See Vol. IX, page 139, of the Society *Transactions*.

In order to eliminate any weakness on account of lack of data in the various tables two combinations have been made, the summaries of which are given below.

It will be noted that the first is composed of the Yale Graduates, graduating principally during the last half of the 19th Century but excluding the graduates of the Yale Medical School, also that the tables in this group have all shown somewhat similar results. The even increase of the ratio of actual to probable deaths in this combination should be noted.

Combination of the Experience.

Academic Department.....1850–1900 inclusive.
Sheffield Scientific School.....1852–1901 "
Yale Law School.....1843–1901 "

Age Attained.	Years of Exposure.	Actual Deaths.	By the American Table. Expected Deaths.	Ratio of Actual.	By the Modified Healthy English Table. Expected Deaths.	Ratio of Actual.
15–25	39,709	163.	316.79	51.45	298.81	54.54
26–35	86,353.5	553.	730.68	75.68	708.28	78.07
36–45	51,151	394.	506.06	77.95	486.09	81.15
46–55	28,983	330.5	408.86	80.83	387.06	85.38
56–65	13,869	309.	375.87	82.20	345.05	89.55
66–75	4,134	205.5	239.42	85.83	215.85	95.20
76–End	244.5	27.	28.92	93.36	25.47	106.00
Total,	224,444.	1,982.	2,606.60	76.04	2,466.61	80.35

Total number of entrants is 11,290.

The second and last table in this paper combines the experience of all the divisions worked up for this paper and is practically the mortality experience of all Yale Graduates from 1792 to 1901.[1]

The high mortality during the earlier ages is due to the effect of the experience of the Graduates of the Academic Department 1792 to 1850 and to the Graduates of the Medical School. This combination is only made as showing the aggregate experience of this paper. It has no immediate practical comparison.

Combination of the Experience.

Academic Department.....1792–1900 inclusive.
Sheffield Scientific School.....1852–1901 "
Yale Law School.....1843–1901 "
Yale Medical School.....1814–1901 "

[1] The principal exception is the graduates of the Yale Divinity School.

Age Attained.	Years of Exposure.	Actual Deaths.	By the American Table. Expected Deaths.	Ratio of Actual.	By the Modified Healthy English Table. Expected Deaths.	Ratio of Actual.
15–25	59,228.	310.5	472.04	65.78%	444.64	69.83 %
26–35	130,622.5	1,073.	1,106.70	96.95	1,072.72	100.03
36–45	90,420.	871.	897.24	97.08	861.79	101.07
46–55	62,758	822.5	895.64	91.83	847.25	97.08
56–65	41,483	976.	1,154.45	84.54	1,059.25	92.14
66–75	23,290.5	1,186.	1,465.46	80.93	1,315.55	90.15
76–End	9,300.	1,157.	1,612.11	71.77	1,246.89	92.79
Total,	417,102.	6,396.	7,603.64	84.12	6,848.09	93.40

Total number of entrants 16,406.

A comparison of the character of the Yale experience as a whole will indicate the regularity with which the tables progress,—that there are no heavy fluctuations from age to age in the columns of exposures or actual deaths. This is perhaps unusual in an experience of relatively so small a number of entrants, but is accounted for by the fact that all entrants remain in the experience until death or the termination of the experience. The tables could for this reason be easily graduated.

A study of the several tables of this Experience, together with that worked up by Mr. Gaines and published in the Yale Annals (4th series), would indicate a general improvement in mortality amongst college graduates during the last two centuries.

A further subdivision of the tables in this paper has therefore been made to show if possible if such improvement in longevity has been continuous. The following tables clearly illustrate the division made. It should be noted that the principal exception to the gradual improvement is shown in the mortality of the graduates from 1850 to 1874 inclusive. Here there is a slight increase in mortality for the table over the mortality as shown for the graduates in the Classes of 1825 to 1849 inclusive. This increase can possibly be accounted for by the excess mortality due to the Civil War—and the considerable increase in the ratio of actual to probable deaths for the group of ages 26 to 35 inclusive would seem to substantiate this point.

RATIO OF ACTUAL TO EXPECTED DEATHS, AMERICAN TABLE
YALE MORTALITY EXPERIENCE BY CERTAIN PERIODS OF GRADUATION.

	1701-1791 Academic Dept.[1]			Academic Dept. 1792-1824 Inclusive.		
Age Attained	Actual Deaths	Expected Deaths	Ratio	Actual Deaths	Expected Deaths	Ratio
15-25	109	73.80	148 %	56.0	65.85	85 %
26-35	179	149.80	119	117.5	129.87	137
36-45	200	158.67	126	175.5	136.42	129
46-55	247	199.82	124	174.5	171.20	102
56-65	332	308.42	108	250.5	279.18	90
66-75	403	468.11	86	376.	440.58	85
75-End	510	746.09	68	466.	660.88	70
Total,	1,980	2,104.71	94	1,676.0	1,883.98	89

	Academic Dept. 1825-1849 Inclusive.			Academic Dept. 1850-1874 Inclusive. Sheffield Scientific School 1852-1874 Incl. Law School 1842-1844 Inclusive.		
15-25	69.5	70.41	99 %	67.5	93.94	72
26-35	236.	164.28	144	256.5	254.19	101
36-45	206.5	173.80	119	240.	277.41	87
46-55	217.5	222.22	98	281.	352.38	80
56-65	272.5	368.58	74	304.5	373.74	81
65-75	464.	614.95	75	205.5	238.46	86
75-End	524.	753.34	70	27.	28.92	93
Total,	1,990	2,367.58	84	1,382.	1,619.04	85

Academic Department 1875-1900 Inclusive.
Sheffield Scientific School 1875-1901 Inclusive.
Yale Law School 1875-1901 Inclusive.

Age Attained	Actual Deaths	Expected Deaths	Ratio
15-25	95.5	222.85	43 %
26-35	296.5	476.49	62
36-45	154.	228.65	67
46-55	49.5	56.48	88
56-65	4.5	2.13	211
66-75		.96	
75-End			
Total,	600.	907.56	61

Annexed to this paper are tables giving the experience which has been summarized therein, for each age group.

In conclusion, the principal points which this experience has suggested to the writer are as follows:

First, that the mortality of Yale Graduates has been constantly improving, that the nineteenth century mortality is better than that of the eighteenth century—that the experience of the last fifty years

[1] From Yale Annals.

of the nineteenth century is better than the first fifty years. This substantiates the results of other experiences, that the mortality of healthy classes is improving.

Second, that this improvement in mortality is especially shown in the younger and middle ages.

Third, that the mortality among physicians is much higher than that of other graduates.

Fourth, that a body of men of comfortable environment will show mortality fully as favorable as that experienced as a whole by any life insurance company.

YALE MORTALITY EXPERIENCE.

ACADEMIC DEPARTMENT—(YALE COLLEGE).

Graduates of Classes 1850-1900 inclusive.

Exposures Continued to June 30, 1905.

Age Attained.	Exposures.	Actual Deaths.	Expected Deaths. American Table.	Expected Deaths. Modified Healthy English Table.
17	1		.01	.01
18	13		.10	.09
19	116		.90	.82
20	603	1.5	4.76	4.32
21	2025.5	5	15.91	14.68
22	4190.5	17.5	33.13	30.68
23	5617.5	23.5	44.70	42.02
24	6324.5	31.5	50.67	48.19
25	6660	34.5	53.71	51.62
26	6760.5	36.5	54.96	53.00
27	6679	37.5	54.75	52.96
28	6474	40	53.50	51.92
29	6205	43	51.78	50.38
30	5930.5	39.5	49.98	48.69
31	5666.5	39	48.22	46.92
32	5369.5	38.5	46.22	44.84
33	5072.5	42.5	44.22	42.87
34	4862.5	40	42.94	41.43
35	4729	31.5	42.31	40.91
36	4432	31	40.28	38.82
37	4245	30.5	39.20	37.74
38	4082.5	26.5	38.41	36.86
39	3930.5	27	37.68	36.08
40	3781.5	27	37.04	35.47
41	3611.5	23.5	36.14	34.67
42	3445	28	35.32	33.93
43	3299.5	32	34.70	33.32
44	3065	33.5	33.19	31.85
45	3014.5	30.5	33.65	32.44
46	2865	26	33.13	31.83
47	2734	25	32.81	31.52
48	2586	27	32.35	31.03
49	2441.5	28	32.00	30.47
50	2314.5	25.5	31.90	30.11

(Academic Department Continued.)

Age Attained.	Exposure.	Actual Deaths.	Expected Deaths. American Table.	Expected Deaths. Modified Healthy English Table.
51	2178	22	31.67	29.71
52	2080	26	32.01	30.01
53	1971.5	25	32.20	30.22
54	1832.5	20	31.88	29.87
55	1717.5	22	31.90	29.82
56	1616	20.5	32.13	29.93
57	1509.5	24	32.21	29.87
58	1407.5	28	32.28	29.81
59	1303.5	30.5	32.22	29.56
60	1196	31	31.92	29.11
61	1135	30	32.78	29.68
62	990	27	30.98	28.22
63	888	25	30.14	27.60
64	822	23	30.31	27.85
65	743	25	29.82	27.34
66	663	31.5	28.98	26.51
67	572.5	25	27.28	24.86
68	482.5	19.5	25.09	22.77
69	416	23	23.61	21.32
70	347	20	21.51	19.31
71	282	14	19.08	17.03
72	210	11.5	15.48	13.79
73	157.5	10	12.63	11.23
74	118.5	7.5	10.31	9.17
75	80	6	7.55	6.72
76	53	4.5	5.42	4.83
77	37.5	3.5	4.16	3.70
78	23	2.5	2.78	2.46
79	16.5	2	2.17	1.91
80	7.5	1.5	1.08	.94
81	3		.48	.40
82	1		.17	.15
83	1	.5	.19	.16
84	.5	.5	.11	.08
Total	158,010.	1,484.	1,903.10	1,798.43

YALE MORTALITY EXPERIENCE.

Graduates of Sheffield Scientific School 1852-1901 inclusive.

Exposures Continued to June 30, 1905.

17	2		.02	.01
18	9		.07	.06
19	109	.5	.85	.77
20	491.5	1	3.84	3.52
21	1227.5	3	9.65	8.90
22	2037.5	8	16.12	14.91
23	2383.5	9.5	18.97	17.83
24	2505	8	20.07	19.09
25	2498	9.5	20.16	19.36
26	2412.5	12.5	19.61	18.91
27	2296	13.5	18.83	18.21

(Sheffield Scientific School Continued.)

Age Attained.	Exposures.	Actual Deaths.	Expected Deaths. American Table.	Expected Deaths. Modified Healthy English Table.
28	2171	12	17.93	17.41
29	2013.5	10.5	16.81	16.35
30	1858	11	15.66	15.25
31	1690.5	8.5	14.39	14.00
32	1538	11	13.24	12.84
33	1393	7	12.15	11.77
34	1290.5	7.5	11.40	11.00
35	1193	11.5	10.68	10.32
36	1108.5	10	10.08	9.71
37	1031.5	5.5	9.52	9.17
38	946	5.5	8.90	8.54
39	882.5	9.5	8.46	8.10
40	818	10.5	8.01	7.67
41	746.5	6	7.47	7.17
42	691.5	4	7.09	6.81
43	645.5	5.5	6.79	6.52
44	603	4	6.53	6.27
45	557	2	6.22	5.99
46	514	2.5	5.94	5.71
47	462.5	4	5.55	5.33
48	427.5	7	5.29	5.07
49	356	6	4.67	4.44
50	322.5	3	4.44	4.20
51	297	4.5	4.33	4.06
52	266	5	4.09	3.84
53	242	3	3.95	3.71
54	224	2.5	3.90	3.65
55	197.5	3.5	3.67	3.43
56	173	2.5	3.44	3.20
57	145.5	1.5	3.10	2.88
58	131	2.5	3.01	2.77
59	110	4.5	2.72	2.49
60	91	4	2.43	2.21
61	78	1	2.25	2.04
62	73	0	2.28	2.08
63	64	.5	2.17	1.99
64	63.5	1.5	2.34	2.15
65	58	1	2.33	2.13
66	53	0	2.32	2.12
67	46	.5	2.19	2.00
68	43.5	1.5	2.26	2.05
69	35	2.5	1.99	1.79
70	26.5	2.5	1.64	1.47
71	20	1	1.35	1.21
72	10	.5	.74	.66
73	7.5	1	.60	.53
74	4.5	.5	.39	.35
75	4		.38	.34
76	2		.20	.18
77	1		.11	.10
78	1		.12	.11
Total	41699.	277.	405.71	386.75

YALE MORTALITY EXPERIENCE.

Graduates of the Law School 1843-1901 inclusive.

Exposures Continued to June 30, 1905.

Age Attained.	Exposures.	Actual Deaths.	Expected Deaths. American Table.	Expected Deaths. Modified Healthy English Table.
18	2		.02	.01
19	12		.09	.09
20	40		.31	.29
21	180	1.5	1.41	1.31
22	352.5	.5	2.79	2.58
23	549	1.5	4.37	4.11
24	768.5	3.5	6.16	5.86
25	991	3	8.00	7.68
26	1110	4.5	9.02	8.70
27	1167.5	7	9.57	9.26
28	1187.5	7	9.81	9.52
29	1172.5	6	9.79	9.52
30	1160.5	5.5	9.78	9.53
31	1108	5	9.43	9.17
32	1046	4	9.01	8.73
33	993	6.5	8.66	8.39
34	931.5	8.5	8.23	7.94
35	872	6	7.80	7.54
36	815	7	7.41	7.14
37	767	8	7.08	6.82
38	706	4.5	6.64	6.38
39	657.5	4	6.31	6.04
40	624.5	3	6.11	5.86
41	583.5	2	5.84	5.60
42	562.5	3.5	5.77	5.54
43	527	3	5.54	5.32
44	502	2.5	5.44	5.22
45	469.5	5	5.24	5.05
46	422.5	5	4.88	4.69
47	395.5	4	4.75	4.56
48	360.5	4.5	4.51	4.33
49	331	3	4.34	4.13
50	305	4.5	4.20	3.97
51	275.5	5	4.00	3.76
52	245.5	4.5	3.78	3.54
53	226	5	3.69	3.46
54	204 .	3.5	3.55	3.33
55	187.5	4	3.48	3.26
56	168.5	4.5	3.35	3.12
57	155	2	3.31	3.07
58	144	.5	3.30	3.05
59	135.5	2	3.35	3.07
60	131.5	2	3.51	3.20
61	121.5	2.5	3.51	3.18
62	114	3	3.57	3.25
63	107	3.5	3.63	3.33
64	102.5	3.5	3.78	3.47
65	92	2.5	3.69	3.39
66	89.5	4	3.91	3.58
67	80.5	3.5	3.84	3.50

(Law School Continued.)

Age Attained.	Exposures.	Actual Deaths.	Expected Deaths. American Table.	Expected Deaths. Modified Healthy English Table.
68	71	3	3.69	3.35
69	61	3.5	3.46	3.13
70	55.5	2.5	3.44	3.09
71	51	2.5	3.45	3.08
72	42.5	2	3.13	2.79
73	37.5	1	3.01	2.67
74	35.5	2.5	3.09	2.75
75	31	3	2.93	2.60
76	28	2.5	2.86	2.55
77	22.5	3	2.50	2.22
78	15.5	2	1.87	1.66
79	11.5	1.5	1.51	1.33
80	7	0	1.01	.87
81	7	1.5	1.11	.94
82	4.5	1	.78	.65
83	1.5	.5	.29	.23
Total	24735.	221.	297.69	281.35

YALE MORTALITY EXPERIENCE.

Graduates Yale Medical School 1814-1901 inclusive.

Exposures Continued to June 30, 1905.

16	1		.01	.01
17	1		.01	.01
18	2		.02	.01
19	6		.05	.04
20	19		.15	.14
21	98		.77	.71
22	288	1	2.28	2.11
23	497	3.5	3.96	3.72
24	657.5	6	5.27	5.01
25	801.5	11.5	6.47	6.21
26	908	14	7.38	7.12
27	962	11.5	7.89	7.63
28	990.5	11	8.18	7.94
29	1006.5	9.5	8.40	8.17
30	1003	10	8.46	8.23
31	992	10.5	8.44	8.21
32	974.5	9	8.39	8.14
33	956.5	10.5	8.34	8.08
34	935	11.5	8.26	7.97
35	908.5	9	8.13	7.86
36	892.5	8.5	8.11	7.82
37	872	9.5	8.05	7.75
38	855.5	9.5	8.05	7.73
39	839	9	8.05	7.70
40	818	10	8.01	7.67
41	798	9.5	7.99	7.66
42	783.5	6.5	8.03	7.72
43	774	8.5	8.14	7.82

(Yale Medical School Continued.)

Age Attained.	Exposures.	Actual Deaths.	Expected Deaths. American Table.	Expected Deaths. Modified Healthy English Table.
44	764.5	13.5	8.28	7.94
45	739	10.5	8.25	7.95
46	726.5	8	8.40	8.07
47	712.5	11.5	8.55	8.22
48	690	10	8.63	8.28
49	673	8	8.82	8.40
50	659	11	9.08	8.57
51	640	13	9.31	8.73
52	621	10	9.56	8.96
53	606	7.5	9.90	9.29
54	594.5	8.5	10.34	9.69
55	580	12.5	10.77	10.07
56	559.5	13	11.13	10.36
57	542.5	11	11.58	10.74
58	528.5	16.5	12.12	11.19
59	505	18.5	12.48	11.45
60	481.5	14.5	12.85	11.72
61	464	15.5	13.40	12.13
62	439.5	15.5	13.75	12.53
63	414	14	14.05	12.87
64	392	13	14.45	13.28
65	374	12.5	15.01	13.76
66	356.5	12.5	15.58	14.26
67	338	13	16.11	14.68
68	319	16.5	16.59	15.05
69	296.5	15.5	16.83	15.19
70	274	11	16.99	15.25
71	259	12	17.53	15.64
72	243	12.5	17.92	15.96
73	229.5	17.5	18.40	16.36
74	203	17	17.67	15.72
75	179	13	16.89	15.04
76	163	14.5	16.68	14.84
77	147.5	14	16.38	14.55
78	130.5	13	15.77	13.95
79	114.5	10	15.08	13.23
80	101.5	12.5	14.66	12.68
81	89	16.5	14.12	11.99
82	70.5	10	12.29	10.24
83	56.5	7.5	10.82	8.84
84	48	9	10.15	8.06
85	37	7.5	8.72	6.67
86	29.5	6	7.84	5.70
87	21.5	4.5	6.51	4.44
88	16	4.5	5.55	3.53
89	11.5	4	4.55	2.71
90	6.5	3	2.95	1.63
91	3.5	1	1.86	.93
92	2.5	1	1.59	.70
93	1.5	.5	1.10	.45
94	1		.86	.31
95	1	.5	1.00	.34
96	.5	.5	.50	.35
Total	35097.	748.	745.49	672.68

YALE MORTALITY EXPERIENCE.

Graduates of Academic Department (The College) 1792-1849 inclusive.

Age Attained.	Exposures.	Actual Deaths.	Expected Deaths. American Table.	Expected Deaths. Modified Healthy English Table.
15	2		.02	.01
16	8		.06	.06
17	48		.37	.34
18	206	1	1.59	1.45
19	727	2.5	5.65	5.17
20	1458.5	4.5	11.39	10.46
21	2214	13	17.40	16.05
22	2754	21	21.78	20.16
23	3046	22.5	24.25	22.78
24	3265.5	25.5	26.16	24.88
25	3419	35.5	27.59	26.50
26	3472.5	40.5	28.23	27.22
27	3539	41.5	29.02	28.06
28	3557.5	40	29.38	28.53
29	3550	41	29.64	28.83
30	3518	42.5	29.66	28.88
31	3483	46	29.64	28.84
32	3439	45	29.61	28.72
33	3397	44.5	29.62	28.70
34	3356	37	29.63	28.59
35	3320.5	35.5	29.72	28.72
36	3284	36.5	29.85	28.77
37	3249.5	33	29.99	28.89
38	3214.5	39.5	30.25	29.03
39	3174	41	30.44	29.14
40	3133	41.5	30.67	29.39
41	3093.5	39.5	30.97	29.70
42	3053	35.5	31.29	30.07
43	3016.5	39	31.73	30.41
44	2972.5	38.5	32.19	30.88
45	2942.5	38	32.84	31.66
46	2900	36	33.52	32.22
47	2865	37.5	34.38	33.03
48	2827.5	37	35.37	33.93
49	2790.5	36.5	36.58	34.83
50	2754	42.5	37.95	35.83
51	2711.5	44.5	39.43	36.98
52	2667.5	45	41.05	38.49
53	2623	38	42.83	40.21
54	2584.5	35.5	44.97	42.13
55	2549	39.5	47.34	44.26
56	2509.5	43.5	49.91	46.48
57	2466.5	50	52.64	48.81
58	2416.5	50	55.43	51.18
59	2361.5	45.5	58.38	53.56
60	2320	48	61.92	56.47
61	2272	53	65.62	59.41
62	2217.5	46.5	69.39	63.20
63	2172.5	53	73.73	67.52
64	2119.5	71	78.15	71.81
65	2058	62.5	82.59	75.73
66	1986	48	86.81	79.42

(Academic Department Continued.)

Age Attained.	Exposures.	Actual Deaths.	Expected Deaths. American Table.	Modified Healthy English Table.
67	1928.5	62	91.89	83.75
68	1865	82.5	96.98	87.01
69	1793	87.5	101.77	91.87
70	1709.5	87	105.97	95.12
71	1619.5	89	109.59	97.82
72	1530.5	88	112.84	100.51
73	1442.5	96	115.66	102.81
74	1346.5	105.5	117.19	104.25
75	1238	94.5	116.83	103.99
76	1137	94.5	116.33	103.55
77	1027	97.5	114.06	101.28
78	912.5	93	110.26	97.54
79	816.5	89	107.56	94.32
80	720.5	77	104.09	90.01
81	642.5	67	101.91	86.54
82	548	62	95.52	79.59
83	471	67	90.22	73.68
84	388	63	82.01	65.16
85	319	53	75.14	57.48
86	250.5	47	66.55	48.44
87	200.	38.5	60.60	41.31
88	154.5	34.5	53.56	34.11
89	117	27	46.32	27.57
90	86	17.5	39.09	21.56
91	66.5	16	35.41	17.66
92	49.5	15	31.40	13.95
93	34.5	11.5	25.33	10.39
94	22	6.5	18.86	6.92
95	14.5	3.5	14.5	4.86
96	11.5	3.5	11.5	4.01
97	8	3.5	8	2.95
98	4	1.5	4	1.58
99	1.5	.5	1.5	.61
100	.5	.5	.5	.21
Total	157561.	3666.	4251.56	3708.80

YALE GRADUATE MORTALITY EXPERIENCE—COMBINED EXPERIENCE.

Academic Department, 1850-1900.
Sheffield Scientific School, 1852-1901.
Yale Law School, 1843-1901.

Academic Department, 1792-1900.
Sheffield Scientific School, 1852-1901.
Yale Law School, 1843-1901.
Yale Medical School, 1814-1901.

Age Attained.	Exposures.	Actual Deaths.	Exposures.	Actual Deaths.
15			2	
16			9	
17	3		52	
18	24		232	1
19	237	.5	970	3
20	1134.5	2.5	2612	7
21	3433	9.5	5745	22.5
22	6580.5	26	9622.5	48
23	8550	34.5	12093	60.5

(Combined Experience Continued.)

Age Attained.	Exposures.	Actual Deaths.	Exposures.	Actual Deaths.
24	9598	43	13521	74.5
25	10149	47	14369.5	94
26	10283	53.5	14663.5	108
27	10142.5	58	14643.5	111
28	9832.5	59	14380.5	110
29	9391	59.5	13947.5	110
30	8949	56	13470	108.5
31	8465	52.5	12940	109
32	7953.5	53.5	12367	107.5
33	7458.5	56	11812	111
34	7084.5	56	11375.5	104.5
35	6794	49	11023	93.5
36	6355.5	48	10532	93
37	6043.5	44	10165	86.5
38	5734.5	36	9804.5	85
39	5470 .	40.5	9483	90.5
40	5224	40.5	9175	92
41	4941.5	31.5	8833	77.5
42	4699	35.5	8535.5	77.5
43	4472	40.5	8262.5	88
44	4170	40	7907	92
45	4041	37.5	7722.5	86
46	3801.5	33.5	7428	77.5
47	3592	33	7169.5	82
48	3374	38.5	6891.5	85.5
49	3128.5	37	6592	81.5
50	2942	33	6355	86.5
51	2751	31.5	6102.5	89
52	2591.5	35.5	5880	90.5
53	2439.5	33	5668.5	78.5
54	2260.5	26	5439.5	70
55	2102.5	29.5	5231.5	81.5
56	1957.5	27.5	5026.5	84
57	1810	27.5	4819	88.5
58	1682.5	31	4627.5	97.5
59	1549	37	4415.5	101
60	1418.5	37	4220	99.5
61	1334.5	33.5	4070.5	102
62	1177	30	3834	92
63	1059	29	3645.5	96
64	988	28 .	3499.5	112
65	893	28.5	3325	103.5
66	805.5	35.5	3148	96
67	699	29	2965.5	104
68	597	24	2781	123
69	. 512	29	2601.5	132
70	429	25	2412.5	123
71	353	17.5	2231.5	118.5
72	262.5	14	2036	114.5
73	202.5	12	.1874.5	125.5
74	158.5	10.5	1708	133
75	115	9	1532	116.5
76	28	3.5	959	102.5
77	61	6.5	1235.5	118
78	39.5	4.5	1082.5	110.5
79	28	3.5	959	102.5

(Combined Experience Continued.)

Age Attained.	Exposures.	Actual Deaths.	Exposures.	Actual Deaths.
80	14.5	1.5	836.5	91
81	10	1.5	741.5	85
82	5.5	1	624	73
83	2.5	1	533	75.5
84	.5	.5	436.5	72.5
85			356	60.5
86			280	53
87			221.5	43
88			170.5	39
89			128.5	31
90			92.5	20.5
91			70	17
92			52	16
93			36	12
94			23	6.5
95			15.5	4
96			12	4
97			8	3.5
98			4	1.5
99			1.5	1
100			.5	.5
Total	224444.	1982	417102.	6396.

(Presented by Mr. M. M. Dawson.)

Mortality Table for Female Beneficiaries in Survivorship Annuities.

BY CHRISTIAN JENSEN.

There does not exist any mortality experience for female beneficiaries in survivorship annuities in this country or in England, I assume, the number of survivorship annuity policies issued being small, while there is quite a large mortality experience for this kind of lives in the Danish State's Life Insurance Institution, "Statsanstalten for Livsforsikring." I have, therefore, tried to construct a mortality table from "Statsanstalten's" mortality experience, hoping that it, combined with Mr. Hunter's Makehamized American Experience, may be suitable for use in the calculation of premiums and reserves for survivorship annuities with a man as insured and a woman as beneficiary, in this country, "Statsanstalten" issues both voluntary and obligatory survivorship annuities, the latter for Government Civil servants. I have only used the experience for voluntary survivorship annuities. The beneficiaries are exclusively Danish women.

Period of observation is 1842–1900.

Only observations of female beneficiaries above 40 years of age are used in the graduation, and in these ages the number of deaths is 4041.

Lives are observed. Several policies simultaneously in force on the same life are counted only as one and the observation lasts from the date the first policy is issued until the last policy for the beneficiary is terminated, as long as she is beneficiary in at least one policy.

The method of observation is the exact age method. A beneficiary is taken under observation from first birthday after entry and observed from birthday to birthday as long as she is beneficiary. In case of death the observation ceases first birthday after death. In case of surrender there is given a paid-up policy, unless the paid-up amount is below a very small fixed sum, and the observation is continued. When no paid-up policy is given in the period 1842–1895, the observation ceases nearest foregoing birthday, while in the period 1895–1900 the observation is considered to cease one-half year after

last birthday. In "Statsanstalten" the number of surrendered without paid-up policies is considered insignificant. The number of such withdrawals in ages above 40 may safely be taken as immaterial, and I have used these ages only in the graduation. I have, therefore, also used the observations 1895–1900 in order to get a larger mass of data though withdrawals by surrender in this period are not dealt with in the same way as in 1842–1895, as stated.[1]

Policies in force on the birthday of the beneficiary in 1900 are considered to terminate then.

For the graduation of the mortality experience I have used Makeham's formula, $-\log p_x = a + bc^x$. Only observations in ages above 40 are used to determine the constants herein. The mortality in 1842–1880 for ages below 41 is a good deal heavier than in 1880–1900, but the mortality for these ages in 1880–1900 agrees well enough with the mortality table based on the constants derived from the observations in the higher ages. I used experimentally the c which was found by Mr. Hunter for Makehamized American Experience and the two other constants were determined according to "Method of Least Squares."[2] The foremost Danish Life Insurance mathematicians have used "Method of Least Squares" in graduation by formula of mortality experiences. It was first used by Professor Opperman, 1870, for the graduation of "Statsanstalten's" mortality tables for insured men and for women.

The formula used by him was:

$$\mu_x = (\alpha + \beta x)e^{-\kappa x} + \gamma e^{+\lambda x}.$$

Here are 5 constants to be determined. e is the basis of the natural logarithms. For women this formula was used for ages 14 and upwards. For ages below 14 was used:

$$\mu_x = \frac{a}{\sqrt{x}} + b + c\sqrt{x}.*$$

It is evident that the observed p_x's are not equally well determined

[1] It may be observed that there are only four .5's in "exposed to risk" in the ages I have used in the graduation.

The number of female beneficiaries is diminishing. It was in 1885—6472; 1890—6251; 1895—5818; 1900—5378.

[2] Boersch und Simon: Abhandlungen Zur Methode der kleinsten Quadrate von Carl Friederich Gauss, Berlin. T. N. Thiele: Theory of Observations, London.

*Tidsskrift for Mathematik—Journal of Mathematics—1884, pag. 113, pp. Copenhagen.

for all ages. Observed p_x is not so well determined when it is derived from few as when derived from many exposed to risk and, therefore in determining the constants it must have larger influence, greater weight, in the latter case than in the former. The weight is in "Method of Least Squares" determined as inversely proportional to the square of the mean error. The square of the mean error is the sum of the squares of the errors each multiplied by its probability. Let, for age x, n denote the number of exposed to risk, t number of deaths which may have all values from o to n and p_x and q_x have their true values. The square of the mean error of observed q_x is then :

$$M_e^2 oq_x = \sum_{t=0}^{t=n} \left(\frac{t}{n} - q_x\right)^2 C_{n.t} p_x^{n-t} q_x^t \; *$$

$$= \sum_{t=0}^{t=n} \left(\frac{t(t-1)+t}{n^2} - 2\frac{tq_x}{n} + q_x^2\right) C_{n.t} p_x^{n-t} q_x^t$$

but

$$\sum_{t=0}^{t=n} \frac{t(t-1)}{n^2} C_{n.t} p_x^{n-t} q_x^t$$

$$= \frac{n(n-1)}{n^2} q_x^2 \sum_{t=2}^{t=n} C_{n-2.t-2} p_x^{n-2-(t-2)} q_x^{t-2} = \frac{n(n-1)}{n^2} q_x^2$$

$$\sum_{t=0}^{t=n} \frac{t}{n^2} C_{n.t} \cdot p_x^{n-t} q_x^t$$

$$= \frac{n}{n^2} q_x \sum_{t=1}^{t=n} C_{n-1.t-1} p_x^{n-1-(t-1)} q_x^{t-1} = \frac{q_x}{n}$$

$$\sum_{t=0}^{t=n} -\frac{2tq_x}{n} C_{n.t} p_x^{n-t} q_x^t = -2q_x^2$$

$$\sum_{t=0}^{t=n} q_x^2 C_{n.t} p_x^{n-t} q_x^t = q_x^2$$

hence $$M_e^2 oq_x = \frac{p_x q_x}{n}.$$

op_x stands for observed p_x.

$$M_e^2 op_x = M_e^2(1 - oq_x) = M_e^2 oq_x$$

log stands for $\log_{10}$

*The symbol $\sum_{t=0}^{t=n}$ represents the sum of the terms from $t=0$ to $t=n$, t being the variable—$C_{n.t} = \frac{n(n-1)\ldots(n-t+1)}{1.2\ldots t}$.

As $$f(p_x + \epsilon) = f(p_x) + \epsilon f'(p_x)$$
$$\log\,(op_x) = \log\,(p_x + \epsilon) = \log p_x + \epsilon \frac{\log e}{p_x}$$

leaving out second and higher powers of ϵ, the error. Hence

$$M_e^2(-\log op_x) = \frac{(\log e)^2}{p_x^2} M_e^2 op_x = (\log e)^2 \frac{q_x}{n \cdot p_x}.$$

As weight for $-\log op_x$ is used

$$w_x = \frac{n \cdot p_x}{q_x}$$

The first set of weights may be calculated from the observations. The graduated p_x's and q_x's may be used in recalculating the weights and a new graduation undertaken until the graduated quantities will be reproduced.

Let ϵ_x now be the error defined by

$$-\log op_x - (-\log p_x) = \epsilon_x$$

In Makeham's formula, $-\log p_x = a + bc^x$, the two constants a and b are to be determined according to "Method of Least Squares" by the two equations which represent the conditions of minimum of

$$\sum_{x=41}^{x=94} w_x \epsilon_x^2 = \sum_{x=41}^{x=94} w_x [-\log op_x - (-\log p_x)]^2$$
$$= \sum_{x=41}^{x=94} w_x [-\log op_x - (a + bc^x)]^2$$

namely $$\sum_{x=41}^{x=94} w_x(-\log op_x) = \sum_{x=41}^{x=94} w_x \cdot a + \sum_{x=41}^{x=94} w_x c^x \cdot b$$
$$\sum_{x=41}^{x=94} w_x c^x(-\log op_x) = \sum_{x=41}^{x=94} w_x c^x \cdot a + \sum_{x=41}^{x=94} w_x c^x c^x \cdot b$$

Or inserting numerical values

$$78865.8 = 14320600\ a + 4475460000\ b$$
$$55913700 = 4475460000\ a + 4389660000000\ b$$

hence $\log b = \overline{5}.0192656$

$a = .0022402$

Assumed $\log c = .04579609$

If the mortality table is to be of any use in this country, the same c as in Makehamized American Experience ought to be used. Beforehand, it was doubtful whether it was justifiable; but the result justifies it, I think.

It may be seen from Table I that for age 41 and upward the signs of the errors vary with sufficient frequency, the errors measured by the corresponding mean errors are not too large and the total sum of the squares of the errors, each one divided by the square of the corresponding mean error, is not larger than can co-exist with a good graduation. The total sum ought to be :

$$54 - 2 \pm \sqrt{2 \cdot (54 - 2)} = 52 \pm \sqrt{104}$$

and it is 57.3. I also calculated the total sum of the squares of the errors of op_x each one divided by the square of the corresponding mean error, and found that to be 57.5. This latter test is the fundamental test of "Method of Least Squares" for the formula used with its constants to represent the observations with sufficient accuracy. Tables II and IIa show that number of actual and expected deaths agree well enough.

In Tables III and IIIa are given actual and expected deaths for 1890–1900.

Tables IVa, b, c show that for ages 20–40, there has been a decrease in the mortality. The mortality in 1842–1880 was much heavier than in 1880–1900. The actual deaths in 1880–1900 agree well enough with the expected deaths and the mortality in this period may be expected to be reproduced approximately in the future. In 1890–1900 the number of actual deaths in these ages is smaller than number of expected deaths probably on account of the small number of observations and selection in first years of insurance. Actual deaths of female beneficiaries in obligatory survivorship annuities in ages 20–40 agree fairly well with expected deaths according to my mortality table in the period 1880–1900 according to the figures in "Statsanstalten's" 6′ Quinquennial Statement. Makeham's formula practically always holds for standard lives above age 30 with sufficient accuracy for practical purposes. The mortality it represents will very often not differ so much from actual mortality down to age 20, that it may not be used in the calculation of premiums and reserves. It seems to be corroborated by the observations 1880–1900 in ages 20–40 that the present table may be used down to age 20 of the beneficiary.

From the formulas :

$$a_{y \mid x} = \Sigma v^t {}_tq_y (1 - {}_tq_x)$$

$$\text{Annual Premium} = \frac{a_{y \mid x}}{1 + a_{xy}}$$

may be seen that the adverse effect of selection during the first years of insurance of the beneficiaries is easily offset by the favorable mortality for the insured except in old ages, and that even a table that represents the expected future mortality only fairly well, may be used. In order to get a safety addition for old ages "Statsanstalten" reduces the age of the beneficiary a little when her age at entry is 70 or above in calculating the premium.

Table V gives a comparison of q_x for female beneficiaries with q_x in Messrs. McClintock's and Finlaison's tables for female annuitants.

Table VI gives a comparison of the relative mortality of women with whole life insurances, women as beneficiaries in voluntary survivorship annuities and women as annuitants in the period 1880–1900 in "Statsanstalten." Herefrom it may be seen that the mortality is heavier for women as beneficiaries in survivorship annuities than for women as annuitants as might be expected on account of selection. That the ratios are the same in ages 50–59 is accidental, of course, on account of the small number of female annuitants in these ages. The ratio of actual to expected deaths for female annuitants in ages 50–59 in the period 1868–1900 is .79 ; in the period 1885–1900 it is .78.

The mortality in my mortality table is heavier than in Mr. McClintock's, as it ought to be, except for ages 53–65, where it is slightly lighter. It may, however, be noticed that in ages 55–64 the number of expected deaths according to Mr. McClintock's table is larger than the number of actual deaths as well according to 15 Am. Co.'s annuity experience, as to British Annuitants, "New Annuities," and there is also in these ages a considerable depression in the ratios of actual deaths of French Annuitants to expected deaths according to Mr. McClintock's table. (*Transactions*, Vol. VI, pages 18, 19, 141, 142.)

The accented letters are the constants in Makehamized American Experience, the letters not accented are the constants in my table.

$$-\log p_{x+t.y+t} = a + a' + (bc^x + b'c^y)c^t$$
$$-\log p_{w+t.w+t} = a + a' + (b + b')c^w \cdot c^t$$

hence

$$c^w = \frac{bc^x + b'c^y}{b + b'}$$

from which formula the table of "uniform seniority" is calculated.

There being no table available of $\log l_x$ according to Makehamized American Experience I calculated it from the constants :

$$\begin{aligned} \log k' &= \quad 5.03370116 \\ \log s' &= - \quad .003296862 \\ \log g' &= - \quad .00013205 \\ \log c &= \quad .04579609 \end{aligned}$$

I used 8 decimal places in calculating $\log p_x$ and left out the eighth decimal place in the result. This $\log p_x$ I used in calculating $\log l_x$ starting with $\log l_{10}$ calculated directly from the constants and left out seventh decimal place in the result. Every tenth $\log l_x$ was also calculated directly from the constants and the discrepancy was less than a unit in sixth decimal place. Log l_x for female beneficiaries was calculated in a similar way starting with $\log l_{10} = 5.000000$. Every calculation was made twice and was checked off by differences and otherwise. $(1 + a)$ is found before sixth decimal place is left out in $\log (1 + a)$.

As $$Pa_{y \mid x} = \frac{1 + a_x - (1 + a_{w.w})}{1 + a_{w.w.}} = \frac{1 + a_x}{1 + a_{ww}} - 1$$

$$\log (1 + Pa_{y \mid x}) = \log (1 + a_x) - \log (1 + a_{ww})$$

it will be easiest and most accurate in calculating the annual premium to interpolate on $\log (1 + a_{ww})$ making a correction for second difference by inspection if necessary, and not on $1 + a_{ww}$.

The reserves will generally be negative only when the age of the beneficiary is considerably higher than the age of the insured.

The condition for nth reserve being positive is, both (x) and (y) being alive,

$$1 + a_{x+n} - 1 + a_{w+n.w+n} - \left(\frac{1 + a_x}{1 + a_{ww}} - 1\right)\left(1 + a_{w+n.w+n}\right) > 0$$

$$\frac{1 + a_{x+n}}{1 + a_{w+n.w+n}} - 1 > \frac{1 + a_x}{1 + a_{ww}} - 1$$

$$Pa_{y+n \mid x+n} > Pa_{y \mid x}$$

$$\log (1 + a_{x+n}) - \log (1 + a_{w+n.w+n}) > \log (1 + a_x) - \log (1 + a_{ww})$$

or $$\log (1 + a_{ww}) - \log (1 + a_{w+n.w+n}) > \log (1 + a_x) - \log (1 + a_{x+n})$$

$$\sum_{t=0}^{t=n-1} - \Delta \log (1 + a_{w+t.w+t}) > \sum_{t=0}^{t=n-1} - \Delta \log (1 + a_{x+t})$$

for $n = 1$

$$-\Delta \log (1 + a_{ww}) > -\Delta \log (1 + a_x)$$

It may, therefore, be seen from the tabulated $-\Delta \log (1 + a_x)$ and $-\Delta \log (1 + a_{ww})$ when there will be a negative reserve.*

In case of negative reserves "Statsanstalten" makes the premium payable in so short a term that the reserves will become positive. "Statsanstalten" does not grant the return premium feature except when the consideration is paid as a single premium.

It is often stated that the continuous instalment policy is a much better policy than the survivorship annuity policy. When the purpose is to secure only a certain person the beneficiary an annual income during the lifetime of same from the death of the insured if the beneficiary be then alive a survivorship annuity policy answers the purpose and is much cheaper than a continuous instalment policy. While Mr. Hunter's Makehamized American Experience may be used as well for insured and beneficiary when the beneficiary is a man, a special table for female beneficiaries combined with Makehamized American Experience ought to be used when beneficiary is a woman.

* If the negative reserves appear only in ages near age 100 they may be left out of consideration.

TABLE I.

Age.	$-\log op_x$	$-\log p_x$	$\epsilon_x = -\log op_x - (-\log p_x)$	$\frac{\epsilon_x^2}{M_e^2(-\log op_x)}$
41	.0034010	.0030289	+.00037	0.71
42	.0028646	.0031166	—.00025	0.32
43	.0037508	.0032141	+.00054	1.43
44	0027221	.0033224	—.00060	1.71
45	.0041154	.0034427	+.00067	2.06
46	.0029272	.0035764	—.00065	1.86
47	.0044925	.0037250	+.00077	2.48
48	.0038933	.0038902	.00000	0.00
49	.0045264	.0040737	+.00045	0.76
50	.0029335	.0042776	—.00134	6.29
51	.0047398	.0045041	+.00024	0.19
52	.0053877	.0047559	+.00063	1.21
53	.0057935	.0050357	+.00076	1.63
54	.0052090	.0053466	—.00014	0.05
55	.0048087	.0056920	—.00088	1.84
56	.0051739	.0060759	—.00090	1.75
57	.0070402	.0065025	+.00054	0.57
58	.0070533	.0069765	+.00008	0.01
59	.0069893	.0075032	—.00051	0.41
60	.0064795	.0080885	—.00161	3.65
61	.0076226	.0087389	—.00112	1.58
62	.0095912	.0094616	+.00013	0.02
63	.0109395	.0102647	+.00067	0.44
64	.0112975	.0111571	+.00014	0.02
65	.0109812	.0121488	—.00117	1.00
66	.0128478	.0132507	—.00040	0.10
67	.0155596	.0144752	+.00108	0.64
68	.0171445	.0158358	+.00131	0.80
69	.0155301	.0173478	—.00182	1.32
70	.0192499	.0190279	+.00022	0.02
71	.0234902	.0208949	+.00260	1.92
72	.0272808	.0229695	+.00431	4.35
73	.0227648	.0252748	—.00251	1.20
74	.0296931	.0278364	+.00186	0.54
75	.0329346	.0306830	+.00225	0.64
76	.0318974	.0338461	—.00195	0.38
77	.0357436	.0373610	—.00162	0.21
78	.0435053	.0412668	+.00224	0.31
79	.0411253	.0456069	—.00448	0.94
80	.0522003	.0504297	+.00177	0.11
81	.0640024	.0557889	+.00821	1.79
82	.0660198	.0617440	+.00428	0.35
83	.0743152	.0683614	+.00595	0.47
84	.0810000	.0757148	+.00529	0.26
85	.0584807	.0838859	—.02541	4.08
86	.0836817	.0929657	—.00928	0.37
87	.0881361	.1030553	—.01492	0.65
88	.0898645	.1142669	—.02440	1.12
89	.1156387	.1267254	—.01109	0.15
90	.1026624	.1405693	—.03791	1.14
91	.1461280	.1559529	—.00982	0.05
92	.2388820	.1730473	+.06583	1.22
93	.2218488	.1920427	+.02981	0.13
94	.2430380	.2131506	+.02989	0.05

Age.	TABLE II. 1842–1900 Exposed to Risk.	Actual Deaths.	Expected Deaths.	Actual less Expected.	TABLE III. 1890–1900 Exposed to Risk.	Actual Deaths.	Expected Deaths.
41	6794.5	53	47.2	+ 5.8	933.5	8	6.5
42	6845	45	49.0	— 4.0	998	6	7.1
43	6861	59	50.6	+ 8.4	1059	12	7.8
44	6882	43	52.4	— 9.4	1103	5	8.4
45	6892	65	54.4	+ 10.6	1151	6	9.1
46	6848	46	56.2	— 10.2	1201	15	9.8
47	6802	70	58.1	+ 11.9	1218	15	10.4
48	6723	60	60.0	0.0	1242	11	11.1
49	6655	69	62.2	+ 6.8	1301	19	12.1
50	6536	44	64.0	— 20.0	1317	10	12.9
51	6449	70	66.6	+ 3.4	1348	13	13.9
52	6326.5	78	68.9	+ 9.1	1374.5	27	15.0
53	6188	82	71.4	+ 10.6	1392	10	16.0
54	6039	72	73.9	— 1.9	1424	14	17.4
55	5903	65	76.9	— 11.9	1468	16	19.1
56	5742	68	79.8	— 11.8	1476	12	20.5
57	5597	90	83.2	+ 6.8	1492	21	22.2
58	5400.5	87	86.1	+ 0.9	1461.5	15	23.3
59	5199	83	89.0	— 6.0	1424	24	24.4
60	4997	74	92.2	— 18.2	1414	30	26.1
61	4828	84	96.2	— 12.2	1392	22	27.7
62	4624	101	99.6	+ 1.4	1375	24	29.6
63	4382	109	102.3	+ 6.7	1345	37	31.4
64	4167	107	105.7	+ 1.3	1337	33	33.9
65	3925	98	108.2	— 10.2	1272	18	35.1
66	3705	108	111.4	— 3.4	1283	37	38.6
67	3495	123	114.6	+ 8.4	1270	51	41.6
68	3281	127	117.5	+ 9.5	1253	49	44.9
69	3074.5	108	120.4	— 12.4	1224.5	48	47.9
70	2860	124	122.6	+ 1.4	1177	48	50.5
71	2640	139	124.0	+ 15.0	1134	69	53.3
72	2398	146	123.5	+ 22.5	1058	66	54.5
73	2154	110	121.7	— 11.7	1009	48	57.0
74	1952	129	121.2	+ 7.8	947	63	58.8
75	1739	127	118.7	+ 8.3	888	70	60.6
76	1511	107	113.3	— 6.3	788	50	59.1
77	1329	105	109.5	— 4.5	723	48	59.6
78	1154	110	104.6	+ 5.4	651	56	59.0
79	974	88	97.1	— 9.1	562	53	56.0
80	830	94	91.0	+ 3.0	481	57	52.7
81	686	94	82.7	+ 11.3	402	56	48.5
82	546	77	72.4	+ 4.6	327	45	43.3
83	426	67	62.0	+ 5.0	258	41	37.6
84	335	57	53.6	+ 3.4	212	35	33.9
85	254	32	44.6	— 12.6	171	17	30.0
86	194	34	37.4	— 3.4	133	19	25.6
87	147	27	31.1	— 4.1	105	18	22.2
88	107	20	24.8	— 4.8	81	15	18.7
89	77	18	19.5	— 1.5	57	13	14.4
90	57	12	15.8	— 3.8	43	9	11.9
91	42	12	12.7	— .7	31	9	9.4
92	26	11	8.5	+ 2.5	18	9	5.9
93	15	6	5.4	+ .6	11	5	3.9
94	7	3	2.7	+ .3	5	1	1.9
95	4	4	1.7	+ 2.3	4	4	1.7

TABLE II a.

1842–1900.

Ages.	Actual Deaths.	Expected Deaths.
41-50	554	554.1
51-60	769	788.0
61-70	1089	1098.5
71-80	1155	1124.6
81-	474	474.9
Total	4041	4040.1

TABLE III a.

1890–1900.

Actual Deaths.	Expected Deaths.
107	95.2
182	197.9
367	381.2
580	570.6
296	308.9
1532	1553.8

TABLE IV a.

1842–1900.

Age.	Exposed to Risk.	Actual Deaths.	Expected Deaths.
20	1352	9	7.2
21	1655.5	15	8.9
22	1991	21	10.7
23	2367.5	21	12.8
24	2786.5	25	15.2
25	3160.5	24	17.3
26	3584	23	19.8
27	4024.5	36	22.4
28	4405.5	34	24.7
29	4770.5	47	27.0
30	5109	51	29.2
31	5383.5	41	31.1
32	5639.5	60	32.9
33	5911	54	35.0
34	6121	62	36.8
35	6257.5	50	38.2
36	6427	60	39.9
37	6539.5	53	41.4
38	6617	47	42.8
39	6674.5	62	44.1
40	6749	64	45.7
		859	583.1

TABLE IV b.

1880–1900.

Exposed to Risk.	Actual Deaths.	Expected Deaths.
182		1.0
228.5	2	1.2
277		1.5
364.5	2	2.0
457.5	5	2.5
543.5	2	3.0
672	5	3.7
796.5	2	4.4
918.5	6	5.1
1045.5	7	5.9
1182	12	6.7
1300.5	5	7.5
1405.5	8	8.2
1552	11	9.2
1650	10	9.9
1744.5	10	10.6
1848	18	11.5
1931.5	10	12.2
2030	8	13.1
2116.5	14	14.0
2223	18	15.1
	155	148.3

TABLE IV c.

1890–1900.

Exposed to Risk.	Actual Deaths.	Expected Deaths.
66		.4
88.5	1	.5
114		.6
141.5		.8
176.5		1.0
213.5		1.2
256	1	1.4
296.5		1.6
341.5	2	1.9
409.5	1	2.3
471	5	2.7
523.5	3	3.0
537.5	1	3.1
599	3	3.5
629	4	3.8
664.5	3	4.1
713	6	4.4
757.5	0	4.8
803	3	5.2
825.5	5	5.5
886	4	6.0
	42	57.8

TABLE V.

Age.	McClintock's Female Annuitants. q_x	Female Benef. in Surv. Ann. q_x	Finlaison's Ultimate. q_x
20	.00384	.00534	
21	.00388	.00536	
22	.00392	.00539	
23	.00396	.00542	
24	.00401	.00545	
25	.00407	.00548	
26	.00412	.00552	
27	.00419	.00556	
28	.00426	.00560	
29	.00434	.00565	

(Table V Continued.)

Age.	McClintock's Female Annuitants. q_x	Female Benef. in Surv. Ann. q_x	Finlaison's Ultimate. q_x
30	.00443	.00571	
31	.00452	.00577	
32	.00462	.05084	
33	.00474	.00592	
34	.00486	.00601	
35	.00500	.00610	
36	.00515	.00621	
37	.00531	.00633	
38	.00549	.00646	
39	.00569	.00661	
40	.00590	.00677	.0108
41	.00614	.00695	.0110
42	.00640	.00715	.0113
43	.00668	.00737	.0115
44	.00700	.00762	.0117
45	.00734	.00790	.0120
46	.00771	.00820	.0122
47	.00812	.00854	.0125
48	.00857	.00892	.0127
49	.00906	.00934	.0130
50	.00960	.00980	.0132
51	.01020	.01032	.0141
52	.01085	.01089	.0151
53	.01156	.01153	.0162
54	.01234	.01224	.0175
55	.01319	.01302	.0181
56	.01412	.01389	.0185
57	.01515	.01486	.0194
58	.01627	.01594	.0204
59	.01750	.01713	.0209
60	.01884	.01845	.0219
61	.02031	.01992	.0233
62	.02193	.02155	.0248
63	.02369	.02336	.0266
64	.02562	.02536	.0292
65	.02773	.02759	.0320
66	.03005	.03005	.0348
67	.03257	.03278	.0375
68	.03534	.03581	.0403
69	.03836	.03916	.0434
70	.04166	.04287	.0467
71	.04527	.04697	.0511
72	.04921	.05151	.0565
73	.05352	.05654	.0629
74	.05821	.06208	.0697
75	.06334	.06821	.0771
76	.06891	.07497	.0843
77	.07500	.08243	.0921
78	.08163	.09065	.0996
79	.08884	.09969	.1085
80	.09668	.10963	.1176
81	.10520	.12055	.1285
82	.11445	.13253	.1391

(Table V Continued.)

Age.	McClintock's Female Annuitants. q_x	Female Benef. in Surv. Ann. q_x	Finlaison's Ultimate. q_x
83	.12448	.14564	.1521
84	.13535	.15999	.1652
85	.14711	.17565	.1810
86	.15983	.19270	.1951
87	.17355	.21124	.2142
88	.18834	.23134	.2319
89	.20425	.25308	.2515
90	.22134	.27651	.2671
91	.23966	.30169	.2920
92	.25924	.32864	.3074
93	.28014	.35738	.3280
94	.30238	.38786	.3435
95	.32598	.42005	.3798
96	.35093	.45383	.4006
97	.37723	.48906	.4456
98	.40485	.52556	.4902
99	.43372	.56306	.6538

TABLE VI.

Relative Mortality of Women with Whole Life Insurance, Women as Beneficiaries in Survivorship Annuities and Women as Annuitants in "Statsanstalten" 1880–1900, according to its 6th Quinquennial Statement. Expected Deaths are calculated according to its old standard mortality table for women.

	Women with Whole Life Ins.			Female Beneficiaries in Survivorship Annuities.			Female Annuitants.		
Ages.	Actual Deaths.	Expected Deaths.	Actual to Expected.	Actual Deaths.	Expected Deaths.	Actual to Expected.	Actual Deaths.	Expected Deaths.	Actual to Expected.
50-59	50	54	.93	346	414.5	.83	81	97.8	.83
60-69	109	114.8	.95	688	796.3	.86	303	433.9	.70
70-79	166	156.6	1.06	945	1051.1	.90	734	937.8	.78
80-	97	103.6	.94	510	562.4	.91	571	669	.85

TABLE VII.

Female Beneficiaries in Survivorship Annuities.

Interest $3\frac{1}{2}$ Per Cent.

Age.	log l_x	log D_x	log N_x
20	4.977093	4.678286	6.029931
21	974767	661020	010163
22	972431	643743	5.990276
23	970084	626456	970268
24	967726	609158	950131
25	965355	591846	929858
26	962968	574519	909443
27	960566	557177	888880

(Table VII Continued.)

Age.	log l_x	log D_x	log N
28	4.958146	4.539816	5.868159
29	955705	522435	847274
30	953243	505033	826215
31	950755	487604	804974
32	948240	470149	783541
33	945695	452664	761905
34	943115	435143	740057
35	940498	417586	717984
36	937839	399986	695674
37	935133	382340	673115
38	932376	364643	650292
39	929561	346887	627191
40	926682	329068	603796
41	923732	311178	580090
42	920703	293208	556055
43	917586	275151	531673
44	914372	256997	506923
45	911050	238734	481782
46	907607	220351	456229
47	904031	201835	430238
48	900306	183169	403781
49	896415	164338	376829
50	892342	145325	349353
51	888064	126106	321319
52	883560	106662	292689
53	878804	086966	263427
54	873768	066989	233490
55	868422	046703	202833
56	862730	026070	171409
57	856654	005054	139163
58	850151	3.983611	106040
59	843175	961694	071977
60	835672	939251	036909
61	827583	916222	000762
62	818844	892542	4.963457
63	809383	868141	924909
64	799118	842936	885024
65	787961	816838	843701
66	775812	789749	800828
67	762561	761558	756285
68	748086	732142	709937
69	732250	701366	661641
70	714903	669079	611237
71	695875	635110	558550
72	674980	599275	503389
73	652010	561364	445543
74	626735	521149	384782
75	598899	478373	320848
76	568216	432749	253462
77	534370	383963	182315
78	497009	331662	107064
79	455742	275454	027330
80	410135	214907	3.942697
81	359706	149538	852703
82	303917	078808	756835
83	242173	002124	654528

(Table VII Continued.)

Age.	log l_x	log D_x	log N_x
84	4.173811	2.918822	3.545152
85	098096	828166	428012
86	014211	729341	302333
87	3.921245	621435	167255
88	818190	503439	021827
89	703923	374232	2.864986
90	577197	232566	695553
91	436628	077056	512219
92	280675	1.906163	313523
93	107628	718175	097840
94	2.915585	511192	1.863369
95	2.702434	283101	608091
96	2.465829	031555	329768
97	2.203159	0.753945	025904
98	1.911527	0.447373	0.693718
99	1.587711	0.108616	0.330109
100	1.228134	$\overline{1}$.734099	
101	0.828817	$\overline{1}$.319842	
102	0.385341	$\overline{2}$.861425	
103	$\overline{1}$.892795	$\overline{2}$.353939	
104	$\overline{1}$.345723	$\overline{3}$.791927	
105	$\overline{2}$.738059	$\overline{3}$.169322	
106	$\overline{2}$.063067	$\overline{4}$.479390	

TABLE VIII.

Female Beneficiaries in Survivorship Annuities.

Interest $3\frac{1}{2}$ Per Cent.

Age.	D_x	N_x	$1 + a_x$
20	47674.5	1071350.2	22.472
21	45816.3	1023675.7	22.343
22	44029.4	977859.4	22.209
23	42311.3	933830.0	22.070
24	40659.1	891518.7	21.927
25	39070.2	850859.6	21.778
26	37542.1	811789.4	21.623
27	36072.6	774247.3	21.464
28	34659.0	738174.7	21.298
29	33299.3	703515.7	21.127
30	31991.4	670216.4	20.950
31	30732.9	638225.0	20.767
32	29522.2	607492.1	20.577
33	28357.2	577969.9	20.382
34	27236.0	549612.7	20.180
35	26156.9	522376.7	19.971
36	25118.1	496219.8	19.756
37	24117.9	471101.7	19.533
38	23154.9	446983.8	19.304
39	22227.3	423828.9	19.068
40	21333.8	401601.6	18.825

(Table VIII Continued.)

Age.	D_x	N_x	$1 + a_x$
41	20472.8	380267.8	18.574
42	19643.0	359795.0	18.317
43	18843.0	340152.0	18.052
44	18071.6	321309.0	17.780
45	17327.4	303237.4	17.500
46	16609.3	285910.0	17.214
47	15916.0	269300.7	16.920
48	15246.5	253384.7	16.619
49	14599.5	238138.2	16.311
50	13974.1	223538.7	15.997
51	13369.2	209564.6	15.675
52	12783.9	196195.4	15.347
53	12217.0	183411.5	15.013
54	11667.8	171194.5	14.672
55	11135.3	159526.7	14.326
56	10618.7	148391.4	13.975
57	10117.1	137772.7	13.618
58	9629.66	127655.58	13.257
59	9155.75	118025.92	12.891
60	8694.63	108870.17	12.522
61	8245.59	100175.54	12.149
62	7808.04	91929.95	11.774
63	7381.44	84121.91	11.396
64	6965.24	76740.47	11.018
65	6559.01	69775.23	10.638
66	6162.39	63216.22	10.258
67	5775.08	57053.83	9.879
68	5396.87	51278.75	9.502
69	5027.66	45881.88	9.126
70	4667.44	40854.22	8.753
71	4316.28	36186.78	8.384
72	3974.43	31870.50	8.019
73	3642.20	27896.07	7.659
74	3320.08	24253.87	7.305
75	3008.66	20933.79	6.958
76	2708.63	17925.13	6.618
77	2420.82	15216.50	6.286
78	2146.16	12795.68	5.962
79	1885.62	10649.52	5.648
80	1640.24	8763.90	5.343
81	1411.04	7123.66	5.049
82	1198.97	5712.62	4.765
83	1004.90	4513.65	4.492
84	829.511	3508.754	4.230
85	673.234	2679.243	3.980
86	536.218	2006.009	3.741
87	418.249	1469.791	3.514
88	318.742	1051.542	3.299
89	236.718	732.800	3.096
90	170.831	496.082	2.904
91	119.414	325.251	2.724
92	80.5681	205.8365	2.555
93	52.2607	125.2684	2.397
94	32.4483	73.0077	2.250
95	19.1912	40.5594	2.113
96	10.7536	21.3682	1.987

(Table VIII Continued.)

Age.	D_x	N_x	$1 + a_x$
97	5.6747	10.6146	1.871
98	2.8014	4.9399	1.763
99	1.2842	2.1385	1.665
100	0.5421	0.8543	
101	.2089	.3122	
102	.0727	.1033	
103	.0226	.0306	
104	.0062	.0080	
105	.0015	.0018	
106	.0003	.0003	

TABLE IX.

Two Joint Lives.

Mortality of Makehamized American Experience and Female Beneficiaries in Survivorship Annuities.

Interest at $3\frac{1}{2}$ per cent.

Age.	log l_x in Makeh. Am. Exp.	log D_{xx}	log N_{xx}
20	4.966676	9.644962	10.921080
21	963258	624278	897452
22	959827	603570	873658
23	956380	582836	849691
24	952917	562075	825539
25	949436	541282	801192
26	945934	520453	776640
27	942409	499586	751870
28	938859	478675	726871
29	935281	457716	701627
30	931672	436705	676125
31	928028	415632	650351
32	924345	394494	624288
33	920619	373283	597918
34	916845	351988	571223
35	913019	330605	544183
36	909134	309120	516779
37	905183	287523	488985
38	901159	265802	460778
39	897055	243942	432132
40	892861	221929	403021
41	888567	199745	373412
42	884162	177370	343277
43	879634	154785	312575
44	874969	131966	281275
45	870152	108886	249335
46	865166	085517	216709
47	859992	061827	183355
48	854609	037778	149219
49	848994	013332	114247

(Table IX Continued.)

Age.	log l_x in Makeh. Am. Exp.	log D_{xx}	log N_{xx}
50	4.843122	8.988447	10.078382
51	836963	963069	041558
52	830485	937147	003707
53	823654	910620	9.964753
54	816430	883419	924614
55	808770	855473	883201
56	800624	826694	840418
57	791938	796992	796159
58	782654	766265	750309
59	772703	734397	702741
60	762013	701264	653320
61	750500	666722	601893
62	738074	630616	548297
63	724632	592773	492350
64	710063	552999	433854
65	694239	511077	372590
66	677023	466772	308321
67	658258	419816	240779
68	637774	369916	169677
69	615378	316744	094691
70	590858	259937	015468
71	563977	199087	8.931618
72	534474	133749	842708
73	502057	063421	748260
74	466401	7.987550	647744
75	427146	905519	540574
76	383893	816642	426100
77	336196	720159	303598
78	283561	615223	172270
79	225439	500893	031228
80	161221	376128	7.879483
81	090227	239765	715937
82	011705	090513	539371
83	3.924816	6.926940	348428
84	828632	747454	141597
85	722118	550284	6.917197
86	604125	333466	673360
87	473376	094811	408001
88	328454	5.831893	118800
89	167781	542013	5.803174
90	2.989607	222173	458240
91	791985	4.869041	080792
92	572751	4.478914	4.667248
93	329504	4.047679	4.213624
94	059571	3.570763	3.715469
95	1.759986	3.043087	3.167822
96	1.427451	2.459006	2.565155
97	1.058302	1.812247	1.901289
98	0.648466	1.095839	1.169339
99	0.193419	0.302035	0.361615
100	$\bar{1}$.688132	$\bar{1}$.422231	
101	$\bar{1}$.127020	$\bar{2}$.446862	
102	$\bar{2}$.503873	$\bar{3}$.365298	
103	$\bar{3}$.811792	$\bar{4}$.165731	

TABLE X.

Two Joint Lives.

Mortality of Makehamized American Experience and Female Beneficiaries in Survivorship Annuities.

Interest at $3\frac{1}{2}$ per cent.

Age.	D_{xx}	N_{xx}	$1 + a_{xx}$
20	4415320000	83383410000	18.885
21	4209960000	78968090000	18.757
22	4013930000	74758130000	18.625
23	3826800000	70744200000	18.487
24	3648170000	66917400000	18.343
25	3477620000	63269230000	18.193
26	3314770000	59791610000	18.038
27	3159260000	56476840000	17.877
28	3010750000	53317580000	17.709
29	2868900000	50306830000	17.535
30	2733410000	47437930000	17.355
31	2603950000	44704520000	17.168
32	2480240000	42100570000	16.974
33	2362020000	39620330000	16.774
34	2248990000	37258310000	16.567
35	2140940000	35009320000	16.352
36	2037610000	32868380000	16.131
37	1938760000	30830770000	15.902
38	1844170000	28892010000	15.667
39	1753650000	27047840000	15.424
40	1666970000	25294190000	15.174
41	1583960000	23627220000	14.917
42	1504420000	22043260000	14.652
43	1428190000	20538840000	14.381
44	1355080000	19110650000	14.103
45	1284950000	17755570000	13.818
46	1217630000	16470620000	13.527
47	1152990000	15252990000	13.229
48	1090880000	14100000000	12.925
49	1031170000	13009120000	12.616
50	973749000	11977947000	12.301
51	918479000	11004198000	11.981
52	865261000	10085719000	11.656
53	813992000	9220458000	11.327
54	764573000	8406466000	10.995
55	716924000	7641893000	10.659
56	670956000	6924969000	10.321
57	626602000	6254013000	9.981
58	583801000	5627411000	9.639
59	542497000	5043610000	9.297
60	502648000	4501113000	8.955
61	464218000	3998465000	8.613
62	427185000	3534247000	8.273
63	391537000	3107062000	7.936
64	357272000	2715525000	7.601
65	324397000	2358253000	7.270
66	292935000	2033856000	6.943
67	262915000	1740921000	6.622
68	234378000	1478006000	6.306
69	207369000	1243628000	5.997
70	181944000	1036259000	5.695

(Table X Continued)

Age.	D_{xx}	N_{xx}	$1 + a_{xx}$
71	158156000	854315000	5.402
72	136066000	696159000	5.116
73	115723000	560093000	4.840
74	97174000	444369500	4.573
75	80448700	347195500	4.316
76	65560500	266746800	4.069
77	52500000	201186300	3.832
78	41230900	148686300	3.606
79	31687900	107455400	3.391
80	23775400	75767500	3.187
81	17368600	51992100	2.993
82	12317200	34623500	2.811
83	8451620	22306290	2.639
84	5590540	13854670	2.478
85	3550450	8264130	2.328
86	2155090	4713680	2.187
87	1243970	2558590	2.057
88	679036	1314621	1.936
89	348348	635585	1.825
90	166791	287237	1.722
91	73967.5	120445.6	1.628
92	30124.1	46478.1	1.543
93	11160.4	16354.0	1.465
94	3721.89	5193.60	1.395
95	1104.30	1471.71	1.333
96	287.744	367.413	1.277
97	64.9003	79.6689	1.228
98	12.4692	14.7686	1.184
99	2.0046	2.2994	1.147
100	0.2644	0.2948	
101	.0280	.0304	
102	.0023	.0024	
103	.0001	.0001	

TABLE XI.

Number of Years to be added to age of Younger of Two Joint Lives to obtain equivalent Equal Ages.

Difference in Age.	Beneficiary the Younger.	Insured the Younger.
1	0.597	0.429
2	1.219	0.883
3	1.865	1.364
4	2.535	1.872
5	3.228	2.405
6	3.943	2.965
7	4.679	3.550
8	5.435	4.161
9	6.210	4.797
10	7.003	5.457
11	7.812	6.139
12	8.638	6.845
13	9.478	7.571
14	10.331	8.319
15	11.197	9.085

(Table XI Continued.)

Difference in Age.	Beneficiary the Younger.	Insured the Younger.
16	12.076	9.870
17	12.964	10.672
18	13.863	11.491
19	14.771	12.324
20	15.688	13.172
21	16.612	14.032
22	17.543	14.905
23	18.481	15.789
24	19.425	16.684
25	20.374	17.588
26	21.328	18.501
27	22.286	19.422
28	23.248	20.350
29	24.214	21.285
30	25.183	22.226
31	26.156	23.173
32	27.130	24.124
33	28.108	25.081
34	29.087	26.041
35	30.069	27.006
36	31.052	27.973
37	32.038	28.944
38	33.024	29.918
39	34.012	30.894
40	35.001	31.873

TABLE XII.

Logarithms of Annuities on One Life based on mortality of Female Beneficiaries in Survivorship Annuities and logarithms of Annuities on Two Joint Lives based on mortality of Makehamized American Experience and Female Beneficiaries in Survivorship Annuities. Interest $3\frac{1}{2}$ per cent.

Age.	$\log (1 + a_x)$	$-\Delta \log (1 + a_x)$	$\log (1 + a_{xx})$	$-\Delta \log (1 + a_{xx})$
20	1.35165	.00251	1.27612	.00295
21	34914	.00261	27317	.00308
22	34653	.00272	27009	.00323
23	34381	.00284	26686	.00340
24	34097	.00296	26346	.00355
25	33801	.00309	25991	.00372
26	33492	.00322	25619	.00391
27	33170	.00336	25228	.00408
28	32834	.00350	24820	.00429
29	32484	.00366	24391	.00449
30	32118	.00381	23942	.00470
31	31737	.00398	23472	.00493
32	31339	.00415	22979	.00515
33	30924	.00433	22464	.00540
34	30491	.00451	21924	.00566
35	30040	.00471	21358	.00592
36	29569	.00491	20766	.00620
37	29078	.00513	20146	.00648
38	28565	.00535	19498	.00679
39	28030	.00557	18819	.00710
40	27473	.00582	18109	.00742

(Table XII Continued.)

Age.	$\log (1 + a_x) - \Delta \log (1 + a_x)$		$\log (1 + a_{xx}) - \Delta \log (1 + a_{xx})$	
41	1.26891	.00606	1.17367	.00776
42	26285	.00633	16591	.00812
43	25652	.00659	15779	.00848
44	24993	.00688	14931	.00886
45	24305	.00717	14045	.00926
46	23588	.00748	13119	.00966
47	22840	.00779	12153	.01009
48	22061	.00812	11144	.01052
49	21249	.00846	10092	.01098
50	20403	.00882	08994	.01145
51	19521	.00918	07849	.01193
52	18603	.00957	06656	.01243
53	17646	.00996	05413	.01293
54	16650	.01037	04120	.01347
55	15613	.01079	02773	.01401
56	14534	.01123	01372	.01455
57	13411	.01168	0.99917	.01513
58	12243	.01215	98404	.01570
59	11028	.01262	96834	.01628
60	09766	.01312	95206	.01689
61	08454	.01362	93517	.01749
62	07092	.01415	91768	.01810
63	05677	.01468	89958	.01872
64	04209	.01523	88086	.01935
65	02686	.01578	86151	.01996
66	01108	.01635	84155	.02059
67	0.99473	.01693	82096	.02120
68	97780	.01752	79976	.02181
69	96028	.01812	77795	.02242
70	94216	.01872	75553	.02300
71	92344	.01933	73253	.02357
72	90411	.01993	70896	.02412
73	88418	.02055	68484	.02465
74	86363	.02115	66019	.02513
75	84248	.02177	63506	.02560
76	82071	.02236	60946	.02602
77	79835	.02295	58344	.02639
78	77540	.02352	55705	.02671
79	75188	.02409	53034	.02698
80	72779	.02462	50336	.02719
81	70317	.02514	47617	.02731
82	67803	.02563	44886	.02737
83	65240	.02607	42149	.02735
84	62633	.02648	39414	.02723
85	59985	.02686	36691	.02702
86	57299	.02717	33989	.02670
87	54582	.02743	31319	.02628
88	51839	.02764	28691	.02575
89	49075	.02776	26116	.02509
90	46299	.02783	23607	.02432
91	43516	.02780	21175	.02342
92	40736	.02769	18833	.02238
93	37967	.02749	16595	.02124
94	35218	.02719	14471	.01997
95	32499	.02678	12474	.01859
96	29821	.02625	10615	.01711
97	27196	.02561	08904	.01554
98	24635	.02486	07350	.01392
99	22149		05958	

Staff Pension Funds with Special Reference to a Retirement Plan for United States Civil Service Employees.

BENEDICT D. FLYNN.

American actuaries have given but little attention to the subject of old age pension plans, chiefly for the reason that the idea of providing for the aged poor of the state, and for the old and faithful employee has not yet seriously taken hold of the American people. It is true that many of the great railway systems of the United States and Canada, and a few of the large banks and commercial houses, have established funds for the relief of their old employees, but when one considers the large number of institutions throughout the country in which this idea could be put into effect, the number of such funds existing is seen to be comparatively small. There are signs, however, that the subject is beginning to occupy public attention. The appointment recently by the Governor of Massachusetts of a commission to investigate and consider the various plans of old age pensions, with a view to establishing an old age insurance system in the Commonwealth, and the interest which has been taken in a retirement scheme recently proposed for the United States Civil Service employees, indicate that the time is not far distant when the question of National Old Age Pensions will be fully discussed, and the justice and advantages of staff pensions will be more clearly understood.

The part which the actuary must take as this idea of old age provision develops, is shown by a study of the history of old age pensions in foreign countries to be both important and difficult. Actuarial advice will undoubtedly be sought in attempts to obtain a satisfactory solution to the problem of National Old Age Pensions, but it is in the valuation and readjusting of existing staff pension funds and in the establishing of new retirement plans that the services of the actuaries of this country will be more often required. It is advisable, therefore, that more attention be given to this branch of the subject in preparation for the responsibilities

which may later fall upon the members of this Society. Recognizing the necessity for this, the Society at its last meeting added to its requirements for admission as a Fellow, a knowledge of the methods of construction and of valuation of pension funds. The subject is important and most interesting and it is with the purpose of bringing it before the Society for consideration and discussion that this paper is submitted.

But few articles have been written on the technical side of the subject of Staff Pension Funds from an actuarial standpoint, for the reason, probably, that one can become an authority upon the subject only after years of thought and practical experience, and also that the few papers written have covered the subject ably and thoroughly. Mr. Henry W. Manly, in his masterly work "On the Valuation of Staff Pension Funds" (*J. I. A.*, Vol. 36, page 209; Vol. 37, page 193) first placed the study of Staff Pension Funds upon a sound and scientific basis. Prior to the appearance of this paper, although some of the methods described by him had been used by other actuaries, nothing had appeared in print and most of the problems met in the valuation of these funds had been solved by methods of approximation. In a previous paper read before the Second International Congress of Actuaries (*Trans. of Sec. Int. Cong.*, page 860) this gentlemen had given solutions for many of the simpler problems which arise in connection with these funds, but in this later work he takes up those of a more intricate and practical nature and gives solutions which are mathematically exact. Nothing else of particular value appeared upon the subject, with the exception of an article by Mr. David Carment, "Practical Notes on the Valuation of Pension Funds" (*Trans. of Fac. of Act.*, Vol. 1, page 305), until Mr. George King contributed a paper on "Staff Pension Funds" (*J. I. A.*, Vol. 29, page 129) which has often been referred to as a text-book upon the subject. Although covering much the same ground as Mr. Manly, Mr. King treats the subject from a more practical standpoint and introduces some minor changes and improvements in the methods. These papers, with remarks and discussions upon them, if carefully read by the student, should give a good idea of the technical construction and of the methods of valuation of these funds.

A brief outline of the methods employed by these actuaries in the treatment of this problem may be of interest. The best way to

gain an idea of the general construction of a staff pension fund is to take up, as does Mr. King, the valuation of an existing fund. As the statistics of one fund cannot be safely used for another, the first step is to collect data from which to obtain rates of withdrawal, mortality and superannuation of members on the active list. From these statistics a service table is then made, which differs from a mortality table only in that there are three columns of decrements,—Withdrawals, Deaths and Pensioned—instead of one. The average salary of the members at each age is also obtained and adjusted to represent the probable future average salaries of the fund at each age, if it has not yet reached a stationary state, and the result used as a model scale of salary or column from which the rate of increase of salary can be obtained. The valuation of the future contributions and the various benefits of the fund is made by means of factors, which in turn are obtained by the use of various sets of commutation columns. The factor which represents the value of the contribution of 1 per cent. of future salaries, for instance, by an employee at age x, present salary \$100 and increasing according to scale, takes the form in somewhat similar notation of $\frac{N_x}{D_x}$. The commutation symbol in the numerator of this expression is based upon the l_x column of the service table, which is subject to the three decrements before mentioned, and also upon the salary scale which is introduced at each age. As salaries are assumed to be payable uniformly throughout the year of age, and the withdrawals, deaths and retirements to be distributed in a similar manner, the numerator of this expression and of those which follow is a continuous function. The denominators in every case are based upon the deaths, but in factors for the valuation of benefits depending upon future contributions, this function is modified slightly for ease in calculation.

In making the valuation of an existing fund, as the past contributions are known, the method followed is to consider separately the value of benefits based upon past and upon future contributions. Taking up the valuation of the simpler benefits, such as the return upon withdrawal of total contributions without interest, we have in respect to past contributions the total of such payments multiplied by a factor of the form $\frac{M_x'}{D_x}$. The numerator of this expression is based upon the withdrawals just as the regular M_x is based upon

the column of deaths. Similarly, the factor to be used to obtain the value at age x of the return upon withdrawal of 1 per cent. of future salaries, present salary \$100 and increasing according to scale, is similar to $\frac{R_x}{D_x}$. The numerator of this expression is formed by using the column of withdrawals and also considering the salary scale. The factor for return upon death before pensioned is calculated upon the same principle for past and future contributions as those upon withdrawal, the difference being simply that the deaths are used instead of the number of withdrawals. The special forms of return upon death, such as total contributions with compound interest at valuation rate, with interest other than at valuation rate, and at simple interest, are obtained by employing special commutation columns to obtain the valuation factors.

Taking up next the valuation of the pensions, a table of continuous annuities applicable to the fund under observation must be prepared. The annuity values obtained from the experience of the fund can be used for the early ages, but in the later ages it is safer to employ some other table. Considering the simplest form of pension benefit, that based upon average salary from commencement of membership, upon retirement at any time, we have in respect to past contributions a factor of the form $\frac{M_x}{D_x}$, and for those of the future $\frac{R_x}{D_x}$. The numerators of each of these expressions are based upon the column of retirements and in the case of the factor for use in future contributions the salary scale is also introduced. This last factor gives the value of a pension of 1 per cent. of average salary for each year of membership in respect of a salary of \$100, increasing according to scale. In practice, many variations in this benefit are met, such as pensions based upon terminal salaries or upon varying percentages not directly based upon the number of years of service, and these are solved by obtaining factors based upon new commutation columns. It can be seen, therefore, that although many of the problems arising from the rules of funds met in practice are very complicated, their solution is much simplified by this extended use of commutation columns.

Although the American actuary will be called upon to value existing funds, it will be chiefly in the inauguration of new

pension systems that his services will be required. It is therefore important to investigate the principal methods of construction of Staff Pension Funds in order to determine which plan is best adapted to certain conditions and requirements.

From the above general outline, a fair idea of the elaborate structure of most of the funds in existence in European countries and the United States can be obtained. In the papers previously mentioned and throughout the remarks upon them, reference is often made to the practical difficulties encountered in establishing a fund along these lines and the danger of weakness resulting from the unreliable nature of the materials upon which this framework is based. I quote from the remarks of Mr. G. J. Lidstone (*J. I. A.*, Vol. 36, page 279) upon the latter phase of the subject.

"It must be remembered that, however much time and care might be taken in erecting an elegant and refined edifice, it could never have any more stability than the materials upon which it was founded. In the present case the materials were not only uncertain, but were often actually shifting, and not the same from time to time. Considering first of all the rate of withdrawal, which was assumed in the formula to be known and stationary, obviously the rate might vary within wide limits at different periods, owing to the trade conditions and a number of other circumstances, and therefore the experience of the fund might not be a safeguard in the future. An even greater difficulty arose when dealing with a new fund. One had then to utilize the experience of withdrawals before the fund was formed, but it was one of the principal objects of those funds—certainly one of their effects—to steady the service and prevent withdrawals to a certain extent, for a man belonging to such a fund thought twice before shifting his employment and thus sacrificing years of pension service. It might therefore be that the rate of withdrawal deduced from the experience amongst the officials before the fund commenced would be quite unreliable for the valuation of the fund when it was once in existence."

What Mr. Lidstone has said in regard to the great difficulty of obtaining a true measure of the rate of withdrawal applies, to a large extent, to the rates of mortality and retirement involved. And the salary scale, which is probably of greater importance in determining the amount of contribution than any other element,

is most difficult to determine with reasonable accuracy. The presence of high salaried officials in the older ages of the staff and the probability of change in the methods of advancement, call for the greatest care and judgment in the adjustment of the salary scale. Even when the work is completed in the most skillful manner, there is a grave question as to whether the result gives a fair estimate of the rate of increase in salaries to be experienced in the future.

The fact that these assumptions with regard to the rates of withdrawal, retirement and salary increase cannot be made with accuracy, however, would not of itself be of great moment, provided the errors of judgment did not place the fund in an unsafe condition, if it were not for the fact that in such a fund individuals or certain classes are not treated equitably. This point is set forth ably in remarks by Mr. William Sutton before a select Committee of the House of Commons in 1891 as follows:

"It may be said that, with a few exceptions, Superannuation Funds as generally constituted are radically wrong in principle when looked at from the actuarial point of view. Instead of resting content with the introduction of as few assumptions as possible, they are made to involve not only assumptions as to the rates of mortality to be experienced among the members and as to the rate of interest to be earned by the accumulated funds—these may be fairly said to be indispensable—but they are also made to depend upon such capricious elements as the rates of secession of members (that is, of members leaving active service otherwise than by death or retirement) and the rates of salary the members will receive, and on which the nature and amount of their contributions to the Fund will depend, as well as the amount of the pension they will receive. It thus follows that in bringing into the question rates of secession and rates of salary, matters which cannot be prognosticated with any certainty for any length of time, classes of members get lumped together whose real circumstances and conditions in respect of these matters are as different as possible."

The percentage contribution of a member to a fund formed on these lines can be looked upon as a rate in determining which the savings from various sources, such as deaths or withdrawals, have been anticipated and which has been based upon the salary which the member will receive in future. If, therefore, the experience on certain classes differs greatly from the assumptions made, especially

in regard to such uncertain and important elements as rates of withdrawals and salary increase, the result must be inequitable treatment of the members. It might be possible, in theory, to improve this situation by dividing the whole membership into homogeneous groups and to modify the rates for each. The task of obtaining standards of measurement for the various groups which would be reasonably safe indices of future results would be, however, too great for practical solution.

Mr. Manly in his article upon Staff Pension Funds states that he endeavors to persuade the employer to guarantee a certain rate of pension based upon years of service and average salary which is to be understood to be a reward for faithful service,—that is, that the institution create and support its own pension fund. In cases such as this, where the employer contributes the whole of the cost of the pension scheme, the use of such a fund as outlined above is permissible, for the reason that the right of the employer to give to some larger pensions than to others cannot be denied. Where part of the cost is contributed by the employer and the rest by the employees, however, although the latter cannot complain if they receive ample return for their own contributions, it will probably save much annoyance and discontent if some other method be adopted. In case the pension system is based entirely upon the contributions of the members, some other plan would seem to be necessary.

Mr. King states (*J. I. A.*, Vol. 37, page 44) that wherever possible he has recommended a system of deferred annuities combined with whatever other benefits are desired, the annuity to be purchased by a percentage deduction from the salary. In case of increase in salary the pension payable is obtained by entering the table of annuity values at the age attained, the same percentage of the increase in salary being used. This method is undoubtedly an improvement upon the other plan, in that it removes the troublesome element of rate of increase in future salaries, but the assumption with regard to the rate of withdrawal remains.

Before going farther with the question of the proper plan to use in the organization of a pension system, the demands of the employees as shown in the rules of these plans should be considered. It is probably because of human nature that practically all pension schemes which depend in any degree upon compulsory contribu-

tions from members, contain in their rules the privilege of return of the whole or part of the contributions with or without interest in case of withdrawal or death before age of retirement is reached. Mr. Manly states this point clearly in his remarks (*J. I. A.*, Vol. 37, page 43) :—

"In nearly all the schemes the men try to get all their money back somehow, as well as a pension; and he had before him now a case where they wanted their money back with compound interest and to have a good pension as well."

In view of the fact, therefore, that the rules of pension schemes require the return of the whole or the greater part of the contributions of the members, and oftentimes with interest, the question which naturally arises is what necessity there may be for introducing the elements of mortality and withdrawal and of the erection of this elaborate statistical structure. Why not eliminate these assumptions entirely in so far as active members are concerned and simply accumulate the contributions at compound interest? This Savings Bank idea, although referred to at various times throughout the discussion of Staff Pension Funds, has never been given the consideration that would seem to be its due.

In cases where the rules of the plan call for return of contributions with tabular interest upon withdrawal, early retirement or death, the deposit required upon this plan would be exactly correct. Where the rules allow simply return of contributions without interest or at a rate of interest lower than the tabular for any of the above modes of retirement, the rates upon this plan will be larger than if profits were anticipated and a surplus account formed, its size being measured by the degree of liberality shown to the retiring members. Some idea of the difference in rates of a plan in which the contributions are simply accumulated at compound interest and of one in which the savings are anticipated can be gained from the following table. The figures for Columns 1 and 2 are taken from the tables illustrating Problems 7a and 9a of Mr. H. W. Manly's paper "On the Valuation of Staff Pension Funds" (*J. I. A.*, Vol. 36, pages 224 and 227).

The basis of these figures is Mr. Manly's "Hypothetical Pension Fund Experience Table, No. 4," which takes into consideration rates of death, withdrawal and early retirement. The rate of interest used is 4 per cent.

ANNUAL PREMIUMS TO PROVIDE AN ANNUITY OF $65 - x$ AT THE AGE OF 65 WITH THE CONDITION THAT THE WHOLE OF PREMIUMS PAID BE RETURNED UPON DEATH, WITHDRAWAL OR RETIREMENT BEFORE REACHING AGE 65.

Age x	COLUMN 1. Without Interest	COLUMN 2. With int. at tabular rate (4%) in case of death or retirement and without interest upon withdrawal	COLUMN 3. With interest at tabular rate (4% (Savings Bank Plan)
15	.971	1.750	2.531
20	1.278	2.331	2.873
25	1.632	2.905	3.253
30	2.054	3.474	3.672
35	2.563	4.036	4.134
40	3.181	4.603	4.639
45	3.927	5.180	5.190
50	4.818	5.777	5.789
55	5.849	6.428	6.437
60	6.942	7.120	7.134

The rates in Column 2, which will probably apply more nearly to the rules of the Compulsory Staff Pension Fund of to-day, can be seen to be but slightly smaller than the compound interest deposit in Column 3 at an average age of entry of about thirty. The slightly larger premium of the compound interest plan insures solvency and it should not be a difficult task to devise an equitable method of apportioning at certain intervals the profits which will accrue, basing the computations upon the experience of the members by classes. The results under such a plan should be more satisfactory than by a method which in its rates anticipates the gains of the fund by making use of a single set of assumptions more or less inaccurate.

It is in the ease with which a plan based upon the Savings Bank idea can be started and operated, however, that its chief advantage lies. An account can be kept for each member and the proper return upon death or withdrawal or the amount of pension upon retirement can be determined with accuracy. Another advantage which this plan possesses is in the case of change in the rules of the plan. Mr. Manly states that in his experience rules were changed about every five years, and oftentimes without the knowledge and advice of the actuary who made the original calculations. Funds which start with simple benefits very often assume obligations of a more costly nature without a corresponding increase being made in the contributions required. In cases where such changes have been made and actuarial advice either ignored or not

sought at all, the funds have become insolvent with consequent loss to members. The result of increasing the benefits to the members under the Savings Bank Plan would simply be to cut down the gains to the fund and to reduce the surplus to be divided among the members—a simple adjustment compared with the situation in a fund built upon assumptions. It can be said, therefore, if the rules of a plan require the return of contributions with interest, as will most plans of to-day which make payments by members compulsory, that the use of the Savings Bank plan as outlined above has many advantages and that, even in cases where the benefits to members are more restricted, it will prove a safe and desirable method for starting and carrying the scheme until a reliable experience can be obtained.

The value and adaptability of this method can best be shown in detail by considering a plan which has been proposed recently for the retirement of the employees of the Classified Branch of the United States Civil Service. This scheme was devised by Mr. Herbert D. Brown, formerly a Special Examiner for the Bureau of Corporations of the Department of Commerce and Labor, to fit the conditions peculiar to the United States Government Service, and the writer was later consulted especially in regard to the actuarial problems involved.

The Classified Civil Service of the United States is made up of about 150,000 employees in many different lines of work, situated in various parts of the country and subject to widely varying conditions. Railway Postal Clerks, for instance, although required to pass a rigid physical examination upon entrance, are subjected to special hazards and conditions incident to their work, which place them in a class by themselves. Various other branches such as City Letter Carriers, because of the requirements of the work, experience rates of mortality and withdrawal differing greatly from the average for the whole service. Again, about 8 per cent. of the whole number of employees are females—a most disturbing element in a fund and one which is most difficult to deal with. It can be seen, therefore, that the establishment of a fund in which the contributions and benefits of these widely differing classes are figured upon the same basis with regard to rates of mortality, withdrawal and increase of salary would be manifestly unwise and unjust. It should be stated that, because of public sentiment

against civil pension lists, it was necessary that the scheme be supported almost entirely by the contributions of members.

The foundation of the proposed retirement plan is a compulsory savings account for each government employee, to provide for himself in old age or in event of physical incapacity prior to date of retirement. A certain percentage of the monthly wage of each person in the classified branch of the Civil Service, sufficient to provide a fund upon retirement, of 1½ per cent. of his annual salary for every full year of service, shall be withheld and deposited in the United States Treasury at 4 per cent. compound interest per annum, in the name of and to the credit of the employee. Upon absolute separation from the Service before reaching the age of retirement, and only in such event, the employee shall have the privilege of withdrawing his accumulated savings in one sum, or, if the amount of the fund to his credit be at least $1000, he shall have the option of using his savings to provide an annuity at his attained age.

In case of the death of an employee while in the service, the amount to his credit shall be paid to his estate. Upon attaining the age of retirement, the employee may withdraw his savings in one of the following methods.

Option I.	In one sum.
Option II.	An annuity payable quarterly throughout life.
Option III.	An annuity payable quarterly throughout life, with the provision that in case of the death of the annuitant before he has received in annuities the amount of his savings, the balance shall be paid to his estate plus interest credited thereon.
Option IV.	An income payable certain for a limited term of years.

The proper age for compulsory retirement in the Civil Service differs according to the particular division which is under observation. There are provided, therefore, three groups into which various branches of the service are to be divided by the President of the United States, the age for Group (I) being sixty years, for Group (II) sixty-five years and for Group (III) seventy years.

As it is often to the advantage of the service to retain an old

employee because of his expert knowledge, it is provided in the rules that in such a case the employee may be retained, if he be willing, after the age of retirement, for two years and for successive periods of two years each, as long as, in the opinion of the head of the department, he continues efficient in the work upon which he is engaged.

In order that the object of the scheme may not be defeated in the case of old employees who will not be able in the years remaining before retirement to save enough to provide a proper pension, it is necessary for the government to supplement the savings of these old employees, in order that they may be retired upon a comfortable competence. To attain this end, the plan contains the provision that an appropriation be made annually from funds in the United States Treasury, which will allow to Pensioners in Group (I), for each year of service prior to the date of introduction of the system of savings, 1 per cent. of the average salary received during the ten (10) years preceding that date, 1¼ per cent. for Group (II) and 1½ per cent. for Group (III). Some provision similar to the above is needed in the formation of every staff retirement plan where a constant percentage deduction is made from the salaries at all ages, in order that the younger members may not be unduly taxed to provide the pensions of the older men who will soon retire.

Stating the plan briefly, therefore, the employee is required to provide for himself in old age by systematic saving during his years of service, and the Federal Government, as an aid to the scheme, guarantees an attractive rate of interest, agrees to stand behind the plan and to meet the expense of operating it, and offers to help its present old employees who will not be able in the years remaining before retirement to provide pensions for themselves through their own efforts.

A brief reference to the actuarial work involved in establishing this plan may be of interest. As the selection exercised by the employee in choosing Option II will be nearly, if not quite, as great as that against an insurance company by a person purchasing an annuity, the British Offices Life Annuity Tables—Males and Females—with, however, no loading for expense, have been used in computing the charges for this option. In Option III, partly because of the advanced age of the employee at retirement and the consequent weight of the insurance element involved, and also for

the reason that the selection exercised by the employee would not be as severe as in Option II, the British Offices Life Table (O^M) was selected as the basis of this charge. Compound interest at 4 per cent., which is to be guaranteed by the Federal Government as an aid to the plan, was used in computing the figures for Options II, III, and IV.

Census Bulletin No. 12, "The Executive Civil Service of the United States," giving statistics as of July 1st, 1903, was used as a basis for determining the amount of annual appropriation by the Federal Government to provide annuities for old employees. As these data were not considered sufficiently recent, the Census Bureau has compiled new statistics as of July 1st, 1907, which will be used this Winter in revising the figures already obtained.

The chief requirement in determining the annual appropriations was that the amounts be the maximum required to provide these annuities. For this reason, although the right of an employee to an annuity from the Government is to be relinquished upon withdrawal or death, the withdrawals were disregarded and deaths alone considered in obtaining these figures. The method pursued is best shown by taking a particular age attained, as age forty-five in Group I, the age of retirement being sixty years. The total of the annuity payments based upon 1 per cent. for each year of service of the average salary for the past ten years was obtained for each age, and then discounted by the probability of dying before reaching the age of retirement. As the ages given in the Census Bulletin were for last birthday, and, consequently, the average age understated six months, and, as the annuity at retirement is to be payable quarterly, the factor $\frac{l_{60\frac{1}{2}}}{l_{45\frac{1}{2}}}$ was considered to be a safe approximation. To obtain the amount of annuity payments for employees now aged forty-five during the second year and after retirement, the above result was multiplied by the factor $\frac{l_{61\frac{1}{2}}}{l_{60\frac{1}{2}}}$ and so on each year to the end of the mortality table. When these calculations had been completed for all ages and groups, the results were arranged and the total appropriations necessary to provide annuity payments in 1908, 1909, and later years were obtained.

The mortality table selected as the basis of these computations was the American Experience Table of Mortality. This decision was reached after consideration of the effect of the conditions in-

volved in this part of the proposed plan upon the amounts of appropriation and all other information obtainable which would throw light upon the character of the class under observation from the standpoint of probable longevity. As stated before, withdrawals, other than by death which take place before the retirement age is attained, will tend to decrease the amount of annuity payments each year. These amounts will also be diminished because of the fact that there are many employees at the older ages, such as old soldiers, who have been in the service but a short time and who are receiving small salaries—virtually pensions—for unimportant work, who will be allowed to retain their positions rather than be retired upon an annuity which would be but a small percentage of their annual salary. For these reasons, and because of the standard of the mortality table adopted, the appropriations determined would seem to be a safe outside estimate of the amount necessary to provide the annuity payments.

To establish and maintain this savings system, although it might require considerable clerical work, should not prove a difficult task. From a table giving the amount of $1.00 per month at 4 per cent. compound interest for certain terms of years, the amount of accumulated savings at retirement could be easily determined for any age at entrance. At each increase in salary, the table could be entered for the amount of monthly increase for the term of years remaining before attaining the age at retirement, and the value thus obtained added to the amount of accumulation which was being provided for under the old salary. A card could be written for each employee upon which could be kept his account and, in addition, other data which could be used in obtaining the mortality experience, rate of withdrawal, and other valuable information in regard to the members of the Classified Civil Service. Because of the large number under observation, the experience could safely be subdivided to show the mortality of employees in the District of Columbia, as distinguished from the other main division of the Service, elsewhere; also the experience in certain branches of the Service, such as Postal Railway Clerks and City Letter Carriers, which are affected by conditions peculiar to these classes of employees. The experience thus obtained would not only prove of considerable statistical value, but would serve as a guide in future valuations or adjustments of the plan.

An Instructive Mortality Experience.

M. M. DAWSON.

Perhaps insufficient attention has so far been given by actuaries to an investigation of the period in life when the gradual decrease of the death-rate, which sets in from birth, comes to an end, normally, and the gradual increase of the death-rate commences which becomes more and more marked as old age advances.

Dr. Farr's famous English Life Table No. 3 showed this period at age 13, for males and females.

The general supposition has been that it is not above age 12 or that, in any event, it is not beyond early maturity.

The recent British investigation of insured lives (up to 1893) seemed to show, however, that the period may even be above the age of legal majority.

As to mortality among women, all investigations or nearly all show that, whatever the relation to death-rates among men at the same age, there is a marked increase during the ages when maternity is most common and a gradual subsidence. And as maternity is possible and not infrequent before complete maturity is attained, it follows that the decrease may yield to an increase prematurely, because of the intrusion of this extra hazard.

There are *a priori* reasons, however, for believing that, under especially good conditions as to race, heredity, habits, climate, hygiene and sanitary surroundings, the lowest death-rate among men should be when they are in their prime and just before the powers begin to fail, by reason of decline into old age. What has already been cited, tends also to confirm this notion.

Recently I have investigated a mortality experience among insured lives, covering in all 61,564 lives and 403,466 years of life, and a period of observation of 30 years. The experience is that of the Minnesota Grand Lodge of the Ancient Order of United Workmen, up to and including 1906. In this experience, a most extraordinary demonstration of the foregoing proposition has been found.

Discarding all exposures and deaths within the calendar year of admission and the four calendar years next thereafter, the statistics for the remaining calendar years, embracing 28,033 lives and 202,634 years of exposure, were united at attained ages to form an ultimate table. This, graduated by the Graphic Method applied to central death-rate of groups of five ages each, showed the lowest death-rate at age 32.

That this is not accidental, also, nor peculiar merely to the ultimate section of the table, is shown by the fact that the lowest death-rate for each of the first five calendar years of duration was found at the following ages, respectively, viz:

First year after entry	54,967 lives. 51,297 years of exposure.	Age 31.
Second year after entry	45,030 lives. 42,723 years of exposure.	Age 37.
Third year after entry	37,486 lives. 36,088 years of exposure.	Age 29.
Fourth year after entry	31,994 lives. 30,963 years of exposure.	Age 31.

For the year of entry, the graduation of the experience, embracing 61,564 lives and 29,671 years of exposure, showed, on the contrary, an increasing rate of mortality, from age 18, upward.

Various theories to explain this departure might be advanced, such as that deaths within so short a time after medical examination, are either accidental or in consequence of the sudden onslaught of fatal disease, the liability to both slightly increasing as one approaches middle age. The increase is very slight and gradual, however, from .00175 at 20 to .00195 at 30, .00213 at 35 and .00233 at 40. This is for an average period slightly under six months; the mortality indicated is only moderately low for freshly selected risks, and there is no indication of remarkably close and discriminating medical selection.

The population of Minnesota is very largely of Scandinavian origin; the race and heredity, therefore, are unusually free from weaknesses. The habits of the people are good and very rarely effeminate in any case. The climate is with an abundant rainfall but free from humidity, cold in winter but rarely chilly and damp. The sanitary conditions, especially as to water and drainage, are

good throughout the state; and pure air and cleanliness have constituted singularly simple but effectual hygienic measures.

The censuses have long shown a most remarkably low death-rate for the entire state and also for its largest cities.

The experience seems to show, conclusively, that under such conditions the weaknesses which predispose to early disease and death are not present and that, left thereby to its normal and reasonable course, the death-rate tends to diminish throughout youth and to the very verge of middle age.

This experience is also accompanied by an extraordinarily favorable death-rate, even during and after middle age, tending to show that, under these conditions, when a very unusually large proportion of the lives have reached age 35, they set out from that age with vigor little sapped by vices of the constitution, irregularities of habit or inroads of disease, and with greatly improved chances for attaining a ripe old age.

It should be said, before leaving the subject, that the experience investigated is composed of lives which were insured on the "flat assessment" or "equal levy" plan, *i. e.*, with each member, without regard to age upon admission or attained age, paying the same assessment as every other member. This plan was still in use at the close of the observation, the net mortality cost to each member in 1906 being $11.00 per $1000 of insurance. The number of assessments has gradually increased for some years. Apparently, as would be expected under conditions so favorable,—so deceptively favorable,—there has been little or no adverse selection of any kind.

The graduation was made by Mr. C. W. Jackson, A. A. S., under my direction.

TABLE 1.

UNADJUSTED CENTRAL DEATH-RATES.

Age.	Year 0.	Age.	Year 1.	Age.	Year 2.	Age.	Year 3.
20	.00154	21	.00504	22	.00583	23	.00355
25	.00165	26	.00394	27	.00513	24	.00419
30	.00240	31	.00368	32	.00497	28	.00441
35	.00197	36	.00424	37	.00487	33	.00382
40	.00216	41	.00539	39	.00502	38	.00467
45	.00261	46	.00607	42	.00437	43	.00569
				44	.00617	48	.00604
				47	.00786		

Age.	Year 4.	Age.	Ultimate.
24	.00485	25	.00583
29	.00428	30	.00394
34	.00299	35	.00520
39	.00564	40	.00556
44	.00527	45	.00722
49	.00768	50	.00916
		55	.01359
		60	.01854
		65	.03327

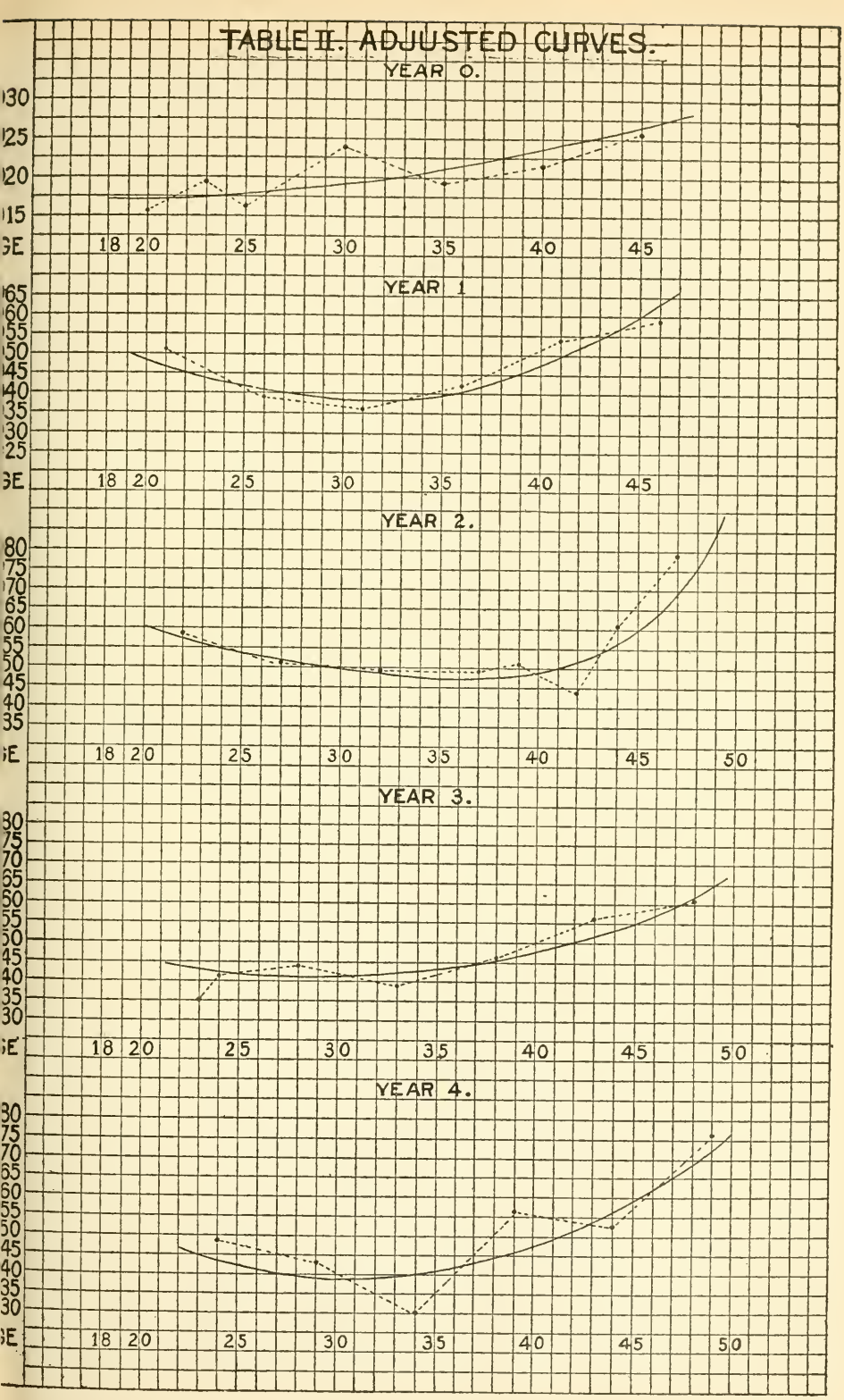


TABLE II. ADJUSTED CURVES.
YEAR 0.
18 20 25 30 35 40 45
YEAR 1
18 20 25 30 35 40 45
YEAR 2.
18 20 25 30 35 40 45 50
YEAR 3.
18 20 25 30 35 40 45 50
YEAR 4.
18 20 25 30 35 40 45 50

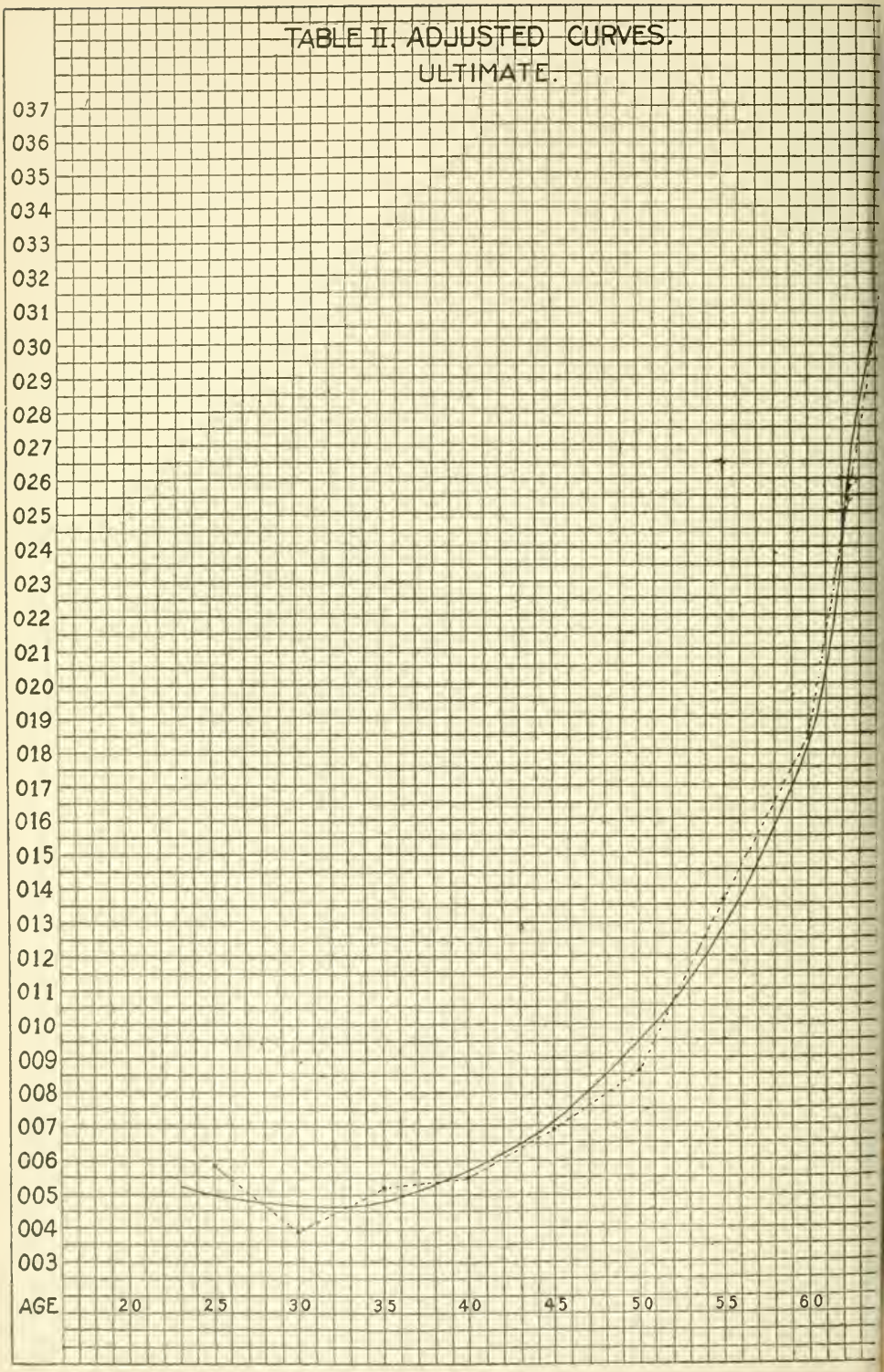


TABLE II. ADJUSTED CURVES.
ULTIMATE.
037
036
035
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021
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006
005
004
003
AGE
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25
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60

TABLE 3.

ADJUSTED DEATH-RATES. (GRAPHIC.)

YEAR 0.

Age.	q_x	Expected Deaths.	Actual Deaths.	Deviation.	Accumulated Deviation.
18	.00174	1	0	+1	+1
19	.00175	1	2	—1	0
20	.00175	2	0	+2	+2
21	.00176	3	2	+1	+3
22	.00177	3	5	—2	+1
23	.00178	3	2	+1	+2
24	.00179	4	3	+1	+3
25	.00180	4	2	+2	+5
26	.00182	4	8	—4	+1
27	.00185	5	3	+2	+3
28	.00188	5	5	0	+3
29	.00192	5	3	+2	+5
30	.00195	5	12	—7	—2
31	.00198	5	4	+1	—1
32	.00202	5	7	—2	—3
33	.00206	5	5	0	—3
34	.00209	5	4	+1	—2
35	.00213	5	5	0	—2
36	.00217	5	5	0	—2
37	.00221	5	5	0	—2
38	.00225	5	4	+1	—1
39	.00229	5	7	—2	—3
40	.00233	5	5	0	—3
41	.00237	5	1	+4	+1
42	.00242	5	6	—1	0
43	.00248	5	6	—1	—1
44	.00254	5	7	—1	—2
45	.00260	2	2	—0	—2
46	.00270	3	1	—0	0
47	.00278	3	0	+2	+2
48	.00288				

TABLE 3.

ADJUSTED DEATH-RATES. (GRAPHIC).

YEAR 1.

Age.	q_x	Expected Deaths.	Actual Deaths.	Deviation.	Accumulated Deviation.
18	.00485	3	5	—2	—2
19	.00473	3	3	0	—2
20	.00461	3	4	—1	—3
21	.00450	5	7	—2	—5
22	.00438	6	3	+3	—2
23	.00426	6	7	—1	—3
24	.00415	7	6	+1	—2
25	.00406	7	8	—1	—3
26	.00399	8	10	—2	—5
27	.00394	8	4	+4	—1
28	.00391	9	12	—3	—4

(Table 3.—Year 1 Continued.)

Age.	q_x	Expected Deaths.	Actual Deaths.	Deviation.	Accumulated Deviation.
29	.00390	9	4	+5	+1
30	.00390	9	6	+3	+4
31	.00392	9	7	+2	+6
32	.00395	9	12	—3	+3
33	.00399	9	6	+3	+6
34	.00406	9	13	—4	+2
35	.00415	10	7	+3	+5
36	.00430	9	7	+2	+7
37	.00450	10	13	—3	+4
38	.00472	10	11	—1	+3
39	.00495	10	5	+5	+8
40	.00517	10	13	—3	+5
41	.00538	10	9	+1	+6
42	.00560	10	14	—4	+2
43	.00583	11	16	—5	—3
44	.00606	14	13	+1	—2
45	.00630	3	2	+1	—1

TABLE 3.

ADJUSTED DEATH-RATES. (GRAPHIC.)

YEAR 2.

Age.	q_x	Expected Deaths.	Actual Deaths.	Deviation.	Accumulated Deviation.
20	.00625	3	2	+1	+1
21	.00600	2	5	—3	—2
22	.00580	2	1	+1	—1
23	.00565	5	3	+2	+1
24	.00552	5	7	—2	—1
25	.00540	6	6	0	—1
26	.00529	7	6	+1	0
27	.00519	7	5	+2	+2
28	.00510	8	5	+3	+5
29	.00502	8	3	—5	0
30	.00495	9	6	+3	+3
31	.00489	9	11	—2	+1
32	.00486	9	10	—1	0
33	.00485	9	9	0	0
34	.00484	9	10	—1	—1
35	.00483	9	6	+3	+2
36	.00482	9	5	+4	+6
37	.00482	9	12	—3	+3
38	.00485	9	11	—2	+1
39	.00490	9	11	—2	—1
40	.00495	9	4	+5	+4
41	.00503	9	8	+1	+5
42	.00515	8	6	+2	+7
43	.00550	9	8	+1	+8
44	.00580	9	11	—2	+6
45	.00620	11	15	—4	+2
46	.00660	14	13	+1	+3
47	.00730	3	3	0	+3

TABLE 3.

ADJUSTED DEATH-RATES. (GRAPHIC.)

YEAR 3.

Age.	q_x	Expected Deaths.	Actual Deaths.	Deviation.	Accumulated Deviation.
21	.00440	1	1	0	0
22	.00434	1	0	+1	+1
23	.00429	1	1	0	+1
24	.00424	3	4	—1	0
25	.00420	3	2	+1	+1
26	.00418	3	5	—2	—1
27	.00417	4	4	0	—1
28	.00416	4	4	0	—1
29	.00415	5	5	0	—1
30	.00416	6	6	0	—1
31	.00418	6	7	—1	—2
32	.00421	7	3	+4	+2
33	.00426	7	7	0	+2
34	.00432	7	8	—1	+1
35	.00440	7	5	+2	+3
36	.00449	7	6	+1	+4
37	.00458	7	5	+2	+6
38	.00469	8	9	—1	+5
39	.00481	7	9	—2	+3
40	.00495	8	8	0	+3
41	.00509	8	8	0	+3
42	.00523	8	9	—1	+2
43	.00537	8	7	+1	+3
44	.00551	7	10	—1	0
45	.00565	8	8	0	0
46	.00581	9	9	0	0
47	.00599	11	10	+1	+1
48	.00620	2	2	0	+1

TABLE 3.

UNADJUSTED DEATH-RATES. (GRAPHIC).[1]

YEAR 4.

Age.	q_x	Expected Deaths.	Actual Deaths.	Deviation.	Accumulated Deviation.
22	.00465	1	1	0	0
23	.00455	1	0	+1	+1
24	.00447	1	0	+1	+2
25	.00438	2	2	0	+2
26	.00430	2	5	—3	—1
27	.00420	3	4	—1	—2

[1] For the use of the jurisdiction, an ultimate table was made by a somewhat different process, terminating in a graduation by Makeham's formula. Under it, the lowest rate appears to be at 35. The graphic graduation, here given, is more accurate.

(Table 3.—Year 4 Continued.)

Age.	q_x	Expected Deaths.	Actual Deaths.	Deviation.	Accumulated Deviation.
28	.00405	3	3	0	—2
29	.00398	3	4	—1	—3
30	.00395	4	3	+1	—2
31	.00394	4	5	—1	—3
32	.00395	5	2	+3	0
33	.00400	5	5	0	0
34	.00407	6	6	0	0
35	.00415	6	4	+2	+2
36	.00425	6	3	+3	+5
37	.00438	6	11	—5	0
38	.00454	6	8	—2	—2
39	.00473	7	8	—1	—3
40	.00495	7	5	+2	—1
41	.00516	7	7	0	—1
42	.00537	8	6	+2	+1
43	.00558	8	12	—4	—3
44	.00579	7	3	+4	+1
45	.00600	7	6	+1	+2
46	.00624	8	7	+1	+3
47	.00651	9	4	+5	+8
48	.00681	12	17	—5	+3
49	.00714	2	3	—1	+2
50	.00750				

TABLE 3.

Adjusted Death-Rates. (Graphic.)

Ultimate.

Age.	q_x	Expected Deaths.	Actual Deaths.	Deviation.	Accumulated Deviation.
23	.00525	1	0	+1	+1
24	.00515	1	1	0	+1
25	.00510	2	1	+1	+2
26	.00502	3	3	0	+2
27	.00495	5	8	—3	—1
28	.00489	6	5	+1	0
29	.00484	8	6	+2	+2
30	.00480	10	9	+1	+3
31	.00477	13	12	+1	+4
32	.00476	16	12	+4	+8
33	.00477	18	25	—7	+1
34	.00479	22	25	—3	—2
35	.00481	25	27	—2	—4
36	.00489	28	25	+3	—1
37	.00503	32	32	0	—1
38	.00520	35	34	+1	0
39	.00540	39	35	+4	+4
40	.00565	43	50	—7	—3
41	.00592	47	40	+7	+4
42	.00621	51	50	+1	+5
43	.00652	55	57	—2	+3
44	.00686	58	52	+6	+9
45	.00722	62	72	—10	—1

(Table 3.—Ultimate Continued.)

Age.	q_x	Expected Deaths.	Actual Deaths.	Deviation.	Accumulated Deviation.
46	.00761	64	57	+7	+6
47	.00802	67	67	0	+6
48	.00845	71	76	—5	+1
49	.00891	78	68	+10	+11
50	.00940	74	80	—6	+5
51	.00995	70	70	0	+5
52	.01058	68	59	+9	+14
53	.01132	67	73	—6	+8
54	.01215	67	69	—2	+6
55	.01305	63	79	—16	—10
56	.01395	58	52	+6	—4
57	.01480	54	54	0	—4
58	.01580	51	53	—2	—6
59	.01690	49	46	+3	—3
60	.01825	47	41	+6	+3

Abstract of the Discussion of Papers Read at the Previous Meeting.

ACTUARIAL NOTE ON SUMMATION FORMULAS—D. E. KILGOUR.

Vol. X, Page 1.

Written Discussion.

Mr. Henderson: This paper brings out very clearly an analogy between Lubbock's and Woolhouse's formulas and exhibits a very direct method of deriving the constants in those formulas. Unfortunately, however, the clearness of this analogy is somewhat obscured before they can be put in their final form and this would seem to suggest that a somewhat different viewpoint might make the analogy clearer. I think also that the symbolic method enables the essentials of the various operations to be more clearly exhibited.

The problem of both formulas is to express in convenient form the value of

$$\frac{1}{n}(u_0+u_{\frac{1}{n}}+u_{\frac{2}{n}}+\ldots+u_{\frac{m-1}{n}})-(u_0+u_1+\ldots+u_{m-1})$$

In other words, we desire to find the correction to be applied to the sum for unit intervals in order to obtain the sum for fractional intervals. In Lubbock's formula this correction is expressed in terms of finite differences, whereas, in Woolhouse's it is expressed in differential co-efficients and the two expressions being different forms of the same function necessarily reduce to one another.

The correction may be expressed symbolically as follows:

$$\left\{(1+\Delta)^m-1\right\}\left[\frac{1}{n\left\{(1+\Delta)^{\frac{1}{n}}-1\right\}}-\frac{1}{\Delta}\right]u_0$$

The expansion of the expression $\dfrac{1}{n\left\{(1+\Delta)^{\frac{1}{n}}-1\right\}}-\dfrac{1}{\Delta}$ in ascending powers of Δ gives us immediately Lubbock's formula. This expansion is easily made by expanding $n\left\{(1+\Delta^{\frac{1}{n}})-1\right\}$ by the binominal theorem and dividing into unity.

If, now, we express this operator in terms of differentials, we have

$$\frac{1}{n(e^{\frac{D}{n}}-1)}-\frac{1}{e^D-1}$$

which may be expanded in ascending powers of D and gives Woolhouse's formula.

The symbolic method suggests the derivation of a summation formula in terms of central differences. Using the notation of my recent paper on interpolation the operator takes the form

$$\frac{1}{2n}\left(\frac{\cosh\frac{D}{2n}}{\sinh\frac{D}{2n}}-1\right)-\left(\frac{\sigma}{\rho}-\frac{1}{2}\right).$$

Expanding now $\cosh\frac{D}{2n}$ and $\sinh\frac{D}{2n}$ in ascending powers of $\sinh\frac{D}{2}$ or $\frac{\rho}{2}$ we get

$$\frac{1}{2n}\left[\frac{\sigma\left\{1+\frac{\frac{1^2}{n^2}-1}{\lfloor 2}\cdot\frac{\rho^2}{2^2}+\frac{\left(\frac{1}{n^2}-1\right)\left(\frac{1}{n^2}-9\right)}{\lfloor 4}\cdot\frac{\rho^4}{2^4}+\ldots\right\}}{\frac{1}{n}\cdot\frac{\rho}{2}+\frac{\frac{1}{n}\left(\frac{1}{n^2}-1\right)}{\lfloor 3}\cdot\frac{\rho^3}{2^3}+\frac{\frac{1}{n}\left(\frac{1}{n^2}-1\right)\left(\frac{1}{n^2}-9\right)}{\lfloor 5}\cdot\frac{\rho^5}{2^5}+\ldots}-1\right]$$

$$-\left(\frac{\sigma}{\rho}-\frac{1}{2}\right)$$

$$=\frac{\sigma}{\rho}\cdot\frac{1-\frac{n^2-1}{2n^2}\cdot\frac{\rho^2}{2^2}+\frac{(n^2-1)(9n^2-1)}{24n^4}\cdot\frac{\rho^4}{2^4}-\ldots.}{1-\frac{n^2-1}{6n^2}\cdot\frac{\rho^2}{2^2}+\frac{(n^2-1)(9n^2-1)}{120n^4}\cdot\frac{\rho^4}{2^4}-\ldots.}\cdot-\frac{\sigma}{\rho}+\frac{n-1}{2n}$$

$$=\frac{\sigma}{\rho}\left\{1-\frac{n^2-1}{3n^2}\cdot\frac{\rho^2}{2^2}+\frac{(n^2-1)(11n^2+1)}{45n^4}\cdot\frac{\rho^4}{2^4}-\&c\right\}-\frac{\sigma}{\rho}+\frac{n-1}{2n}$$

$$=\frac{n-1}{2n}-\frac{n^2-1}{12n^2}\cdot\sigma\rho+\frac{(n^2-1)(11n^2+1)}{720n^4}\sigma\rho^3-\&c.$$

or $$\frac{1}{n}\left(u_0+u_{\frac{1}{n}}+u_{\frac{2}{n}}-\ldots.u_{\frac{m-1}{n}}\right)$$

$$=(u_0+u_1+\ldots.u_{m-1})+\frac{n-1}{2n}(u_m-u_0)-\frac{n^2-1}{12n^2}\sigma\rho(u_m-u_0)$$

$$+\frac{(n^2-1)(11n^2+1)}{720n^4}\sigma\rho^3(u_m-u_0)\ \&c.$$

where it will be remembered that $\sigma\rho$ expresses the operation of forming the first central difference and $\sigma\rho^3$ that of forming the third.

Where the upper limit of summation is such that u_m and all its differences vanish, we have

$$\frac{1}{n}\left(u_0+u_{\frac{1}{n}}+u_{\frac{2}{n}}+\&c\right)=(u_0+u_1+\ldots.\ \&c)-\frac{n-1}{2n}u_0+\frac{n^2-1}{12n^2}\sigma\rho u_0$$

$$-\frac{(n^2-1)(11n^2+1)}{720n^4}\sigma\rho^3u_0+\&c.$$

By putting $$\sigma\rho = \Delta - \frac{1}{2}\Delta^2 + \frac{1}{2}\Delta^3 - \frac{1}{2}\Delta^4 + \&c.$$

and $$\sigma\rho^3 = \Delta^3 - \frac{3}{2}\Delta^4 + \&c.$$

this may be transformed into Lubbock's formula, and by putting

$$\sigma\rho = D + \frac{1}{6}D^3 + \&c.$$

and $$\sigma\rho^3 = D^3 + \&c.$$

it transforms into Woolhouse's.

This formula shares with Woolhouse's the advantage that the second and fourth difference corrections vanish, but it requires the computation of values outside the range of summation.

Oral Discussion.

Mr. Blackadar: I have not anything prepared on this subject, but I desire to congratulate the students here present in having the advantage of Mr. Kilgour's proofs of these two formulas—Lubbock's and Woolhouse's. If they have them before them in their studies they will find them of the greatest benefit, both from the directness and similarity of the proofs.

Mr. Dawson: Mr. President, I merely want to say that when these proofs were brought to my attention, before Mr. Kilgour had become a Fellow, they were submitted by me to the Council; I was immediately struck—as one who is not so very far from having been a student himself for examination—with the great simplicity that marks these particular demonstrations, and I do believe that students will find it exceedingly wise to use them instead of the demonstrations given in the Institute text-book.

DOUBLE INTERPOLATION—H. N. SHEPPARD.

VOL. X, PAGE 5.

WRITTEN DISCUSSION.

MR. BRADSHAW: In this paper Mr. Sheppard has ingeniously developed a formula involving two variables which may be applied in the calculation of functions where only one life is involved, but which will probably be found more useful where more complicated benefits are concerned. It has occurred to me, however, that the resultant formula might be obtained more readily and the following demonstration is submitted in which it will appear that Finite Differences alone are used, thus avoiding the complications arising from the introduction of the Differential Calculus. Adopting the usual notation and expanding the functions in the ordinary way we have, assuming the intervals of differencing to be unity,

$$
\begin{aligned}
&f(x+\alpha,\ y+\beta)\\
&\quad = f(x+\alpha,\ y) + \beta\Delta' f(x+\alpha,\ y) + \frac{\beta(\beta-1)}{\lfloor 2}\Delta'^2 f(x+\alpha,\ y)\\
&\qquad\qquad + \frac{\beta(\beta-1)(\beta-2)}{\lfloor 3}\Delta'^3 f(x+\alpha, y) + \ldots\ldots\\
&\quad = f(x,\ y) + \alpha\Delta f(x,\ y) + \frac{\alpha(\alpha-1)}{\lfloor 2}\Delta^2 f(x,\ y) + \frac{\alpha(\alpha-1)(\alpha-2)}{\lfloor 3}\Delta^3 f(x,\ y)\\
&\qquad\qquad + \ldots\ldots\\
&\qquad + \beta\Delta' f(x,\ y) + \alpha\beta\Delta'\Delta f(x,\ y) + \frac{\beta\alpha(\alpha-1)}{\lfloor 2}\Delta'\Delta^2 f(x,\ y)\\
&\qquad\qquad + \ldots\ldots\\
&\qquad\quad + \frac{\beta(\beta-1)}{\lfloor 2}\Delta'^2 f(x,\ y) + \frac{\alpha\beta(\beta-1)}{\lfloor 2}\Delta'^2\Delta f(x,\ y)\\
&\qquad\qquad + \ldots\ldots\\
&\qquad\qquad\quad + \frac{\beta(\beta-1)(\beta-2)}{\lfloor 3}\Delta'^3 f(x,\ y)\\
&\qquad\qquad + \ldots\ldots\\
&\quad = f(x+\alpha,\ y) + f(x,\ y+\beta) - f(x,\ y)\\
&\quad + \alpha\beta\Delta'\Delta f(x,\ y) + \frac{\alpha\beta}{\lfloor 2}\left\{(\alpha-1)\Delta'\Delta^2 + (\beta-1)\Delta'^2\Delta\right\} f(x,\ y)
\end{aligned}
$$

where differences of order higher than the third are neglected.

If the intervals of differencing are m and n respectively, we have

$$
\begin{aligned}
&f(x+\alpha,\ y+\beta)\\
&\quad = f(x+\alpha,\ y) + f(x,\ y+\beta) - f(x,\ y)\\
&\quad + \frac{\alpha\beta}{mn}\Delta'\Delta f(x,\ y) + \frac{\alpha\beta}{2mn}\left\{\left(\frac{\alpha}{m}-1\right)\Delta'\Delta^2 + \left(\frac{\beta}{m}-1\right)\Delta'^2\Delta\right\} f(x,\ y)
\end{aligned}
$$

Mr. Strong: In passing from simple to double interpolation the complexity of the problems is enormously increased; this increase is probably greater than in the corresponding transition in Calculus because in the latter the standard Taylor's formula can be readily extended, theoretically at least, to the field of two independent variables while the corresponding Finite Difference formulas do not to the same degree lend themselves to such an extension. When in addition to the difficulty of double interpolation we consider that most of the practical problems to which it would be applied can be solved more or less accurately by linear interpolation, we find a reason why the literature of the subject should be so meagre.

The one paper on this subject which has in the past been presented before this society was that of Mr. Hardcastle; read in October, 1905. This paper, by using finite difference methods upon a matrix of elements, derived a difference expression of order $(n+m-2)$ where n and m are the dimensions of the matrix; that is, of an order equal to the sum of the orders to be obtained by differencing rows and columns.

The present paper in many respects is in contrast to Mr. Hardcastle's. Instead of using finite difference methods, its basis is the Taylor calculus formula for two variables—this is transformed into finite differences by the third difference method of the Institute Text-Book and then simplified to a final formula; instead of making use of a matrix of elements it uses a triangle only, and instead of the difference equation obtained being of higher order than the linear difference equations to be obtained from the same triangle, it is of the same order. The formula given is for third differences only; to extend the method to a higher order, a new formula must be had by retaining more terms in the Taylor formula and then using more differences in the transformation formulas. As a test of the accuracy of the results the formula gives, I interpolated the joint annuity for the ages 33 and 23 from the annuities at five year intervals using the Mutual Benefit 3 per cent. American Experience Tables. The result obtained was 16.76568 while the correct figure is 16.76584 which shows a good degree of accuracy.

In order to reach Mr. Sheppard's results it is not however necessary to employ double interpolation since the formula he derived may be obtained as an analytical expression of a special case of a general method of successive linear interpolation. This method may be briefly described thus; form a triangle of values like Mr. Sheppard's; interpolate horizontally for $u_{h.0}$, then from this interpolate vertically in the column $x=h$ for u_{hk}, having first adjusted the successive differences so that they will be the correct differences for this column.

More in detail the method is as follows:

Form the triangle of known values

$$
\begin{array}{l}
u_{0,0}, u_{0,1} \cdots\cdots\cdots\cdots u_{(n-1)0}, u_{n0} \\
u_{0,1}\ u_{1,1}\ \cdots\cdots\cdots\cdots u_{(n-1)1} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
u_{0(n-1)}, u_{1(n-1)} \\
u_{0n}
\end{array}
$$

Interpolating the top row we get $u_{h,0}$. We then wish to interpolate vertically in the same manner along the column $x=h$ to u_{hk}. To do this it is necessary to adjust the successive differences so that they may be the differences for this column; which can be done by interpolation of the differences. If the triangle of w's is differenced by column throughout, a triangle of first differences, w, is obtained: interpolation of the top row of the w triangle gives $w_{h0}=\bar{\Delta}u_{h,0}$, the adjusted first difference we wish. By differencing the w triangle and proceeding in the same manner we can get the adjusted second difference $\bar{\Delta}^2u_{h0}$, etc. It is evident that the order of the interpolating equation decreases by unity with the increase of unity in the order of the difference which is being interpolated so that we shall have finally $\bar{\Delta}^nu_{h0}=\Delta^nu_{h0}$. Having thus the adjusted differences it remains only to use them by the ordinary method for interpolating to u_{hk}.

As stated above this method analytically expressed for third differences may be put in the form of Mr. Sheppard's formula.

Where h is large relative to k, or if the differences in rows are large relative to the differences in columns, the accuracy may be increased without a corresponding increase in the work by interpolating by row for u_{h0} to a higher number of differences than is used in the later interpolation by column to u_{hk}.

It is evident that u_{hk} should be contiguous to u_{00}, where possible.

THE ULTIMATE TABLE, $O^{M(5)}$—M. S. HALLMAN.

VOL. X, PAGE 9.

WRITTEN DISCUSSION.

MR. MACAULAY: I am personally under a deep obligation to Mr. Hallman for introducing this important subject, and not merely giving us the benefit of his own views in regard to it, but collecting for us the opinions of authorities on both sides of the Atlantic. If we are ever to change the official table of mortality of the Dominion, the natural time to do so would be when passing the new general act which will come before parliament at its next session. The discussion of the merits of Select, Aggregate and Ultimate tables is thus peculiarly appropriate. The objections to an aggregate table, such as our present standard, the H^M, are of course well known. The mortality predicted by it at any particular age is not that which may be expected either among lives that have been long on the company's books, or among those which have been recently selected. It is the mortality which prevailed in the middle of the last century among a special group of lives of that age, assured for varying durations from the initial year upward, the heavy mortality among the long assured lives being counterbalanced by the favorable rate among those fresh from the doctor's hands, the result being a mere average. Such a table cannot exactly represent the experience of any office, unless the percentage of lives assured under policies of different durations should happen to precisely correspond with that of the British experience on which the table was based. If a company's business is comparatively new, its experience would certainly be much better than that of the H^M table, even though its mortality should conform exactly to that of the newly selected lives contained in the original statistics. On the other hand, if a company should cease to issue new policies, and its mortality should conform exactly to the original experience on long assured lives, it would be in excess of that predicted by the H^M table. Moreover the percentage of these new lives in the foundation statistics was not constant, those under five years duration varying from nearly ninety per cent. of the total exposure at age twenty-five, to less than five per cent. at age seventy-five. As the proportion of newly selected lives is thus much greater in early life than at the later ages, it follows that the mortality for these younger ages is reduced to a much greater extent than is true of the mortality rates at the higher ages. This distortion of the mortality table has the effect of artifically steepening the mortality curve, and that of course means an artificial increase in the reserves, as compared with those required by any table in which the mortality curve has not been altered by the effects of selection. As Mr. Hallman points out, an aggregate table has now few defenders in actuarial circles.

Two alternative forms of table still remain to be considered, and each of these has its advocates. Select tables are based upon the theory that each group of lives that enter at the same age, should be kept separate

from those entering at other ages, the mortality in each group being reduced during the early years by the effects of the initial selection. The other method is that based on the opposite idea of eliminating theoretically all effects of selection, by using only the experience on policies after their first five or ten years of existence. The difference between these two systems is therefore radical and irreconcilable. There is much to be said on behalf of both contentions. Personally I prefer an ultimate table. If a company in all its assurance calculations assumes the highest rates of mortality which it is ever likely to experience, even when its policies shall have been in force for a quarter of a century, why is it not acting prudently and conservatively? It is true that its managers may hope by careful selection of risks to save a percentage of this mortality, especially in the early years of the assurances, but are they not acting wisely rather than otherwise if they take no account of this hoped for saving, but leave it as a margin for future contingencies? I think the ordinary unsophisticated business man would say so.

I will be reminded, however, that select tables actually bring out heavier reserves than do ultimate ones, and that they therefore must be the better. As Mr. George F. Hardy well says, however, in one of the extracts quoted by Mr. Hallman, this by no means follows. The most suitable basis of valuation is not necessarily that which brings out the highest reserves. If an ultimate table brings out reserves whose absolute sufficiency cannot be reasonably be questioned, what more is required?

It is well, however, to consider for a moment just why the reserves brought out by a select table are higher than those called for on the ultimate basis. This brings us back to the old theory of suspended mortality, of which we heard so much a quarter of a century ago. The basic supposition is that the amounts nominally saved in the early years of policies, as the result of initial selection, are not really saved at all, and that the mortality in later life will be increased to just that extent. This might be true if the comparison were made with an aggregate table which would represent in the total the experience of the particular company in question with absolute accuracy. It is not, however, with any aggregate table that we are making comparisons, much less with an aggregate table of absolute accuracy. The comparison we are now making is with the ultimate table, which provides throughout for the highest rate of mortality which may be expected to prevail even after the effects of medical selection have worn out. Is it necessary then to provide for an extra mortality which will never be experienced? If any company wishes to voluntarily set aside unusually heavy reserves, it should be free to do so, but that does not mean that such reserves are necessary.

It may be contended, however, that it is not fair to any policyholder to calculate his premiums on the supposition that his group of entrants will experience a higher mortality than the one it is actually expected to experience. Since, therefore, it may reasonably be expected that the mortality will be reduced in the early years as the result of medical selection, it is claimed that the assured should get the benefit of that

lower mortality, either by a reduction in his premium, or by a special allowance of profits. But is this argument sound? How is this favorable initial mortality produced? The agent brings new lives to the company, and the doctor eliminates those that are undesirable. If there were no agents, there would be few fresh lives, and if there were no doctors, there would be no selection. The benefit is the result of the joint action of agents and medical examiners. But agents and doctors have to be paid for these services. To whom should this expense be charged? To the old policyholders? Should not the new members, as far as possible, pay their own way? The initial saving in mortality it seems to me is directly due to the expenditure by the company in its agency and medical departments. But for this there would be no such saving at all, and it would appear, therefore, eminently right and proper that it should be considered not as a real saving but merely as an offset to that extent of the expense. It is only by looking solely at the one factor of mortality and closing our eyes to all others, that we can say that a new policyholder is overcharged if we assume the full ultimate rate of mortality in all calculations from the commencement of the policy. When we look at the mortality and expense combined, we find that the cost of a new assurance is actually greater in its early years instead of less. It is moreover not through any action of the new life that the favorable mortality exists, and to give him the benefit of this favorable mortality, while charging the expenditure by which it was secured, or any part thereof, to the old policyholders, would seem to be unjust.

In Great Britain it is customary to calculate the premiums on the select basis, and of course in such a case an argument could be adduced in favor of carrying the same principle into the valuation of the policies, in order to be consistent throughout. On this side of the Atlantic, however, the calculation of premiums on the select basis is rare, and in fact I am not sure that at the present time there is any company on the Continent whose premiums are thus calculated.

The merits of the two plans may perhaps be better understood if we contrast two suppositious companies. We will assume that one is very careless in its selection of risks, and has a mortality throughout which is equal to the ultimate table. The other is particularly careful, and for the first year of its policies has a mortality of only forty per cent. of the ultimate, not reaching the full ultimate rate until its assurances are say ten years in force, its mortality thereafter being the same as that of its neighbor. If the experience of these two companies were combined, the result would be a select table showing a substantial reduction in the mortality for the first ten years, and the full ultimate rate thereafter. The ultimate table thus found would be a thoroughly safe basis for both companies. The suitably of the select table would however hardly be so clear. To penalize the conservative company, and to require that, because it had by its own carefulness succeeded in reducing the mortality in the early years of its policies, it must, on this very account, set aside higher reserves than if its mortality had been equal to the ultimate rate throughout, would hardly seem entirely reasonable.

I myself am strongly of the opinion that as the initial saving in

mortality is due solely to expenditure by the company, the saving should be put against such expenditure and should not be taken into account at all in the calculation of the reserves. This initial saving is, moreover, so uncertain, and varies so much in different companies, that it is unwise to assume that all companies must necessarily have it. I therefore favor the elimination of medical selection entirely from the table to be used as our standard, and agree with Mr. Hallman in preferring one calculated on the ultimate basis.

And now we come to the consideration of individual tables. Mr. Hallman has referred to the fact that the OM(5) table is based upon a much larger experience than the HM. The exact figures are as follows:

	HM	OM(5)
Entrants, - -	130,243	411,340
Exposed to risk,	1,200,400.5	5,324,862
Died, - - -	20,521	129,001

The OM(5) table is thus based on a very much broader foundation than the HM. It also includes the experience of sixty British offices. whereas the HM had but twenty. It, moreover, has the advantage of being more modern and more applicable to these days of improved sanitation and surgery. All exposures before the year 1863 were excluded, the period embraced being the thirty years ending on the anniversaries of the policies in 1893. As ultimate tables the OM(5) and OM(10) certainly take rank as the very best in the world. The OM(10) has in its favor the fact that by excluding the first ten years of each assurance the effects of selection are more completely eliminated, while, on the other hand, the OM(5) has the advantage of greater numbers, and of having many tables based on it already worked out. The difference between them is quite trifling. If we in Canada are to make any change in our legal basis of valuation, I certainly think that it should be to one or other of these ultimate tables.

One of the most remarkable facts in connection with the OM(5) and OM(10) tables is, their wonderful closeness to the HM. The similarity is impressive.

Comparison of q_x per 1000.

Age	HM	OM(5)	OM(10)	Age	HM	OM(5)	OM(10)
20	6.33	6.52	6.64	60	29.68	29.21	29.07
25	6.63	6.89	7.00	65	43.43	42.21	41.92
30	7.72	7.47	7.57	70	62.19	62.19	61.69
35	8.77	8.37	8.46	75	98.36	92.67	91.87
40	10.31	9.78	9.86	80	144.65	138.50	137.20
45	12.19	12.00	12.05	85	209.88	205.69	203.71
50	15.95	15.45	15.46	90	279.45	300.75	
55	21.03	20.83	20.79				

In a comparison which I prepared a year ago for our Royal Commission. I pointed out that the reserves by the $O^{M(10)}$ table, with 3½ per cent. interest, amounted to 99.6 per cent. of those required by the H^{M} table with the same interest, in the case of a typical company five years old; to 99.7 per cent. of such reserves in the case of a typical company ten years old; and to 99.5 per cent. in the case of a similar company twenty years old.

These figures would apparently indicate that the sanitary and other improvements of the last half century have been exactly sufficient to equal the saving in mortality formerly obtained by medical selection, so that lives assured for ten years, and thus as free as possible from the effects of selection, are now almost exactly on a par with the groups of lives of the previous time of the same ages, which included those fresh from the doctor's hands. Moreover, curiously enough, these benefits of modern civilization and increased knowledge would seem to have been distributed throughout the various ages of life in almost precisely the same proportions as the benefits which resulted from the introduction of the varying percentages of new lives in the previous experience. This is certainly remarkable and unexpected. It follows from it, however, that if we in Canada retain the H^{M} table as our standard, we are practically getting the same results as if we were to adopt the $O^{M(5)}$ or the $O^{M(10)}$ table as our basis. In other words our present table is in practice actually equivalent to an ultimate table. I myself however, would rather see Canada get the credit of adopting the modern ultimate standard, even though the actual difference in effect would be but little. That, however, is a question upon which I have no desire to press my views beyond those of my brother actuaries.

Some facts in regard to the O^{M} table itself may not be out of place in this connection. The general objections to it as an aggregate table need not be repeated. There are, however, certain objections which apply to it in particular. Mr. Hallman has pointed out that, unlike the $O^{M(5)}$ table, the O^{M} is not graduated by Makeham's formula. He also quotes Mr. George F. Hardy, to whom the very difficult and important task of graduating the tables was assigned by the committee of actuaries. Mr. Hardy says "with respect to the two tables, the O^{M} and the $O^{M(5)}$, I must confess that I regard the latter as by far the more important." Mr. Hallman is right in stating that the opinion of such an authority as Mr. Hardy, on the merits of the different tables, after subjecting the data necessary to the minutest analysis, is significant and of the very highest importance.

If we examine the basis of the O^{M} table carefully, we may I think discover a special reason why the O^{M} table is of less value than the corresponding ultimate one. The British actuaries, in their desire to get a thoroughly modern experience, excluded everything before the anniversary of each policy in 1863. The previous existence of every policy was ignored. For example, a policy issued in 1853 on a life which in 1863 had reached forty years of age, was not included at all in the experience under age forty, but was then added to the group aged forty

who had been assured ten years. For an ultimate table, and in particular one from which the first ten years were to be excluded, this method was unobjectionable. For an aggregate table, however. such as the Om, it introduced a new and utterly disturbing factor. The objection to an aggregate table, as already mentioned, is that the mortality rates for early life are unduly depressed by the inclusion of a large number of newly selected lives at those ages, thus making the mortality curve unnaturally steep, and increasing the reserves. The effect of the British regulation was to greatly accentuate this objectionable feature in the Om table. If, for example, at age forty a great mass of additional lives were included for the first time, these lives being assured for ten, eleven, twelve, fifteen, twenty or thirty years, it is evident that the proportion of lives at that age which would be subject to the full ultimate mortality would be much greater than in an aggregate experience prepared in the ordinary manner like the Hm table itself. The addition of great numbers of these long assured lives which were included for the first time on their anniversary in 1863, naturally raised the mortality rates for the latter half of life, causing these rates to approach much more rapidly to the ultimate or maximum. The extent of this influence may be seen by the following table.

Proportion of Lives over Five Years Assured.

		Hm				Om			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Age.	Exposed to risk	Number over 5 years ($4\frac{1}{2}$) assured	Per cent of total exposures over 5 ($4\frac{1}{2}$) years assured	Per cent of total exposures under 5 yrs. ($4\frac{1}{2}$) assured	Exposed to risk	Number over 5 years ($4\frac{1}{2}$) assured	Per cent of total exposures over 5 years assured	Per cent of total exposures under 5 years ass'd	Excess of (8) over (4)
20	3293.5	510	15.5	84.5	14,863	1,573	10.6	89.4	4.9
25	13622.5	1477	10.8	89.2	73,296	11,016	15.0	85.0	4.2
30	27112.5	7191	26.5	73.5	141,375	56,577	40.0	60.0	13.5
35	35818.5	15919.5	44.4	55.6	180,582	112,830	62.5	37.5	18.1
40	38195.0	21971.5	57.5	42.5	192,013	144,812	75.4	24.6	17.9
45	84835.5	23298.5	67.0	33.0	187,347	155,870	83.2	16.8	16.2
50	28855.5	21114.0	73.2	26.8	170,869	151,167	88.5	11.5	15.3
55	22170.5	17387.0	78.4	21.6	145,407	134,822	92.7	7.2	14.4
60	15672.5	12952.5	82.6	17.4	116,078	110,795	95.4	3.6	13.8
65	9984.5	8711.5	87.2	12.8	84,720	82,882	97.8	2.1	10.7
70	5622.	5171.5	92.0	8.0	55,038	54,412	98.9	1.1	6.9
75	2693.	2576.5	95.7	4.3	30,211	30,104	99.7	.2	4.1

It will be noticed that the percentage of lives assured for more than five years to the total exposed at each age, increases very much more rapidly in the O^M experience than in the H^M, for the reason already stated. The result is to still further steepen the mortality curve for the first half of life. This in its turn has the effect of still further abnormally increasing the reserves on policies taken out early in life. This is clearly brought out in Mr. Hallman's paper, where it will be noticed that the total reserves by the O^M table are about two per cent. higher than those by the $O^{M(5)}$, and from one to one and a half per cent. higher than by the H^M. If these extra reserves were really required, we of course should at once begin to set them aside, but since they are due solely to a peculiarity in the mode of compiling the statistics on which the table is based, they become merely an evidence of the unsuitability of the table in question. The H^M table which we at present have provides reserves considerably heavier than those required by the American table in use in the United States. The American table, however, is admittedly thoroughly safe, even as an ultimate table for policies of long duration. If the companies of the United States are safe in setting aside merely the reserves called for by the American table, the Canadian companies are still more secure in setting aside, as at present, the reserves called for by the H^M table, and still higher reserves are certainly unnecessary. If we in Canada are to make any change at all in our standard, I think that it should be to one of the ultimate tables—either the $O^{M(5)}$ or the $O^{M(10)}$.

Mr. Nichols: The qualifications which render a mortality table best suited for office purposes will probably continue to be a disputed question with actuaries, so long as opinions differ on those various questions of office methods and theories of the business which underlie it. It can best be answered, if at all, by those who have a wide practical experience in office work. I shall only attempt to point out what seems to me some of the important factors to be considered in reaching any conclusion. If by reason of statutory restrictions, or for any other cause, the abundant adequacy of such a table is relied on as a source of security merely, and expenses are to be provided for by means of premium loadings, we evidently have very different conditions to deal with than in the case of the British offices, where, according to Mr. George King in his recent opinion on the proposed Canadian legislation, the loaded mortality table and low interest rate are looked to as the provision for expenses, while the premiums are loaded simply to provide bonuses. According to this distinguished actuary, the practice of ear-marking the profits derived from any particular source, is like an attempt to decide whether the household expenses should be paid from the investments or earnings of the householder. The legal restriction of expenses to premium loadings is both unwise in practice and unsound in theory. It is a direct encouragement to extravagance in case of an old company loaded with renewals, and a compulsion to unduly raise its rate on the part of a young company with but little old business. The more appropriate Table, therefore,

under the British System, might be totally different from that which would be best adapted for the so-called reform legislation of New York.

Apart from this, I assume that other things being equal, the selection should be determined by the table which is most likely to express the future experience of the company and preserve equity among the various classes of policyholders. As between aggregate, select and ultimate tables, it is obvious that the former at no age represent the actual risk of the individual policyholder. At best they are a mere average of the experience of classes which are distinct as to their policy ages, an average which must vary from the expected experience, according as the percentage and distribution of new business may differ from that of the experience from which the table was compiled, and in which the discrepancy between the actual individual risk and the tabular is greatest at the younger ages. It is both inaccurate and out of proportion as a measure of individual risks. The strongest argument for its use, it seems to me, is that its inaccuracies may be offset by the initial cost of procuring the business of which the ordinary net premium valuation gives no recognition.

The question of equity in the use of such a table must largely depend it would seem, on the practical feasibility of separating the classes. Insurance is based on our ignorance of the individual risk, combined with our knowledge of the class to which it belongs. If the actual after life of the individual were known, or that of his class were unknown, there could be no insurance other than a gamble. Equity requires that the individual should be classified according to our knowledge. There was no injustice done when the early fire underwriters of Philadelphia charged a uniform premium for every building regardless of its character, provided it was free from shade trees, for all they knew was that some shade trees had once prevented the extinguishment of a serious blaze. It would be madness for the modern fire underwriter, with his knowledge of risks, to do the same thing. The moment we learn that any group of risks may be separated in distinct classes, with differing costs of insurance, we are bound in equity to deal with them as separate, if practical conditions will permit, and competition will usually compel it. The chief reason why no consideration of occupation is allowed to enter into the premium rate and age has been made the only factor, is its practical necessity.

But in the case of classes separated by policy years, the same argument no longer holds good. Select mortality tables have been prepared which follow the history of such classes to the end. As between the select and the combined select and ultimate, however, it might again be claimed that selection ceases to exert an important influence after the first five years and thereafter the member merges into the general class represented by the ultimate table. Therefore, a select table need only to be applied to the early policy years and thereafter the ultimate may properly be used. It may be noted in this connection, that where both aggregate and ultimate and select tables are constructed from the same data, the observations included in each separate age table of the latter must necessarily be few as compared with those of the former and hence the risk of fluctu-

ation must be much greater. This objection, however, would only have force where the observations are too few in number to produce a reliable average, and here is one of the advantages of the employment of Makeham's Method of Adjustment. If that law has been found to satisfactorily apply to the aggregate observations, a determined value of q for these is obviously applicable to the elements of the select table and the remaining constants may be left to express the special law of deviation at the special age. Even if the experience be limited, the aggregate results of all ages under a select table will tend to counterbalance the fluctuations of each particular age, and unless the weight of observations in the company's experience at each age is radically different from that on which the select tables were framed, the aggregate results of such a table may be expected to give more accurate results than if an aggregate experience were used. Hence, I am disposed to agree in the main with the views both of Messrs. King and Hardy, that a select table furnishes the best measuring rod, and that the $O^{M(5)}$ table graduated as it was, was much more important than the aggregate O^{M}.

Regarding methods of graduation, whose relative merits have been so widely discussed, there is one fundamental feature in all whose importance seems not to have been sufficiently recognized. The primary object of graduation, as has been so often said, is to remove the accidental fluctuations while leaving the normal changes of curvature undisturbed. If there were any abstract mathematical principle by which this could be done, the problem of the best method would be an easy one. But every attempted elimination of accidental fluctuations involves, as the starting point, some arbitrary assumption based on past experience. Even the well known law of error holds true only on the assumption based on common experience that an average is likely to give the most nearly correct result, or that every error is made up of an infinite number of minor errors each equally likely. The moment these assumptions no longer hold good, the ordinary law of error fails to apply, as may be noted in a multitude of physical phenomena, such as the budding and flowering of plants and the alternations of temperature with the change of the seasons, which are governed by other classes of frequency curves of which the law of error is only a special case.

The ordinary summation formula employed in graduation, such as that of Woolhouse and the rest, all assume an arbitrary weight of relationship between ages approximately near as the result of previous observations and comparisons and deduction. It is a matter of expert judgment rather than accurate determination. Even the tests of the results usually employed are not necessarily conclusive of their accuracy. De Morgan long ago claimed that the most accurate conclusions which could be drawn from a mortality table were not through its graduation at all, but by the employment of the total ungraduated data in the calculation of the annuity. The graphic method is simply the abandonment of all arbitrary mathematical assumptions as to the relationship between the ages and the substitution of expert judgment based on the appearances of the curvature.

Now, the so-called law of mortality, whether represented by that of Gompertz or Makeham's modification, is like the rest based on the results of observations and experience. But they are of a broader and more universal character. The death rate according to age is regulated by causes. The general uniformity of this death rate among large classes observed on, in every civilized country shows that within certain limits these causes must be more or less uniform in their operation. They must be regulated so to speak, by some force or forces having a more or less definite character, and the expression of those supposed forces and their relations in mathematical terms is called a law. I do not believe it is a mere accidental coincidence, that the application of Makeham's formula is found to fit so closely to mortality observations gathered under widely different conditions and in widely separated countries. The very fact that in the case of these British Tables, it is found to be suited to the ultimate, but not to the aggregate experience, and that when employed in youth or extreme age it needs to be supplemented, if used at all, while during the main period of mature life, it expresses the curvature with only such modifications of the constants as may be assumed to be of local and incedental character agree in its form as an expression of the forces involved. Its use is the application of broad general principles that have been found to regulate and direct the forces upon which mortality, according to age' depends. In so far as age factors control that which is to be expected, rather than circumstances which may be merely accidental and temporary with the observations dealt with, its claims to superiority are manifest.

I do not consider that an adjustment by Makeham's Law should be looked on, as it so often is, as one which is entitled to less weight than the ordinary summation methods. On the contrary, it is the most scientific of them all where found to be adequate, since it is an attempted expression of the actual causes at work as deduced from observations that are world wide. This, in addition to its acknowledged practical superiority as an expression of the law of uniform seniority. Had the original statistics, on which the American Experience is based, been so graduated, instead of merely the results of a previous graduation, Mr. Hunter's modification of that Table might be better entitled to the claim of accuracy than the table itself.

The strength of this argument obviously increases as the observations become fewer and more imperfect. We may have a set of observations so limited in number that no summation between ages nearly related will eliminate accidental fluctuations. Here the application of such a law as that of Makeham, embodying the results of a wide experience on a wide range of ages is likely to be far more accurate. On the other hand, we may have observations so enormous in number that their probable accuracy outweighs any law deduced from past experience and the utmost modification called for is between closely related ages. Between these two extremes may lie any number of intermediate conditions, and it is easy to see that a method of summation, which in one case might best express the probable relation between adjacent ages, might not do so

in another. One graduation may best be made by dealing only with three or five adjacent ages, another with seven and so on. For this reason the disputes which have arisen regarding the best method have generally led nowhere. They have failed to take into account that most important factor, the probable fluctuations of the ungraduated facts as measured by the magnitude or other conditions of the observations themselves. As I said before, experience and arbitrary judgment must, in the end, lie at the foundation of any graduation. The superiority of Makeham's, or any other method, must ultimately be judged by the comparison of its results with the ungraduated facts, having due regard to the probability of accidental fluctuations according to the weight of those facts. The enormous number of observations included in the O^M tables are calculated to discredit any method of adjustment which seriously disturbs the facts. Makeham's law as being more general and scientific could only claim superiority according as its results seem to harmonize with the facts, but when such is the case, its superior merit may well be admitted.

The $O^{M(5)}$ Table thus graduated is, by far, the largest aggregate of selected lives ever brought together and covers the observations of the last forty years. The number was nearly as great as that from which Dr. Farr compiled the H^M Tables and nearly six times greater than that which furnished the basis of the Actuaries Table, in which the average date of the observations was some sixty years earlier, when the business was in its developmental stage, and social and economic conditions were totally different.

That it is a more faithful representation of the mortality to be expected than either seems hardly open to dispute. Regardless of the accuracy of the data, a table founded on a census of the general population cannot be strictly representative of insurance classes, from which the lower strata of the people are eliminated and the risks are carefully selected at their initiation. For the mere practical purposes of a government standard, it would seem that either the aggregate or ultimate would sufficiently answer. Solvency of the company is after all the main object of governmental supervision in the case of financial corporations. As a matter of fact every company in its premium loadings and interest rates provides against an unfavorable mortality. Excessive compulsory reserves have sealed the fate of many a life insurance corporation in the United States.

Probably no more instructive investigations of this subject have been made than those of Dr. Sprague some thirty years ago. He compared the results of the aggregate H^M Tables of the Institute with a select table framed by himself from the same data. He found that in case of the ordinary life policy, the net premiums differed in some cases by over ten per cent., according to the age and duration of the payments. In the early ages the premiums by the select table were in excess, diminishing as middle life was approached and thereafter the aggregate premiums showed a corresponding increase. This can mean but one thing. With our knowledge of the effects of selection, premiums based on an aggregate table are favoring the younger applicants at the expense of the older and

when loaded, the inequality is increased. Such a result might naturally be anticipated in view of the greater influence exerted by selection on the younger ages in an aggregate table.

No such differences in the results, however, are shown between the aggregate and ultimate O^M tables, as given by Mr. Hallman. In fact, the comparison shows that there may be little to choose between them. Whether the O^M select would seriously modify the figures, I do not know. The data on which the tables used by Dr. Sprague were based were, I believe, made up of a much larger proportion of newly selected lives than those of the O^M. But it seems plain that other things being equal, a table whose premiums differ so widely as ten per cent. from those which should be equitably charged between the ages, must be inferior to another whose premiums correctly measure the relative risks. Commissioner Barry, in a recent address before the fire agents at Richmond, charged the whole demoralization in that business and the popular antagonism against the companies, to the favoritism shown in the premium rates of what are technically known as preferred risks. The life underwriter can no more afford than the fire underwriter to do a known injustice between his insured classes.

Mr. Sanderson: Mr. Hallman's paper upon the $O^{M(5)}$ table raises several important questions without attempting to deal exhaustively with them. A somewhat disproportionate part of the paper is taken up with tabulating the net $O^{M(5)}$ premiums and comparisons therewith, and also in quoting the opinions of British actuaries. These opinions are well selected, but it would have been of interest to have Mr. Hallman's comments thereon. The tables comprising net premiums by the $O^{M(5)}$ Table and the comparisons with those of other tables are of interest, but the eight pages taken up with comparisons might have been condensed into four by arranging the ages along the top and the names of the Tables down the side. On the other hand, the space given in the paper to the still more important subject of the relative reserves by different tables is confined to one short paragraph; but to the subject of reserves I shall return presently.

Keeping in view the proposed revision of the Canadian Insurance Act, Mr. Hallman brings to our notice the favorable features of the $O^{M(5)}$ table, and after touching briefly upon the O^M and $O^{[M]}$ tables he concludes that the $O^{M(5)}$ table would be the best substitute for the existing Government Standard. Mr. Hallman gives his paper the title "The Ultimate Table $O^{M(5)}$," but it is well known that in the O^M Experience selection is not exhausted at the end of five years and consequently the $O^{M(5)}$ table cannot strictly be called an ultimate table, the $O^{M(10)}$ more nearly fulfilling that qualification. Mr. Hallman seems to have arrived at his choice of the $O^{M(5)}$ table in preference to the O^M or the $O^{[M]}$ very largely from the point of view of graduation. He finds that the $O^{M(5)}$ table is graduated by Makeham's method and this involves the principle of uniform senority, which is very convenient in dealing with calculations where two or more lives are in question. On the other hand,

the O^M Table does not involve this principle and in its graduation resort was had to a double frequency curve. The impression left by Mr. Hallman's remarks on the graduation of the three tables previously referred to is that in the graduation of the O^M Table some scientific principles are lacking. It is not easy to follow the reasoning at this point, but I cannot help feeling that the writer of the paper has done less than justice to the skill and scientific principles applied in the graduation of the O^M table as well as in that of the $O^{[M]}$. The fact that a table does not lend itself to graduation by Makeham's formula is not in itself evidence against its suitably. The O^M table runs into the $O^{M(5)}$ table after age 84 and practically after age 80; and when two such twin tables as the O^M and $O^{M(5)}$ are identical at the older ages it is highly improbable, if not impossible, that if the one table is graduated by Makeham's formula with uniform constants throughout, the other can also be graduated. I do not think it is a *sine qua non* that a table should be graduated by Makeham's method in order to be considered a scientifically constructed table. It is certainly a fortunate thing when a table can be so graduated, but to assume that it should be capable of graduation by this formula is to assume that there is a fixed law of mortality represented by some such assumption as that of Gompertz. That such a physiological law absolutely exists at all ages cannot be demonstrated, and in fact a comparison of the $O^{M(5)}$ table at young and old ages with the unadjusted data will show that a graduation by Makeham's formula does not closely follow the facts at these extremities but nevertheless the graduation on the whole is very satisfactory.

The new British life tables are fast working their way into actual use in the valuation of individual British companies. It may be of interest to point out what is the trend of events in Great Britain with regard to the respective tables embraced in the new Experience. From a recent English publication I find that out of forty-six companies which have adopted the new Experience in some form only nine have adopted the $O^{M(5)}$ table throughout. The majority have adopted the aggregate table, O^M, while a number have adopted the Combined O^M and $O^{M(5)}$ which gives a rough approximation to a valuation by the Select table, $O^{[M]}$ Further, of the nine British companies which have adopted the $O^{M(5)}$ table, there is only one that would, I think, take its place among the front rank of active British companies and that company adopted the table on account of a special form of distribution of surplus in use, and it combines the $O^{M(5)}$ table of mortality with the very conservative rate of interest of 2½ per cent. in its valuation. Among the other eight companies which value by the $O^{M(5)}$ table, two neither employ general agents nor pay commission and have not therefore the usual reason for adopting such a table, as the new business transacted is very small. Two others of the eight companies are quite young life companies and one is an industrial company. Only two of the nine companies write new business of any dimensions and that only about $4,000,000 per annum each. Thus up to the present it cannot be said that in the land of its

birth the $O^{M(5)}$ table has met with the same favor as either the aggregate table, O^{M}, or the select table, $O^{[M]}$. Again, none of the three Australian companies transacting business in Great Britain have yet adopted the $O^{M(5)}$ table, although one has adopted the aggregate table throughout, and another has adopted it for the valuation of new business.

Coming now directly to the subject of reserves, Mr. Hallman gives a brief table based on King's Model Office of the reserves for a company twenty-five years old and fifty years old, respectively, by different tables of mortality. I would point out that there is an obvious error in stating the reserves on the O^{M} and $O^{M(5)}$ basis for the office twenty-five years old, the figures given being $10,483,491 in place of $10,355,631. In this case the writer of the paper has used the Combined O^{M} and $O^{M(5)}$ throughout, whilst in the corresponding case of the office fifty years old he has used the O^{M} alone for the first five years. This slip may be noticed from an observation of the grading of the two columns in the Table. The writer appears also to have deduced his valuation not from King's Model Office reserves direct but from the column of ratios given in Mr. King's tables. This affects the last three figures in each case but does not change the lesson to be drawn from the table.

The character of the reserves brought out by a table is in many respects more important than the question of premiums and consequently it would have been appropriate if Mr. Hallman had given some extended tables upon this phase of the subject. The two tables that have been in general use in the United States are the Combined Experience (or 17 Offices) table and the American Experience table, while in Canada the H^{M} table has been and remains the official standard. It will doubtless be of interest to see how the reserves by the new British Experience compare with those of the tables just mentioned for companies of various ages. Mr. George King's tables in connection with his Model Office enable this to be done and I give here a table comparing the reserves by the O^{M}, H^{M}, and $O^{M(5)}$, Combined Experience, and American Experience tables. (I use interest at 4 per cent. throughout, as that is the only rate for which the American Experience table has been applied by Mr. King to a valuation of his Model Office.)

Comparison of Reserves by King's Model Office. 4 Per Cent.

Table.	Age of Office 5 yrs.	10 yrs.	15 yrs.	20 yrs.	25 yrs.
O^{M}	10,000	10,000	10,000	10,000	10,000
H^{M}	9,680	9,729	9,772	9,808	9,837
$O^{M(5)}$	9,581	9,647	9,698	9,738	9,769
17 Offices	9,527	9,609	9,677	9,741	9,781
Amer. Exper.	9,107	9,241	9,358	9,461	9,542

(CONTINUED)

Table.	30 yrs.	35 yrs.	40 yrs.	45 yrs.	50 yrs.
O^M	10,000	10,000	10,000	10,000	10,000
H^M	9,858	9,877	9,892	9,902	9,905
$O^{M(5)}$	9,789	9,807	9,821	9,830	9,834
17 Offices	9,807	9,828	9,843	9,852	9,854
Amer. Exper.	9,600	9,651	9,697	9,729	9,749

The table has been constructed by taking the O^M reserve as a basis at 10,000, thus making it possible to tell at a glance what percentage the reserves by each of the other tables are of the O^M, which is the table coming into most general use in Great Britain.

From this table it will be seen that of the various tables mentioned, the O^M produces the highest reserves throughout and the American Experience the lowest reserves throughout, whilst the H^M runs between the O^M and the $O^{M(5)}$. The reserves by the $O^{M(5)}$ table lie between those of the H^M and the Combined Experience for offices fifteen years of age and under, but for offices twenty years old and upwards the $O^{M(5)}$ falls slightly below the Combined Experiences but is materially higher than the reserves by the American Experience table throughout.

Mr. Hallman speaks very confidently of the accuracy with which Mr. King's Model Office may be used to guage any change in the valuation of any individual company, but I think it quite possible that this may not hold true in every case. In using Mr. King's Model Office it must be remembered,—

1. That it is based on the relative number of new entrants and discontinuances which prevailed in the H^M Experience.
2. That the average age at entry is now materially younger than in the Model Office and this is liable to have a disturbing effect.
3. The Model Office is a type of company which has no reversionary bonuses in force and this will be found to make a material difference.
4. The large amount of endowment business now transacted as compared with the experience from which the Model Office is derived also requires special consideration.

All these disturbing factors are peculiarly applicable to business in America, so that estimates drawn from a valuation based on King's Model Office should keep these factors in view and be used with caution.

I have not included in the table above mentioned the valuation by the O^M table for the reason that the paper under consideration has in view a possible substitute for the present Canadian Government standard and from this point of view the Select table would not, I presume, be considered practicable. The most suitable table, even for office use, is, as Mr.

Hallman correctly points out, a controversial question. The majority of the British companies evidently prefer either a Select Table or an up-to-date aggregate table with a conservative rate of interest, but conditions in Great Britain are radically different from those in America and these conditions must be taken into consideration. A valuation by an aggregate table allows to a certain extent for reserves to be set aside against an expanding business, assuming that the true theoretical reserves are those determined by the Select table. A valuation by an Aggregate Table with reduced reserves in the early years further provides for an expanding business and the initial cost connected therewith. A valuation by an Ultimate table or a semi-ultimate table such as the $O^{M(5)}$, with diminished reserves in the early years, still further departs from the theoretical basis.

At the present time Canadian companies value new business by the Canadian Government standard (H^M 3½ per cent.) and are required to put up reserves materially higher than United States companies which are required to value their new business as a rule by the American Experience 3½ per cent. and which may in many cases also make allowance in valuation for the cost of new business. In view of this actual situation there may be some natural hesitation in suggesting for adoption as a government standard a table such as the O^M, which would bring out reserves higher than the present standard. On the other hand, the Government might hesitate to recommend a table like the $O^{M(5)}$ as an alternative table which brings out reserves slightly less than the present official standard. Moreover the question as to the most suitable table for a Government Standard is linked up to the question of allowance for initial expenses by diminishing the reserves in the early policy years.

The reserves by the H^M table run so closely parallel to those brought out by the new British tables that there is no urgent necessity for any change of basis, although looking to the fact that the H^M is fast becoming obsolete in Great Britain there are advantages that would accrue from permitting the use of an up-to-date table. Keeping in view the following considerations,—

1. That legislation is as a rule in the nature of a compromise between extremes;
2. That a Government standard should not be so severe as to promote technical insolvency, nor so low as to encourage companies to value upon a weak basis and thus render reassurance of these companies practically impossible;
3. That the usual official standard in the United States for new business is American 3½ per cent., which calls for reserves materially lower than the present standard in Canada;
4. That apart altogether from the question as to whether the $O^{M(5)}$ table is or is not an Ultimate table, it brings out reserves well graded, and adequate, from the point of view of a Government standard,

I am of the opinion that if any change is to be made in the Canadian Government standard or an alternative table allowed, the $O^{M(5)}$ Table will probably meet with more general acceptance than any other, especially if, as is probable, the continuance of the present conservative rate of interest be maintained in the Canadian Insurance Act.

The Society is indebted to Mr. Hallman for bringing to our notice the merits of the $O^{M(5)}$ table, especially at such an opportune time, when the revision of the Canadian Insurance Act is contemplated.

Oral Discussion.

Mr. Moir: Mr. President and Gentlemen. I am afraid I have to apologise for not being prepared with a written discussion of this paper, but on account of being very busy with my own paper, and on account of other matters coming up I found it quite impossible to do so. There are one or two remarks, however, which I wish to make. It is most useful to have the opinions brought together on a subject of such vital interest to all of us, as to whether an Ultimate, a Select or an Aggregate Table is the best for our use. I had already expressed my opinion in favor of the Ultimate Table and I am very glad indeed to hear Mr. Macaulay put it forward so strongly. I never had such a clear light on this subject as has been afforded by the discussion which he has entered on. There seems to me no doubt whatever that the Ultimate Table is the better for general use, for the reasons that Mr. Macaulay has made so clear, that it throws the saving from mortality against the initial expenses, charges the initial expenses against the new entrants themselves, and does not give that saving from mortality to old members who have not in any sense contributed to it. There are one or two of the extracts to which I would like to call particular attention, especially that from Mr. Hardy's remarks, which I do not remember to have observed until it was quoted by Mr. Hallman. He says, "The saving to an office by the light mortality of the first few years of assurance was generally less than the cost that the office had to incur in getting the business, and it seemed to him"—this is the important part—"there was nothing in principle against the use of a table like the $O^{M(5)}$ for all purposes of the office." With that opinion in general I agree. I think there are only one or two slight exceptions. For example, the charging of Term Premiums for short period policies; Term Insurance for one year, which is very common in Britain, or for three or five years, should, I think, be based on something different from an Ultimate Table, because so much depends on the rapid change in the mortality during the short period the policy is in force. On the other hand if the Term Insurance is of the nature of Renewable Term or Exchangeable Term, I am inclined for that purpose to favor the Ultimate Table also.

One other point here is brought out which I am much interested in. Mr. Hallman gives on page 22 and following pages tables showing the premiums deduced from the $O^{M(5)}$, the American Experience, the $H^{[M]}$ and $O^{[M]}$ Tables. The $H^{[M]}$ and $O^{[M]}$ I would ignore for the reasons Mr.

Macaulay has expressed, but in comparing the American Experience with $O^{M(5)}$ you will observe that for ages 20 and 25 the American Experience rates are higher than $O^{M(5)}$. Then for the period from 25 to 60 the American Experience rates are lower. Then, at the older ages they again become higher than the $O^{M(5)}$. Now, I have very often examined the American Experience Table and have observed the low mortality rates that prevail from about those ages, from 25 to 30, up to 65, and I have never been able to quite make up my mind whether that is a feature of American mortality. If there is anyone here who can give me light on the subject I shall be very glad indeed to have it, as to whether or not that is a peculiar feature of American mortality or whether it is a peculiarity of the American Experience Table which has no right to be there. Of course that is the reason for the lower reserves by the American Experience Table as compared with the $H^{[M]}$ Table to which Mr. Macaulay directed attention.

With these general remarks I merely wish to join others in thanking Mr. Hallman for submitting such an interesting subject for our discussion.

Mr. Sheppard: Mr. President, and Gentlemen, I feel that this is more a Canadian question that is being discussed, but there are one or two things I would like to say. I was much interested in what Mr. Macaulay had to say in support of an Ultimate Table as compared with Select Tables, but I should like to remind him and other members of the Society that what we call an Ultimate Table is really an Aggregate Table, although not a complete one. I was on the staff when those tables were begun, and, between ourselves, we used to call a table like the $O^{M(5)}$ or $H^{M(5)}$ a Truncated Aggregated Table. I think Mr. Ackland was the first who used that expression and although it never received official favor I think it more closely expresses what that table really is than the present expression of Ultimate Table.

My objection to an Ultimate Table as compared with an Aggregate Table is that it brings in another varying factor, as an Aggregate Table, being built up of a series of Select Tables involves the ratio of the entrants, while the Ultimate or Truncated Aggregate Tables also involves the rate of discontinuance in the entrants at different ages. Now, any cause that would tend to upset the rate of discontinuance might not act with equal force on entrants of all ages. I think, probably, this might be found in the United States at the present time when, on account of the New York law, we have a much larger issue of Term Policies. In fact in my own company the Term Policies have increased very largely proportionately with other years. I do not know how the average age at issue of Term Policies compares with that of Life and Endowment, but this increase is not so much a means of selection—at least not in the opinion of men competent to speak—against the company as a result of the prevailing ideas in the United States since the Investigation that what they want is protection at a low cost. The effect of the increase in Term Policies may be, perhaps, to change the rate of discontinuance between different

entrants and that will throw out the Ultimate Table based on the policies taken out at the present time, as compared with the previous experience of the companies. Personally I am of the opinion that on account of the fact that we do not need to value reversions and annuities separately and that it is just as easy to get all our functions from Select Tables as from an Aggregate Table, it would not add to the work in any way if everything were done on the Select Table basis. If that were the case the question of selection favoring one age at entry rather than another, would not come in. All classes would be treated equitably and such a table as the Select and Ultimate would never need to have been introduced. We could then have arrived at a more scientific way of taking account of the initial cost of the business such as is suggested in Canada at the present time, which does not need the idea of the profits from mortality at all.

Mr. Wright: Mr. President, I may say at once that I think the Om(5). Table an excellent table for general standard use, but observations were made by Mr. Macaulay and by Mr. Nichols which prompt me to remind you of a circumstance which I think it may be interesting to recall to your attention, if you already know it. There is a distinguished member of our Society, who is not present to-day, and whose absence I am sure we all regret, who wrote me a short letter on a certain occasion that I prize among the most valued papers I own. In it he expresses his regret that the New York State standard of valuation was ever changed from Farr's Table No. 3. I think myself this was a slightly drastic opinion—an extreme opinion perhaps I should say—but it evinces a philosophical comprehension which was characteristic of the gentleman, and as far as I know he has never altered that opinion. When he suggested the specialized mortality investigation, it was in following his advice and in accordance with the same opinion that Dr. Farr's Healthy Districts Table was taken as the standard of measure. And why? Simply because it is a true or pure table of mortality Mr. Macaulay in the outset of his paper called attention to the difficulty of finding a table of mortality which will suit the circumstances of all companies at all times. That is a circumstance which, much to my amusement, greatly concerns our Fraternal Associations. They think they must have a table which fits their own experience. Now, that is just what they should not have in my judgment. A true table is enough. And so in Mr. Nichols' discussion, for determining the deviations from normal mortality in the specialized mortality investigation, he took a table in which none of the conflicting effects of selection for or against the Company were involved, and philosophically in my judgment.

Just another illustration of that point. We all know that Descartes in his discovery of the mathematical utility of co-ordinates, extended our knowledge of the law of curves very greatly, but we know that the lines, the co-ordinates by which he studied curves were straight lines; and similarly a table of mortality which is derived from population experience makes the best standard. Insured life experience is so affected by the

varying conditions of companies, with the special classes to whom they appeal, the amount or proportion of new business that they do, and all those necessarily very varying circumstances, that finding a table which shall be a true measure in this way, is comparatively impossible. Approximately the very best tables are those like the OM(5) and the old Combined Experience, which I think Mr. Macaulay has not employed with the other figures in his comparison, but I think if he had done so he would have found that it would confirm his conclusion; it is one of the best tables for common use ever employed. And in closing I may say that the old Combined Experience Table does not need Makehamization in order to conform with the law of uniform seniority. In my practice I have frequently used it as according with the law, between the limits of youth, where the curve of mortality changes from high to low, and extreme age, and it works practically perfectly. As Mr. Nichols said, Makeham's principle cannot prove the law of mortality, but the law of mortality proves the correctness of Makeham's principle. Consequently good tables may be known to be good tables if fulfilling Makeham's ideas, or by being suitable to find values according to the law of uniform seniority, without change.

MR. MOIR: Mr. President, may I interpolate just one remark in regard to the word truncated? And point out that the experience was not truncated. It was exactly the opposite process that was applied. They did not cut off the top of the experience, they cut off the bottom. On that account it may be in a sense truncated, but not truncated as that word is generally understood. The use of this word caused much dissatisfaction amongst well-informed actuaries.

MR. MACAULAY: I think most of us will agree that the effects of initial selection will probably never wear out entirely in a properly selected life business; they continue to the very end of the table; but the major effects wear out in from five to ten years and after that the rates reach a uniformity which, I think, justifies us in speaking of them as an Ultimate Table. Ten years is certainly better than five, but a comparison of the figures for five and ten years shows how very close those two tables are.

I wish to express my very great gratification at finding that Mr. Sanderson agrees also in recommending this table. I am very much gratified to hear that.

MR. DAWSON: Mr. President, I expected to review, by invitation, this paper along with another paper. I regret very much Mr. Fackler's absence; it is exceedingly unfortunate under the circumstances, because he could have told us whether the fact which Mr. Moir called attention to, namely the comparatively low rates of mortality exhibited by the American Table at ages of about 30 to 65, was explained by the data or by the more or less arbitrary methods by which it was constructed. Whichever may be the case I have found in graduating tables from actual

experience that there is a good deal of evidence that it is the first. The thing that I referred to this morning was only an extreme illustration of it. I should not be surprised, if we had a wide-spread and thorough-going investigation of experience either in Canada or the United States, or in both, we might discover that this does not occur fortuitously and adventitiously but is really a characteristic of our experience.

Another point I wish to refer to is the name, and I have only a word to say concerning that. The word ultimate seems a very proper word when you take into account that practically all the investigations of the last 25 years have confirmed the view that in the absence of adverse selection the benefits of initial selection, while they do not entirely wear away, are evident in the lives after a certain number of years in about the same proportion, without regard to the number of years that have elapsed since the lives were insured.

There is one objection to the term ultimate. It may convey to some minds that that is a mortality table that cannot be exceeded. I think that is not true of any person who is at all acquainted with our science, but there may be minds to which it conveys that idea. Of course it is not for anything of that sort that such a table is made. My investigation of a number of cases in which adverse selection had set in very seriously would indicate that a very much higher table would have to be constructed to be ultimate in that sense if, indeed, such a thing is at all possible. But in the sense that it does express the normal, reasonable mortality that is to be expected without the influence of a real adverse selection, after the effects of the initial selection have worn off, the word ultimate seems accurate and appropriate.

SOME MODERN METHOD OF VALUATION—COLIN C. FERGUSON.

VOL. X, PAGE 30.

WRITTEN DISCUSSION.

MR. HALLMAN: I wish to congratulate the author on presenting a paper of such interest and profit to the profession.

Mr. Ferguson states: "It is not my wish to become involved in the arguments for or against a departure from the level net premium method of valuation. It will, therefore, be assumed that a company has decided in favor of such departure, and is looking about for the best means of putting its intention into effect."

In consequence of this remark I will not discuss the question as to the merits or demerits of the net premium method of valuation.

The question "Some Modern Method of Valuation," is one that has forced itself to the attention of actuaries ever since the initial expenses of procuring business have been somewhat excessive. The proper method of estimating the liability of a Life Insurance Company under its policies owing to the heavy acquisition expenses was thoroughly discussed and carefully examined in all its phases by the English and German Actuaries forty years ago. It is a remarkable fact that not one of the British Life Companies to-day values its liabilities by a method reducing its reserves in the early years of its policies in order to meet the initial expenses, but on the contrary the tendency has been to make the liabilities more stringent.

There are four methods that have been discussed in the past which are practically the same in principle, namely:

(1) Dr. Zillmar's method of 1863.
(2) Dr. Sprague's method of 1870.
(3) The Select and Ultimate Method by Mr. Dawson.
(4) The method by Mr. Ferguson.

The principles involved in each of the above methods are:

(a) Capitalization of a part of the loading which is borrowed from the first year's reserve, and is replaced by a fixed annual charge of the loading extending over either the whole or part of the premium paying period of the policy.

(b) The present values of the level net premiums, and the net premiums as adjusted, are equal at the issue of the policy.

(1) Dr. Zillmar's Method.

(a) The capitalized loading is $\pi_x - (P - .01)$
The annual fixed charge is $(P - \pi_x)$
The present value of the loadings capitalized $(P - \pi_x)a_x$

To prove that $\pi_x - (P - .01) = (P - \pi_x)a_x$

Given $P = \pi_x + .01(\pi_x + d)$

$$\therefore (P - \pi_x) = .01(\pi_x + d) = \frac{.01}{1 + a_x}$$

$$\therefore (P - \pi_x)(1 + a_x) = .01$$

$$\therefore P - \pi_x - .01 = -(P - \pi_x)a_x$$

$$\therefore \pi_x - (P - .01) = (P - \pi_x)a_x$$

(b) $\pi_x(1 + a_x) = (P - .01) + Pa_x$.

(2) Dr. Sprague's Method.

(a) The capitalized loading is $\pi_x - \pi_{x'\overline{1}|}$

The annual fixed charge is $\pi_{x+1} - \pi_x$

The present value of the loadings capitalized $(\pi_{x+1} - \pi_x)a_x$

To prove that $\pi_x - \pi_{x'\overline{1}|} = (\pi_{x+1} - \pi_x)a_x$

$$A_x = (1+a_x)\pi_x = 1 - d(1+a_x) = v - da_x$$

$$\therefore (1+a_x)\pi_x = v - da_x = v(p_x + q_x) - da_x = vq_x + \frac{a_x}{1+a_{x+1}} - da_x$$

$$= \pi_{x'\overline{1}|} + \left(\frac{1}{1+a_{x+1}} - d\right)a_x = \pi_{x'\overline{1}|} + \pi_{x+1}a_x$$

$$\therefore \pi_x - \pi_{x'\overline{1}|} = (\pi_{x+1} - \pi_x)a_x$$

(b) $(1+a_x)\pi_x = \pi_{x'\overline{1}|} + \pi_{x+1} \cdot a_x$

(3) The Select and Ultimate Method of Mr. Dawson.

(a) The capitalized loading is $-(A_{[x]} - \pi_x \mathbf{a}_{[x]})$

The annual fixed charge is $\pi_x - \pi_{[x]}$)

The present value of the loadings capitalized $(\pi_x - \pi_{[x]})\mathbf{a}_{[x]}$

To prove that

$$-(A_{[x]} - \pi_x \mathbf{a}_{[x]}) = (\pi_x - \pi_{[x]})\mathbf{a}_{[x]}$$

which is self evident.

(b) $\pi_{[x]}(1+a_{[x]}) = \pi_x - (\pi_x - \pi_{[x]})\mathbf{a}_{[x]}$ the 1st year's premium

$+ \pi_x a_{[x]}$ the subsequent year's premium

$= \pi_{[x]}\mathbf{a}_{[x]}$

The capitalized loading is clearly shown in the following table:—

AGE 35 AT 3 %

	Premium—American Experience	Premium—Select Experience	Difference (3)	Life annuity Select, $\mathbf{a}_{[x]}$ and $\mathbf{a}_{[x]\overline{n}|}$ (4)	Capitalized Loading 3) X (4)
Whole Life.....	21.0812	20.5478	.5334	20.1312	10.74
5 Pay Life.....	90.5970	88.6726	1.9244	4.6650	8.98
10 Pay Life.....	49.7252	48.5501	1.1751	8.5201	10.01
15 Pay Life.....	36.3371	35.4499	.8872	11.6687	10.35
20 Pay Life.....	29.8504	29.1103	.7401	14.2098	10.52
10 Year Endowment.	89.3004	88.2428	1.0576	8.5201	9.01
15 Year Endowment.	57.4148	56.5734	.8414	11.6687	9.82
20 Year Endowment.	41.9660	41.2475	.7185	14.2098	10.21

(4) The Method of Mr. Ferguson.

(a) The capitalized loading is $(\pi_x - \pi_{x'\overline{1}|})$

The annual fixed charge $\left(\pi_x + \frac{\pi_x - \pi_{x'\overline{1}|}}{a_{x\overline{4}|}} - \pi_x\right)$

The present value of the loadings capitalized

$$\left(\pi_x + \frac{\pi_x - \pi_{x'\overline{1}|}}{a_{x\overline{4}|}} - \pi_x\right)a_x$$

To prove that

$$(\pi_x - \pi'_{x\overline{1}|}) = (\pi_x + \frac{\pi_x - \pi'_{x\overline{1}|}}{a_{x\overline{4}|}} - \pi_x)a_{x\overline{4}|}$$

which is self evident.

(b) $(1+a_x)\pi_x = \pi'_{x\overline{1}|}$ first year's premium

$$+ \left(\pi_x + \frac{\pi_x - \pi'_{x\overline{1}|}}{a_{x\overline{4}|}}\right) a_{x\overline{4}|} \text{ subsequent four years' premium}$$

$$+ \pi_x \cdot 4/a_x \text{ subsequent premiums after five years}$$

$$= \pi'_{x\overline{1}|} + (\pi_x a_{x\overline{4}|} + \pi_x - \pi'_{x\overline{1}|}) + \pi_x \cdot 4/a_x$$

$$= \pi_x + \pi_x(a_{x\overline{4}|} + 4/a_x)$$

$$= \pi_x + \pi_x \cdot a_x = (1+a_x)\pi_x$$

I have, therefore, shown by (a) and (b) that the four methods are founded on the same principle, being slightly different forms of the same idea.

It is a controversial question which is the best method.

Mr. Ferguson states: "A sound position to take in the matter, is that, while a deduction may properly be made from the full reserves of the early durations, the full valuation should be maintained for policies which have been a reasonable time in force. In order to attain to the full level premium reserve at the end of a reasonable time, say five years, it will now be necessary to increase the level net premiums of the second to fifth years, inclusive." It might be asked why is "five years" a more reasonable time than six or eight or ten years? The five year term is emphasized as a strong feature in favor of this method while others consider this the peculiar weakness of the method and are in favor of Dr. Sprague's Modified Preliminary Term Method where the repayment of the loading is extended over the premium paying period of the policy instead of limiting it to a purely arbitrary period of five years and cramping the operations of a company by restoring the reserve to its normal level in too short a time.

Mr. Ferguson states: "It will be apparent that the plan may be correctly described as an adjusted net premium method, inasmuch as the present values of the level net premiums and the net premiums as adjusted above are equal at the issue of the policy." This, however, is true of the four methods as shown in each case by (b) and they would, therefore, be adjusted net premium methods of valuation. This is a controversial question. Can it not be said of all gross premium methods of valuation that they are adjusted net premium methods?

I greatly appreciate Mr. Ferguson's paper, and I believe it will be a source of considerable discussion which is always profitable.

THE ULTIMATE TABLE Om(5).—M. S. HALLMAN.

VOL. X, PAGE 9.

SOME MODERN METHODS OF VALUATION—C. C. FERGUSON.

VOL. X, PAGE 30.

WRITTEN DISCUSSION.

MR. DAWSON: These papers by younger representative Canadian actuaries should be considered together. They typify the progressive and the reactionary movements, now clashing in the United States and in Canada. At the present moment the reactionary movement is dominant in Canada; but it is not possible that this will be the case, permanently. The champions of progress can afford to smile at the efforts put forth to conceal the true nature of the adjustment which reactionaries are already compelled to make.

Mr. Ferguson's proposal is such an adjustment, with its true nature concealed. A few years ago, the men who now espouse it so warmly, were denying the propriety of a minimum standard lower than net level premium reverses, and were aiding to checkmate the efforts of certain Canadian companies to secure preliminary term valuation. They have progressed perforce since then.

Mr. Ferguson aims at a convenient substitute for "select and ultimate," embracing what he esteems its most valuable feature, that the "full reserve" is made good in five years. He has attained this "without introducing the words 'mortality gains,'" as he puts it; but, he acknowledges, so far as non-participating policies are concerned, not without "anticipation of the general profits of the four subsequent years." In plain English, not without anticipating salvages in mortality, due to fresh selection—and, perhaps, more.

As to participating policies, it is equally true that these salvages are anticipated. If the loading beyond the corresponding non-participating premiums is not for dividends, it is plainly unwarranted. Moreover, if the Canadian companies had no mortality salvages during the first five years, they could not pay renewal and management expenses the second to the fourth policy years, inclusive, after such a deduction from the loading for those years is made. These companies in their reports to the Royal Commission allocated to cost of new business out of their expenses in 1905 an aggregate sum exceeding Mr. Ferguson's proposed allowance by 51 per cent.; yet the remainder of the total expenses amounted to 62.4 per cent. of the loading on renewal premiums. Allocating to cost of new business no more than Mr. Ferguson proposes, the ratio to loading on renewal premiums rises to 85.6 per cent. At age 35, Mr. Ferguson's method results in there being available for expenses the second to the fourth year, inclusive, the following percentages of the loadings:

Life	47 per cent.
20 Payment Life	39 per cent.
20 Year Endowment	33 per cent.

This means that, on the basis of cost of new business limited as proposed by the Royal Commission to Mr. Ferguson's allowances, the net amount available for other expenses the second to the fourth year, inclusive, would be short (taking the 20 Payment Life as the average) about 12 per cent. of the loading, or more than 30 per cent. of the provision which he leaves; and, at the rate of cost of new business prevailing in 1905 in Canadian companies, they would be short about 23.4 per cent. of the loading, or about 60 per cent. of the provision which he leaves. Mr. Ferguson's method, therefore, would not be practical as to participating policies, even if they did not participate for five years, if it were not for the "mortality gains," the words betokening which he successfully evades but not the necessity for relying on the thing, itself.

It is to be remembered, also, that in all companies, perhaps, there is somewhat greater pressure upon the loading the second to the fourth year, inclusive, instead of less, and especially so in new companies.

The foregoing is a concrete demonstration of the fallacy of Mr. Ferguson's paper. The following is an equally conclusive demonstration of it, by recourse to mathematical formulas. The general formula for a gross or office premium is:

$$P = \frac{A+F}{a}(1+k)+c$$

in which F = provision for cost of new business. In Great Britain, it is usually taken at .01 of the sum insured and both A and a by a select table *which latter must be done, indeed, if there is not to be a "concealed loading."* In the United States and Canada, A and a have been by a mortality table which is virtually ultimate and the formula for the gross premium is:

$$P' = \frac{A}{\mathbf{a}}(1+k)+c$$

$$= \frac{A_{[\,]}+(A-A_{[\,]})}{a_{[\,]}+(a-a_{[\,]})}(1+k)+c$$

$$\frac{A_{[\,]}+(A-A_{[\,]})\frac{a_{[\,]}}{\mathbf{a}}}{a_{[\,]}}(1+k)+c$$

Equating with the general formula in order to ascertain the "concealed loading" we have:

$$\frac{A_{[\,]}+F}{a_{[\,]}}(1+k)+c = \frac{\left\{A_{[\,]}+(A-A_{[\,]})\right\}\frac{a_{[\,]}}{\mathbf{a}}}{a}(1+k)+c$$

$$A_{[\,]}+F = \left\{A_{[\,]}+(A-A_{[\,]})\frac{a_{[\,]}}{\mathbf{a}}\right.$$

$$F = \frac{Aa_{[\,]}}{a}-A_{[\,]}$$

This is the "select and ultimate" margin. Mr. Ferguson's proposed provision, $P_x - P'_{x\overline{1}|}$, cannot be deduced in a similar manner and its applicability to the conditions in companies in the United States and Canada which now value on the net level premium basis, without a change in their premiums, rests entirely upon its virtual equivalence to the "select and ultimate" margin. There is no such equivalence under the conditions at present existing in Canada; for the "select and ultimate" margins by the O[M] table exceeds these allowances and the mortality experience of Canadian companies indicates that to use a less favorable select table than the O[M] is still to conceal a considerable amount of the loading.

Of course it would be possible to load for cost of new business the amounts suggested by Mr. Ferguson. I give formulas for this in the second edition of *Practical Lessons in Actuarial Science*, Volume 1, page 235. But to load thus and have the provision made good in five years, is very awkward and it would be hard to explain for what purpose the much larger net loading after the fifth year has been imposed or will be required.

The merits of Mr. Ferguson's method, therefore, for practical use are dependent wholly upon how closely it approximates the "select and ultimate" method. There is really nothing to compare it with, in which the H[M] table can be taken as an ultimate table and the period affected by fresh selection as five years; the O[M(5)] gives lower reserves and the period affected by fresh selection is in this experience found to be really 10 years. The O[M(10)] brings out premiums and reserves very nearly identical with the H[M] but is unavailable because the period is 10 years. Therefore, for purposes of comparison, I have applied Mr. Ferguson's method to the American Experience table and compare it with the "select and ultimate" standard of New York, as follows:

LIFE PLAN, AGE 35.

Initial Margin.

Select and Ultimate	$10.67
Ferguson's Method	11.26

Made Good in year.	Select and Ultimate.	Ferguson's Method.
1	4.39	0.00
2	3.08	3.13
3	2.21	3.13
4	1.34	3.13
5	.45	3.13

RESERVES.

Years.	Select and Ultimate. Mean.	Terminal.	Ferguson's. Mean.	Terminal.
1	7.18	5.12	4.32	0.00
2	22.57	20.11	18.97	14.90
3	37.37	34.72	34.13	30.31
4	51.79	48.94	49.80	46.25
5	65.79	62.73	66.01	62.73

The roughness of the approximations is obvious at a glance. In the early years reserves by Mr. Ferguson's method do not approach the ultimate reserves so rapidly as do reserves by the "select and ultimate;" in later years more rapidly. In the former, less is "made good" than mortality salvages warrant; in the latter, more. Were this reversed, his method would be utterly impractical; as the case stands, it is practicable if the savings in the earlier years are husbanded to eke out the margins in later years; but if the management, not apprehending this want of adaptation to the facts, fails to hold surplus to do this, it may find itself in a trap.

An arbitrary, scheme of "allowances," such as Mr. Ferguson proposes peculiarly conduces to variations. It may be $P_x - P'_{x\overline{1}|}$ as here proposed, or .01 of the sum insured, as in Great Britain, or what you will, no fixed principle guiding the choice. Thus we already have the following variations, actually in use or proposed for use.

Full Preliminary Term.
Modified Preliminary Term.
Ferguson's.
New Jersey.
Committee of Fifteen.
Illinois.
Michigan and Tennessee.
Wisconsin.
Indiana.

When the "select and ultimate" principle is accepted, there can be no variations, except by changing the mortality table or the rate of interest.

Were Mr. Ferguson's methods not subject to these objections and were reserves by it the same as by the "select and ultimate" method, so that the choice was strictly between the same things under different semblances, there would yet be the most cogent objections to his method because of the form it takes. This is as follows: The H^m reserve is the full regular reserve—"really necessary for complete solvency" as it is sure to be put; certain companies, because of weakness, are permitted to deduct certain allowances. "These are not exactly solvent, you know." In other words, it imports an accusation of financial un-

soundness in their cases; instead of an affirmation of superior strength for those which voluntarily carrying a higher reserve than the minimum standard requires. In times of financial stress this moral distinction might react upon a company which had thus discriminated to injure a weaker rival, to its utter ruin in popular esteem, at least. There is a world of difference between having the prudence of the management in holding voluntarily larger reserves than were absolutely necessary, triumphantly vindicated by the event, the extra reserves being called upon and proving an element of strength, and having the folly of the management in setting up under the seal of the state, as "really necessary for complete solvency," a larger reserve than is in fact required, shown conclusively by the company being compelled to drop to the standard which it had discredited.

Mr. Ferguson's method would be cruelly unjust to all companies that avail themselves of it at once; and yet more injurious, though by no means unjust, to companies that later resort to it, because compelled to do so.

All these obvious consequences would be invited merely in order to substitute "tweedle-dum" for an exactly equivalent "tweedle-dee;" and worse, as has been shown, may be expected from the fact that they are not exactly equivalent.

The chief value of Mr. Ferguson's paper from a scientific point of view is the mathematical demonstration of what all discerning minds already knew, that all of these methods, including select and ultimate, are ways of making a provision for cost of new business and are resolvable into modifications of gross premium valuation. This last is true, also, of net level premium valuation. The study of these transformations will be of value to students preparing for examination and the formulas which are given herewith, will supplement them.

The method of making an adjustment in the manner now proposed by Mr. Ferguson is not new. Mr. Wm. Hendry, a veteran Canadian actuary and a charter member of this Society, first suggested virtually this form of allowance and of making it good, in the *Insurance Monitor*, February, 1899. His proposal was that for the four years following the first the reserve be taken as $\frac{5V_x}{4V_{x+1}}({}_{n-1}V_{x+1})$, thus making the reserve good in five years. Mention is made, by Mr. Ferguson, of modified preliminary term, which makes the same allowance, but credit is not given to Dr. Zillmer, Dr. Sprague or Mr. McClintock for the idea of thus limiting it.

Mr. Hallman's paper gives due and generous credit to all those ideas he appropriates, and is manifestly the result of painstaking and very discriminating research and patient consideration. He finds a mortality table in use which must soon be discarded and the continued use of which will be attended by manifold embarrassments; and he gives the following conclusive reasons for his paper:

"As already stated it is not unlikely that the present tables may be superceded in Canada by the O^{M}, or $O^{[M]}$ (Select), and consequently my purpose is to present in this paper the results of my study of the merits and demerits of these tables."

This should be contrasted with the efforts of Mr. Ferguson and others to make arbitrary adjustments, so that the present misfit standard may be preserved.

He first premises that, for convenience, "provided the premiums and reserves brought out were reasonable and practical," a table, representing ultimate experience and graduated by Makeham's law ought to be selected. Such a table he finds the $O^{M(5)}$ to be. As to reserves, he finds upon investigation that this standard, with three per cent. interest, brings out only about five per cent. higher values than the present standard in Canada, H^{M} 3½ per cent.; and he rightly reasons that a transition would not be difficult.

His arguments would have even greater validity, if adduced in support of a proposal to adopt the $O^{M(10)}$ table which is also graduated by Makeham's formula, which is more truly an ultimate table and which corresponds in premiums and reserves, even more closely to the present Canadian standard, than the $O^{M(5)}$.

In a communication that was republished in the *Chronicle* (N. Y.), on September 27, 1906, Mr. Hallman does not mention "select and ultimate" valuation, but he does give as a reason for adopting the $O^{M(5)}$ mortality table, that "the table being an ultimate one takes note of the gains in mortality which may be justly set against the initial expenses of procuring business, thereby returning a large portion of the heavy outlay to the older policyholders who are properly entitled to the benefit." This conforms to the actual practice (though that fact is not always obvious) of the companies of the United States and Canada.

There is no gainsaying that the companies of neither country can long maintain antiquated, inaccurate and misleading mortality tables as the standards for computing premiums and values. The best standards would be mortality tables deduced from insured lives in those countries, respectively, and divided into select and ultimate sections. Pending the preparation of these, the $O^{[M]}$ and the $O^{M(10)}$ tables are the most suitable.

Oral Discussion.

Mr. Ferguson: Eliminating matters which have little concern with the discussion, the only serious criticism which has been made against my paper is to the effect that the loadings in the 2nd to the 4th years are insufficient to make good the amount anticipated in the first year as well as to look after the synchronous expenses. So far as this is stated in reference to non-participating policies I have no objections to make. I am quite willing to admit, as I have done in the paper that in these cases the first year's allowance is based on an anticipation of the subsequent four years' profits. Will my critics contend that such an anticipation is not justifiable? I think they will not. Mr. Dawson states that this

admission proves that my method is identical with select and ultimate. But is it not possible that in certain cases and certain companies the actual savings in mortality will be insufficient to make good the allowances by the select and ultimate method and that recourse must again be had to the "general profits" of the company. This possible failure might suggest to a critic of Mr. Dawson, which I do not mean to be, that the select and ultimate is also an "adjustment with its true nature concealed."

Coming to participating policies, it has been charged that the provision by the Canadian method will again be insufficient in view of the conditions actually prevailing. It must here be remembered that we are attempting to find a standard for the use of the Government, which must not appear even to suggest extravagance to the companies. It is, in my opinion, a merit in the Canadian method that it appears to place a limit on the extent to which expenditure should go. Further, I believe that a true analysis of the expenditure of Canadian offices would show that the expenses to be charged against the first four years' premiums are actually within the limits suggested, but not prescribed by the Canadian method. I do not agree with the view that such expenses should be charged at a higher rate than subsequent ones. Why should they be, when the special cost of procuring the business has already been borne and when no funds to speak of have been accumulated for investment?

The fact that in non-participating policies we may be forced to look to the gains from mortality to assist the loading in performing its functions, does not prove that the Canadian method is identical in form or even in principle with the select and ultimate. The two methods work on altogether different lines, that is, if we adopt Mr. Dawson's interpretation of select and ultimate.

In my paper I have offered another interpretation, the accuracy of which has not been impugned. This being similar to the line of reasoning which suggested the Canadian method, renders a comparison more easy. Let me quote from the paper, page 38:

"The Modified Preliminary Term method fixes the first year's net premium and allows the subsequent ones to take care of themselves. The Select and Ultimate Method determines in advance the renewal net premium and then calculates the first to correspond. The effect of both on the normal level premium reserves is to lower them throughout the whole premium-paying period of the policy. To complete the comparison, we will recall that the Canadian method fixes the first year's net premium in the same manner as does the Modified Preliminary Term method. Instead of adjusting all the remaining premiums, it merely increases four of them so as to bring out normal reserves at the end of the fifth year."

This extract states clearly the difference between the Canadian method and the select and ultimate, and I feel like exclaiming with the poet,

> "Strange that this difference should be
> 'Twixt tweedle-dum and tweedle-dee."

Mr. Dawson apparently has found difficulties in adapting the Canadian method to the expense limitations. Let me say here—and I know I can speak not for myself only but for the Canadian Life Officers' Association as well—that there was no intention of combining the two. We have been, in fact, strongly opposed to the paternalism run mad, which would impose a statutory limitation of expenses. Accordingly the responsibility for the difficulties my critic has encountered must rest on other shoulders.

Mr. Dawson has stated an objection to the Canadian method, namely, that it results in lack of uniformity in the standards employed by different governments. I do not consider this a fatal objection and the consensus of actuarial opinion in a given state or country should be permitted to decide such matters as the amount of the first year's allowance and the term of years within which it must be made good. If this were a proper ground for criticism, the select and ultimate would not escape. Suppose we alter the percentages of the rates of mortality which form the foundation for the select table used by Mr. Dawson, and we would have an infinite variety of "select and ultimates." The reason why this variety has not yet appeared is, perhaps, because few governments have given their approval to select and ultimate by adopting it.

The form which the Canadian method assumed in the report of the Royal Commission has been objected to. The method itself should not be criticised on that ground and, in fact, I am inclined to agree with Mr. Dawson that the form was not well chosen.

I would feel very much grieved if any member of the society should think me guilty of failing to give proper credit to others. My paper is not an historical one, and to mention the names of the gentlemen referred to by Mr. Dawson—gentlemen whose work is so well known that I could not possibly detract from their credit,—would be to carry me back in the history of life insurance for fully a century and would have unduly protracted my remarks.

I am much interested to know of Mr. Hendry's suggestion along lines similar, though by no means identical, with mine. This is an age of improvement when broad-minded men are ever ready to discard old methods in favor of new and better ones. As Mr. Hendry at once gave his approval to the Select and Ultimate method, I have no doubt that he like all other Canadan actuaries, would now be quite willing to give his adherence to the Canadian method as perfecting his own ideals.

MORTALITY AMONG INSURED LIVES ENGAGED IN CERTAIN OCCUPATIONS INVOLVING HAZARD, SUCH AS LIQUOR DEALERS, RAILROAD MEN, MINERS, POLICEMEN AND ELECTRICIANS.—
ARTHUR HUNTER.

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ORAL DISCUSSION.

MR. CRAIG: Mr. President and Gentlemen, I have not any written remarks bearing on this paper. The figures which Mr. Hunter has presented corroborate the results so far as they apply to the specialized investigation on similar cases. In regard to the army officers some of the gentlemen present will, perhaps, remember a few years ago, because of some active if not acrimonious competition between agents of some companies, an effort was made to secure some concession from the companies on the lives of commissioned officers in the army. One company agreed to write these officers on certain forms of policies without extra premium, provided that not less than 51 per cent. of the officers in any army post that took the matter up should become insured. So far as that particular company was concerned that condition killed the proposition. My friend, Mr. Washburne, was particularly interested in that proposition, having himself been a graduate from West Point and exceedingly desirous of having some general plan of action adopted by which army officers should be insured without an extra premium. I think that Mr. Hunter's statistics as printed in this paper show conclusively that such a thing ought not to be done. Whether Mr. Washburne has changed his mind or not, I do not know. He is here and can speak for himself.

There is one paragraph in Mr. Hunter's paper that I hardly concur in. It appears on the 45th page.

"While the Specialized Investigation was a great step in advance, it is not entirely satisfactory, for the reasons which have been pointed out. I hope that in the next few years the Actuarial Society will make another investigation which will be so planned that the types of impairment, whether due to physical condition, family history, personal history or hazardous occupation, will be so sub-divided that deductions may be made therefrom which will serve as a thoroughly reliable guide in the acceptance of risks."

The sub-division that Mr. Hunter here recommends seems to me will involve a vast amount of additional labour and would hardly be justified by the results except from a purely academic point of view. In my mind the point is that too great refinements in the classification of risks in life insurance ought not to be undertaken. Such a thing is not thought of in the ordinary form of insurance, and yet unquestionably there would be found as wide a divergence between risks in different occupations or in different localities or by reason of different environments, a thousand and one different conditions which might be

classified and the results show a great difference in the rate of mortality among the different classes. Yet it is not deemed necessary. It is not considered worth while even to consider the matter. I presume that even a division of risks insured in the Dominion of Canada as distinguished, for instance, from those insured in the Commonwealth of Massachusetts, would show a wide difference in the rate of mortality. Assuming that to be so, should we by reason of that fact, make one rate of premium for the residents of Canada and another rate for the residents of Massachusetts. And if it is not expedient to make such classification among people who are considered ordinary risks, is it expedient to go too far into the details of a classification with those who are considered below the average standard?

Now, our friend Mr. Hunter has, for instance, made four different groups of persons connected either directly or indirectly with the liquor business, and there is a difference in the rate of mortality among all those classes. The rate among bar-tenders was 178 per cent. Among saloon-keepers, 132 per cent. Mr. Hunter must have smiled as he made his comment on the three sub-divisions he has made of the saloon-keepers. Those who attended bar the greatest number of hours showed the smallest rate of mortality and the rate increased as the number of hours attendance upon bar decreased; but the total of the three classes was 132 per cent; and the rate was 148 per cent. among proprietors, managers and clerks of hotels and restaurants with bars, and 147 per cent. among proprietors, managers and clerks of general stores where liquor was sold. Now the general average of these four classes was 142 per cent.

The company with which I am connected has written some substandard policies on liquor dealers; but we make one general class of all those who are connected in any way with the manufacture or sale of liquor, and we have not been so rigid as has the New York Life in undertaking to make a discrimination between those who should be limited to short term endowment and those who were permitted to take other forms of insurance. They were, as a rule, either accepted or rejected. I presume that our method also is different from that of the New York Life in the acceptance or rejection of these risks. It is done by a system of voting, three affirmative votes permitting a man to enter and not more than two of those votes can be from the medical division. There may be five or six votes on a risk before three affirmative votes are secured.

I would like to state our experience on that class of risks. The total number of years at risk was 17,622, and the per cent. of actual to probable was 59 per cent. That means that it was 59 per cent. of double the Actuaries and the American Tables. Double the Actuaries up to 1901 and double the American Table since that time. That is the q_x column, doubled practically all the way through. On the Actuaries Table the rate was doubled up to 60 and then gradually merging into the Combined Experience Table. That table you will find in Vol. I. of the *Transactions*.

There is one thing that occurred to me that I thought might be of interest to bring to the attention of the members. As a rule those per-

sons engaged in or connected in any way with the liquor business knew beforehand that they were to be given what we designate Special Class Policies. The ratio of such policies issued in the year 1905 which were returned not taken was 26 per cent. out of a total of 1954 policies written. Now, if we compare that with another group, the members of which did not know what kind of policy they were going to receive, the group comprised in Family History, for instance, we find that out of 2,337 policies written in 1905, 1,417 were returned as Not Taken, or 60 6-10ths per cent. Family History is one of the groups that Mr. Hunter alluded to. The particular group to which I desire to call your attention is that designated "Impaired Heart." The ratio of Not Takens was 58 per cent. and the rate of mortality was 101 per cent. The point is that presumably we did not get the group of risks in that class that we contemplated getting. Most of them were put in there because of mitral-murmur and with rare exceptions none over 50 years of age. Now I believe that what we call self-selection has entered largely into that group; that when the special class policy was presented to the applicant, for which he did not apply, but to which his attention was directed with the delivery of the policy, if he were not conscious of any heart impairment the probability is that he would refuse to accept the policy; while the man who was conscious of, or had in the past his attention called to, the fact that there was an impairment of the heart would accept his policy and pay the premium. So that, to my mind, the probability is that the vast majority of those persons who refused to accept their Policies were such slightly impaired risks that when they dropped out the mortality on the balance increased very materially; so that we find a rate of 101 per cent. extending over a period of 8 years and involving 6,768 years of risk.

Written Discussion.

Mr. J. K. Gore: So unreliable for insurance office purposes are most of the data available as to the hazard of certain occupations that Mr. Hunter's paper is a welcome contribution. It is to be hoped that as the experience of his company becomes more and more extensive he will be able to give us the benefit of that experience in those classes for which he has not sufficient data at present.

When we remember that, as Mr. Hunter states, the Specialized Mortality investigation represents in the case of most of the hazardous occupations an extraordinary, instead of an ordinary selection on the part of the companies, we realize that it is only from data of the kind given us in this paper that we can learn the safe and equitable method of treating this class of business.

I trust the paper under consideration may be the means indirectly of bringing before the Society the experience of other companies as to hazardous occupations. At the office of the Prudential the causes of death have been investigated among nearly one hundred occupations, occupations at death being taken, instead of at the date of issue of the insurance.

Army Officers, however, are the only class in connection with which we have also ascertained a ratio of actual and expected deaths. Our tabulated experience with Commissioned Officers of the Army begins with 1898 and terminates with policy years ending in 1906. It, therefore, includes the period of the Spanish War. On the basis of the American Experience Table the deaths have been 81 per cent. of the number expected. On the basis of the Compound Progressive Table of the New York Life this percentage is 130. Of course, this percentage cannot be fairly compared with that of the New York Life because most of the period covered has been since the Spanish War, when conditions in the army, sanitary and otherwise, have been at their best.

Among the various occupations referred to by Mr. Hunter our subclassifications are not always the same as those made by the New York Life. The following comparative percentages of deaths from violence and other causes seem interesting enough to be recorded in our *Transactions*. In these tables I have included, where possible, the experience of the Prudential under its Industrial as well as under its Ordinary policies:

ARMY.

Total Number of Deaths.

New York Life	41
Prudential Ordinary	45
Prudential Industrial	462

Percentage of Deaths from Violence.

New York Life	36 per cent.
Prudential Industrial	23 per cent.
Prudential Ordinary	29 per cent.

Typhus, Dysentery and Cholera.

New York Life	22 per cent.
Prudential Ordinary	36 per cent.
Prudential Industrial	32 per cent.

The data of The Prudential Ordinary include a few privates, but are derived mostly from Commissioned Officers. In the Prudential Industrial the data are derived almost entirely from privates.

NAVY.

Total Number of Deaths.

New York Life	35
Prudential Ordinary	9

Violence.

New Lork Life	54 per cent.
Prudential Ordinary	33 per cent.

CITY FIREMEN.

Total Number of Deaths.

New York Life	29
Prudential Ordinary	35

Violence.

New York Life	50 per cent.
Prudential Ordinary	37 per cent.

SERGENTS, POLICEMEN, PATROLMEN, ROUNDSMEN, AND DETECTIVES.

Total Number of Deaths.

New York Life	83
Prudential Ordinary	81
Prudential Industrial	481

Violence.

New York Life	20 per cent.
Prudential Ordinary	14 per cent.
Prudential Industrial	10 per cent.

For Sheriffs, Bailiffs and Marshals we have no separate data.

LOCOMOTIVE ENGINEERS.

Total Number of Deaths.

New York Life	146
Prudential Ordinary	46
Prudential Industrial	117

Violence.

New York Life	50 per cent.
Prudential Ordinary	35 per cent.
Prudential Industrial	32 per cent.

LOCOMOTIVE FIREMEN.

Total Number of Deaths.

New York Life	71
Prudential Ordinary	41
Prudential Industrial	172

Violence.

New York Life	60 per cent.
Prudential Ordinary	59 per cent.
Prudential Industrial	56 per cent.

RAILROAD CONDUCTORS.

Total Number of Deaths.

New York Life	38
Prudential Ordinary	26

Violence.

New York Life	37 per cent.
Prudential Ordinary	35 per cent.

The above New York Life data include only freight conductors. For the Prudential they include both freight and passenger conductors.

MINERS.

Total Number of Deaths.

New York Life	195
Prudential Ordinary	120
Prudential Industrial	1761

Violence.

New York Life	38 per cent.
Prudential Ordinary	41 per cent.
Prudential Industrial	23 per cent.

Of the Prudential Ordinary deaths among miners 68 were of coal miners and 52 of other miners. 49 per cent. of the former were violent and 31 per cent. of the latter. Almost all of the Prudential Industrial deaths were among coal miners. Of the Prudential Ordinary deaths of miners other than coal miners, 21 per cent. were from pneumonia and other acute diseases of the respiratory organs. This percentage is the same as that of the New York Life.

The Prudential has no separate data for Managers, Superintendents, Mining Engineers, Foremen, Captains, Bosses and Overseers.

ELECTRIC CLASS.

Here the classification of the data is not the same in the two companies. In the New York Life there were 19 deaths of "engineers who handle a voltage of 110 and above; dynamo tenders and linemen," of which at least sixteen per cent. were caused by accident. In the Prudential Ordinary there were 18 deaths of electric linemen, of which 11, or 61 per cent. were caused by accident. In the New York Life there occurred 55 deaths among "electrical experts, electrical engineers and stationary engineers who occasionally handle live wires," of which 29 per cent. were accidental. The Prudential Ordinary had 57 deaths among "electrical industries 'other,'" of which 26 per cent. were caused by accident. In the Industrial Department the electrical industries were not sub-divided into groups, but in the entire class there were 716 deaths, of which 23 per cent. were accidental.

JOHN K. GOBE.

Mr. C. W. Jackson (*Presented by M. M. Dawson*): A fraternal organization in Montana, having about one-third of its membership miners,—who were treated as other members, *i. e.*, were not assessed more heavily on account of occupation,—had experienced such heavy mortality recently that it had its mortality experience investigated. The data consisted of. the number of members and amount at risk, at all ages, at the beginning and end of the years 1905 and 1906; also the number of miners and amounts at risk for all ages at the beginning and end of each of the years 1902-3-4-5. In the years 1905 and 1906 there were 5380 years of exposure among all the members and $10,258,000 at risk; the actual experience was 126 deaths and $242,146 loss. According to a table based upon the experience of the whole order which has lodges in nearly every State, the expected deaths were 87 and expected loss $163,908.

As the number of miners in good standing at the end of 1906 was not given it was impossible to separate the experience for that year.

In 1905 the experience of the whole membership was: 2855 years of exposure and $5,507,900 exposed to risk; 46 expected and 67 actual deaths. $87,161 expected and $129,010 actual loss.

The experience among miners in that year was: 719 years of exposure, $1,457,000 at risk, 9.2 expected and 36 actual deaths; $28,903 expected and $70,778 actual loss.

From these figures it will be seen that the extra loss experienced was entirely due to the miners.

In 1905-06, 60 miners died after an average membership of ten years and five months Of these 26 died from consumption, 4 from bronchitis and pneumonia, 9 from accident and the remainder from various causes.

The following table gives the miners' experience for the years 1902-3-4-5 and a comparison between the actual deaths and those expected according to "The table of the order" "Modified Healthy English Table" as used in the Specialized Mortality Investigation and the American Experience Table.

Ages.	Actual Deaths.	Table of Order. Expc'd Deaths.	Table of Order. % of Actual to Expc'd.	Modified Healthy, English. Expc'd Deaths.	Modified Healthy, English. % of Actual to Expc'd.	Am. Exper. Expc'd Deaths.	Am. Exper. % of Actual to Expc'd.
18–28	2	1.883	106	2.161	93	2.269	88
29–42	28	13.698	204	14.140	198	14.681	191
43–56	52	16.498	316	14.991	347	15.760	330
57–72	11	7.405	149	5.548	198	6.063	181
All ages.	93	39.484	236	36.840	252	38.773	240

These results are very similar to those obtained by Mr. Hunter.

Mr. W. C. Wright: It seems hardly useful for me to speak after the careful paper that has been read on the question, but I may simply say that I think I quite agree with Mr. Hunter's general observations and that the paper is, of course, a very valuable accession to the data which are available for determining equitable and reasonable extra premiums for impaired lives. I missed in the paper a summary of the results and I hope Mr. Hunter may supply that. It would be valuable for ready reference in practical use in the future.

Mr. Nichols: Mr. President, Mr. Craig's remarks suggested to me some very interesting lines of research in regard to that whole question of the investigation of these special risks. Of course, as you gentlemen are aware, the more you know of a risk the less valuable becomes your insurance, and the less you insure the more you know of the individual. The more you know of the class and less you know of the individuals, the broader becomes your line of insurance. Fire insurance has gone through that experience of investigating and classifying risks; one class after another; sub-dividing these classes as fresh knowledge came to the fire underwriters. They were driven into it by necessity, because as those facts developed, if one company undertook to charge too much for its risks in connection with what was known to them, of course another company would under cut them. This result follows: the fire company, as you are aware, has become now so sub-divided and that analysis has become so keen that the fire underwriter undertakes to judge of each individual risk, measure it largely as an individual, not simply as a member of any specific class. The result follows that the liability charged against that insurance company has to be measured not according to the value of that risk as ascertained by the statistics of any class, but as measured by the premium charged by the company. The more a company under cuts another in its risks the greater the danger it imposes upon itself; the less actually is the liability charged against that company by the Departments, and as you are aware, the Departments in some cases have made efforts to obtain statistics on which a more equitable basis of liabilities could be established, but so far no satisfactory results have come out of it and it is questionable to my mind whether they will ever be able to secure such a basis as will furnish a satisfactory substitute for the present exceedingly unsatisfactory method of determining the liability of the fire company.

Now it would seem to me that the same result would naturally follow if the life company could, by increasing its knowledge, its investigation, of the various classes of risks, so enlarge its knowledge as to be able to ascertain with reasonable accuracy the difference in premium which must be charged as against one individual rather than another by virtue of his occupation and by virtue of his residence and by virtue of all those other incidents connected with his life which are now disregarded. The life company stands in this peculiar position as compared with the fire company. The fire company issues only risks for a short period of time. The life companies' risks are issued during the life of the individual, or for

the large part of them. Hence the fire company, if it makes a mistake during any one brief period of time can correct it very shortly after. The life company cannot. The fire company can undertake with reasonable security to estimate the value of its individual risks as a matter of expert judgment on the part of the fire underwriter. The life company, on the contrary, up to a certain point is bound to estimate the value of its risks on the basis of broad statistics. I will not undertake to follow that line of argument any further because it hardly unfolds itself to my mind, but you can see the drift of it and how it raises the question in such an investigation as that suggested by Mr. Craig, as to how far these analyses of the occupations and different circumstances connected with the individual risks should be pushed in the general interests of life insurance.

MR. DAWSON: I should not have felt called upon to speak if it had not been for the address of Mr. Nichols, but one thing occurs to me in that connection that it may be of some importance to bring forward. This is that any attempt at fine distinction probably is unwise. The result, however, of the introduction of dealing with impaired risks and with special extra hazards is that a large number of people have been afforded life insurance in the last 10 years that previously could not obtain it. That is important. The investigation, therefore, of the special hazards connected with quite a number of occupations which are deemed virtually uninsurable on ordinary plans, in most of the companies, if the statistics can be had, it seems to me an exceedingly desirable thing for this Society to carry on. The investigation merely of slight distinctions between certain classes; the carrying on of an investigation of occupation hazards to such a point as to discriminate between the carpenter and farmer or a few similar things, perhaps might not be attended by any valuable results; for one reason because all the companies insure carpenters and farmers freely and are glad to get them. It may be that both of them exhibit a slightly higher rate of mortality than clerks, merchants and bankers, but no disposition is evinced to refuse them. On the contrary, however, there are occupations like mining, which ordinarily disqualify for acceptance in most companies, and yet these men have greater need for insurance than the ordinary man, just because they are exposed to this heavy risk. There is very little doubt, also, that if the premiums were placed at a proper figure there would be a very important moral effect produced in the direction, perhaps, of the elimination of many of the dangers that exist in mining. Personally I am quite satisfied that the conditions in mining in the United States are really disgraceful to us as a nation and probably these conditions would be greatly alleviated if we had statistics that enabled us to deal with this question properly and companies, instead of refusing miners on the ground that they did not know what the risk was, were accepting them at proper rates of premium. In this connection I may say that some three years ago the A. O. U. W. proposed to cause an investigation of a number of special hazards among

their membership, then amounting to over 400,000, and those who had been members. I think the aggregate was more than a million lives. It was not carried out, for want of funds which could be applied to that purpose. I suggested at that time that very likely the funds might be contributed by life insurance companies which would be interested. It did not occur to me to suggest that, perhaps, such an investigation might be carried on under the guidance of a committee of this Society. I have the impression that those statistics—which are perishable and may soon pass out of existence—could still be obtained; and if the Council desired it I could take it up with them. I have no doubt that from whomsoever the funds were obtained, if a committee of this Society took it in charge it would be approved of by the Ancient Order. That institution, now nearly 40 years old, has until very recently accepted all classes of occupations without any distinction. It has very large numbers of miners and large numbers of men engaged in other occupations that are perilous. Some occupations, for instance, that do not provide a very large number or would not in a life insurance company of ordinary size, even if it accepted them freely as ordinary risks, are naturally represented, in that membership of a million, pretty freely. I have no doubt that as to some of the classes of risks that are deemed actually uninsurable or virtually so, we could with a minimum of trouble obtain information that might be of very great value.

Mr. Hunter: This discussion has taken a broad range, Mr. Craig having referred not only to risks which are substandard on account of occupation, but to those which are substandard from other causes, such as heart murmurs. One of the matters to which he called attention was the anomaly of the larger mortality among the saloon-keepers who did not attend bar than among the group who stated that they attended bar five hours or more daily. There are probably two causes for this: (a) that some of those among the former group were not strictly accurate in their statement, believing that they would get a better form of policy by stating that they did not attend bar; (b) that the average of temperance may not have been as high in the former as in the latter group. With regard to (b), the ideas of temperance among saloon-keepers is undoubtedly biased by their occupation; they would be apt to consider as a temperate man one who did not get intoxicated, but that would not agree with the ideas of an abstainer.

Mr. Craig does not think it necessary to make a subdivision between (1) Saloon-keepers, (2) Proprietors, Managers and Clerks of hotels and restaurants with bar, and (3) Proprietors, Managers and Clerks of general stores with bar. That subdivision was made for statistical purposes, but, in practice, the New York Life treats these three groups exactly alike. We still deem it advisable to limit all of these types of occupation to 20-Year or shorter Endowments, and bar tenders to 15-Year Endowments. Such limitation has the advantage of eliminat-

ing the shiftless and improvident. There is an additional advantage in that a larger extra mortality can be borne by the Company on the Endowment than on the Ordinary Life or Limited Payment plan. You will find by experiment that, if say, 50 per cent. additional mortality would use up the surplus or dividends on an Ordinary Life policy, it would take 100 per cent. additional mortality to use up all the surplus or dividends on a 15-Year Endowment policy.

I sympathize with Mr. Craig's point of view, that there should not be too great refinement in the classification of risks; but, on the other hand, I do not believe in accepting on equal terms risks showing a wide divergence in prospective longevity. For example, if the Company with which Mr. Craig is connected puts into a special class all those engaged in the manufacture or sale of liquor, then a wholesale liquor dealer would be treated on the same plane as a bar tender, and there would probably be 75 per cent. additional mortality on a group of the latter over a group of the former.

With regard to the treatment of people with impaired hearts, there is the greatest divergence between the mortality among them. For example, a group of people with murmurs at both valves would probably show about three times as high a mortality as a well-selected group of men with functional heart murmurs. Again, the last mentioned group might show a distinct divergence if we took two groups, one of professional men who knew that they had heart murmurs, and took proper care of themselves, and the other, mechanics who were forced by necessity to work hard, and who might not have proper medical attention.

The figures furnished by Mr. Gore are very interesting, and it is rather difficult to see why certain differences should exist between the New York Life and the Prudential figures, although it may be due to the different class of people who are insured in the ordinary branch of the Prudential and in the New York Life.

Mr. Wright suggests that a summary of the investigation would be of advantage. I intentionally left out a summary, because wrong conclusions might be drawn from these statistics unless the text is read.

Mr. Dawson: Mr. President, what has been said concerning the admirable system described reminds me of information given me by Mr. Teece, of the Australian Mutual Provident Society, when here in 1904. It is unfamiliar, I think, to most of us. That company has had very remarkable success in rating up lives, or rather I should qualify that a little by saying that they reject pretty nearly as large a percentage as the American companies, and consequently the rating up is, to quite a large degree, of lives we accept as Select. But notwithstanding, the system Mr. Hunter describes has so many analogies to the one used there that I thought it would be interesting to the members for me to say what Mr. Teece has told me. His company has printed instructions concerning the rating up of lives, concerning every defect they ordinarily rate up for. Those instructions are virtually to the medical examiners. The medical examiner,

in their field work, usually goes with the agent, instead of going after the agent has been there. On the spot he tells the man, after examining him, what the premium will be and what will be done in his case. He does not bind the company absolutely in that; if he has made an error the company is free to reject when it reaches the home office, but in a large majority of the cases it is simply stamped approved; only in a small percentage of the cases is there a real revision of the work of the medical examiner.

MORTALITY EXPERIENCE AMONG ANNUITANTS RESIDENT IN THE UNITED STATES AND CANADA—BY ARTHUR HUNTER.

VOL. X, PAGE 113.

WRITTEN DISCUSSION.

MR. HANN: This paper involves the experience based on the most authentic data obtainable. Its preparation and publication reflects credit on the author and places us under obligation to him and the contributory companies.

It involved considerable labor and thought in sifting the material to eliminate duplicates. Since eight companies furnished 85 per cent. of the cards, three of the United States companies must have issued the bulk of this business. These latter have considerable experience in the acceptance and management of annuities.

The author has the honor of being the pioneer in producing the first important contribution to the mortality of annuitants residing in the United States and Canada.

It was wise to exclude all annuities issued in lieu of dividends, or those granted by way of pensions.

To have made further divisions into countries of birth would have detracted from its usefulness.

We note that the proportion of male to female annuitants is much greater in America than in Europe, that each male life possesses 1.79 annuity to 1.45 annuity for each female and furthermore that the average annuity on males was 60 per cent. greater than that on females. This is natural, because many of the latter are unquestionably further protected by insurance.

The observations include 1933 men and 2873 women. The entrants therefore at each age were few.

It is well to keep in mind that the business has developed rapidly during the later years of the period covered. The experience is young as a whole and only 24½ per cent. of the lives have passed through by death.

Of the 4291 males exposed to risk for the whole period, under age 55 at entry, the deaths were but 43 or one per cent. Of 4225 female exposures for same ages and period there were but 45 deaths or 1.07 per cent.

The experience is altogether too young and too small to afford sufficient reliable data, upon which to base considerations for annuities.

The large rate of interest earned in America in the past, on good security, accounts largely for the small demand for annuities till recently. The decided fall of interest ushered in a more active demand for them. Again women are much less dependent on help in America than in Europe, because the principle of self-help prevails more widely here and labor among crtain classes of the community has a more honored place.

In 1867, 44 offices in the United States issued only 19 annuities.

The latest French and British annuity experiences have been used as a measure of the expected deaths. These tables are nearly identical below age 50, and if the effect of selection were the same in both countries, it is believed the same annuity rates would suit for those ages.

The observations under the French table extend for 70 years from 1819 to close of 1889. The number of lives involved was 67,247, yielding 635,909 years of life. The deaths amounted to 55 per cent. of the number of lives. Nearly 62 per cent. of this experience was from females.

The result, of the investigation, measured by the Weeks-McClintock rate shows, in the aggregate, more appropriate rates between male and female annuitants than are shown by the French table, as a measure of mortality. In the case of males 28 per cent. of the exposures takes place after 5 years, whereas one-third of the exposures occurs among females after 5 years.

I agree with the author that the French table is an unsuitable one upon which to compute annuities for the United States and Canada.

It is instructive to note that, in the aggregate, the experience by amounts runs almost parallel with that by lives. The fluctuations in this experience are between wide limits when considered in relation to the ages.

During the first 5 years, the actual to the expected measured by the B. O. L. A. Table was as follows:

	Lives per cent. of actual to expected.	Amounts per cent. of actual to expected
Men	70	96*
Women	73	79

*Unduly affected by the large loss mentioned, in the first year.

The small numbers involved here give full play to fluctuation.

Considered in the aggregate, the mortality rate is much lower than that for either Great Britain or France. It is to be accounted for largely, from the much younger experience in the United States and Canada, from the greater proportional number of female annuitants purchasing out of their own means and from the absence of annuities provided for in wills without reference to the health of annuitant, as is frequently the case in Great Britain.

Mr. George F. Hardy, has shown from the British Annuity Experience of 1893, "it is not correct to assume that selection ceases after 5 years' or even after 10 years (although for all practical purposes the latter period is accepted), so that annuity values on select lives can be safely based on an aggregate table." The high death rate in the second year of the British Offices' Table of 1903, is not reflected in the United States Experience. It is a very prominent feature in the British Table.

It is proper to keep in mind that in this investigation, as well as in all others of the same kind, the death-rate represents expected release from reserve liability rather than profit or loss from mortality.

Owing to small numbers at certain ages we must be slow in drawing conclusions from the unadjusted data.

Mr. Chatham in the *Journal of the Institute,* deduced the following values of annuities:

Age	Males Unadjusted data.	B. O. L. A. Adjusted data.
40	16.360	17.604
45	14.600	16.061
50	13.867	14.403

and attributed the differences to paucity of data.

The table, giving the expected B. O. L. A., by years for men and women, deals with 569 deaths for males and 756 deaths for females; which considerably exceeds the actual deaths and expected mortality by lives, owing to the annuities taken in different years on the same life, being treated as on different lives. This is referred to in earlier portion of paper, but should be kept in mind when reading lives at head of table.

It has been pointed out that the mortality among annuitants almost without exception, is in the aggregate, lighter than among the assured who are admitted on medical examination.

When we consider that many of these annuitants are also among the experience of assured lives, it appears to point clearly to the fact that intuitive selection is more than a match for medical selection.

Mr. Sheppard: I have some figures that I think may be of interest to the Society. They were suggested by two considerations; first, whether the company with which I am connected should raise its annuity rates at the end of the year, secondly, whether we had a sufficient loading over the net values by Mr. McClintock's Table, on account of that being the New York State legal standard, and on account of the law with regard to the limitation of expenses.

In order to get what may be a net annuity value in this country I made the assumption that the force which raises the annuity value on an assured life as compared with an annuitant, is the same on both sides of the Atlantic, and that the effect of selection by medical examiners is the same. The first column then gives the annuity values by the British Select Annuitants, the second by the British Ultimate Assured, the third gives the ratio, and the fourth gives the values by the American 3½ per cent., which runs very close to the $O^{M(5)}$, the British Ultimate Assured, as Mr. Macaulay said yesterday when he remarked on how close the mortality between those two tables was. The product gives my net values. Comparing these with Mr. McClintock's values they run very close in the case of male lives. The net values were then divided by .925 to get my suggested office rates. I assume that the expenses will be paid out of the excess of the interest earned by the company over the

valuation rate, as I think is done in England in fixing annuity rates. Then I compared the result with the rates charged firstly by three New York companies which increased their rates this year, and secondly with those of fifteen companies which charge now the same rates as those three companies did last year. The result in case of the male annuitants was to give values throughout the Table somewhat less than those charged by the three companies and more than those charged by the fifteen companies; thus coming in between the two. In the case of the female annuitants I took the ratio of the British Female Select Annuitants to the British Male Select Anuitants, which gives a ratio increasing from one per cent. at age 20 up to 10 per cent. at 85 and worked out the values in the same way, assuming that the male and female Ultimate Assured will give the same annuity values in England; I do not think they have got an $O^{M(5)}$ Table calculated, but I assume that the ultimate table would be practically the same as regards the two sexes. Then I have compared the results with Mr. McClintock's Table. The method gives smaller net values at the younger ages but from age 55 it gives greater annuity values. Then the office rate is calculated in the same way, dividing by .925, and gives values that are slightly less than the three companies at the younger ages, but are more from age 35 to age 70 inclusive. It gives greater values even than the increased values of the three New York Companies. If this method is of any account is shows that even the increased charge by those three companies is inadequate on the female lives at those ages. The reason why the net annuity values of the females comes out so much less than Mr. McClintock's I think is due to the fact that Mr. McClintock's probabilities of death on the females are much lower than really they ought to be, on account of the fact that the graduation was done by Makeham's formula and carried right back, although I think the first death was about age 35 both in the case of males and of females. This very low rate of mortality was called to my attention when we had a rather extraordinary request—I think the first we had in the company—to quote rates for temporary annuities on the lives of children, some male and some female, members of one family. As Mr. McClintock's Tables only go back to age 10 we wanted to fit on some other tables to Mr. McClintock's. We took the male table first and found that Farr's Healthy Males fitted on very nicely with slight changes to Mr. McClintock's; but when we came to the female we found it absolutely impossible to fit any mortality experience on the lives of children on to Mr. McClintock's. In order to test whether Mr. McClintock's table is right or probably right on female lives I think it is only necessary to compare Farr's Healthy Males with Farr's Healthy Females. The probabilities of death in Farr's Healthy Females I could not find worked anywhere so I had to work them out myself and I have a table here comparing Farr's Healthy Males with Farr's Healthy Females from age 20 to 85. The Healthy Females show a higher rate of mortality than the Healthy Males up to and including age 45 and then they become less. Now, I think that shows that Mr. McClintock's table is much

too light a table in the case of the female lives and the annuity values that would be obtained from it would show much too heavy a charge. Of course, we are in an unfortunate position, because I believe we ought, if possible, to take into consideration the table on which our reserves are calculated in order to decide on our office rates. I think that is one of the subjects that ought to come before the Society for discussion, as to whether the table of mortality on which we value by State law in these cases ought to be taken into consideration or not; and secondly the question, if Mr. McClintock's table was approved in case of male lives, I think we ought to discuss whether it does not show too low a rate of mortality in the case of the female lives. I think the figures that I submit prove this.

TERMINAL ANNUITY VALUES 3½ PER CENT.

MALES

	British Select Annuitants (1)	British Ultimate Assured $[O^{M(5)}]$ (2)	Ratio (3)	American (4)	Product = Suggested Net (3) × (4)	McClintock
20	20.499	20.299	1.0099	20.144	20.343	20.030
25	19.701	19.489	1.0108	19.442	19.652	19.307
30	18.772	18.542	1.0124	18.605	18.836	18.452
35	17.701	17.449	1.0144	17.614	17.868	17.453
40	16.485	16.204	1.0173	16.446	16.751	16.300
45	15.131	14.810	1.0217	15.087	15.414	14.991
50	13.651	13.282	1.0278	13.535	13.911	13.535
55	12.073	11.649	1.0364	11.830	12.261	11.956
60	10.440	9.953	1.0489	10.032	10.523	10.292
65	8.802	8.253	1.0665	8.219	8.766	8.598
70	7.220	6.614	1.0916	6.482	7.076	6.940
75	5.753	5.101	1.1278	4.900	5.526	5.387
80	4.446	3.770	1.1793	3.437	4.053	4.003
85	3.329	2.657	1.2529	2.082	2.609	2.833

	Suggested Net — McClintock	Suggested Net ÷.925×100 = Office Rate	3 American Companies.*	15 American Companies.*
20	+.313	$2199	$2247.30	$2076
25	.345	2125	2172.30	2007
30	.384	2036	2075.60	1918
35	.415	1932	1962.80	1813
40	.451	1811	1832.60	1693
45	.423	1666	1684.90	1556
50	.376	1504	1520.50	1405
55	.305	1326	1341.80	1240
60	.231	1138	1153.20	1066
65	.168	948	960.80	888
70	.136	765	772.40	742
75	.139	597	630.00	630
80	.050	438	543.00	543
85	—.224	282	468.00	468

* As given by the Spectator Company's Handy Guide.

FEMALES

	British Female Select Annuitants	British Male Select Annuitants	Ratio	Suggested Net = Male Net × Ratio	McClintock	McClintock's q's Males
20	20.718	20.499	1.0107	20.561	21.925	.00785
25	19.964	19.701	1.0133	19.913	21.150	.00815
30	19.090	18.772	1.0169	19.154	20.242	.00863
35	18.110	17.701	1.0231	18.281	19.192	.00938
40	17.040	16.485	1.0337	17.316	17.988	.01056
45	15.890	15.131	1.0502	16.188	16.625	.01244
50	14.647	13.651	1.0730	14.927	15.111	.01542
55	13.264	12.073	1.0986	13.470	13.464	.02011
60	11.691	10.440	1.1198	11.784	11.720	.02750
65	9.941	8.802	1.1294	9.900	9.925	.03911
70	8.137	7.220	1.1270	7.975	8.143	.05721
75	6.439	5.753	1.1192	6.185	6.445	.08522
80	4.945	4.446	1.1122	4.508	4.899	.12790
85	3.686	3.329	1.1072	2.889	3.560	.19151

	Office Rate	3 Am. Cos.*	15 Am. Cos.*	Farr's H.M.q's	Farr's H.F.q's	Females
20	$2223	$2247.30	$2190	.00695	.00766	.00384
25	2153	2172.30	2113	.00775	.00842	.00407
30	2071	2075.60	2025	.00821	.00893	.00443
35	1976	1962.80	1929	.00865	.00942	.00500
40	1872	1832.60	1822	.00938	.01001	.00590
45	1750	1694.00	1694	.01076	.01089	.00734
50	1614	1544.00	1544	.01301	.01221	.00960
55	1456	1374.00	1374	.01656	.01410	.01319
60	1274	1189.00	1189	.02367	.02228	.01884
65	1070	1000.00	1000	.03680	.03387	.02773
70	862	828.00	.828	.05564	.05083	.04166
75	669	690.00	.690	.08400	.07718	.06334
80	487	592.00	.592	.12493	.11624	.09668
85	312	502.00	.502	.18018	.17034	.14711

MR. HENDERSON: In connection with Mr. Hann's discussion, it occurs to me that it may be interesting to the Society to know that an investigation of the annuity experience in different countries of the company with which I am connected led to a result something like this: I did not bring a memorandum of the figures so I am speaking from memory. In Great Britain the actual deaths both male and female very closely followed the Select Table, B. O. L. A., the new institute experience. In the United States and Canada the actual deaths were about 80 per cent. of the expected. In France about 120. In Spain about 125. Those four countries form the bulk or practically all of our annuity experience.

MR. HUTCHESON: Mr. Wells stated that there were two questions before the Society; one as to the mortality which has been experienced in

*As given by the Spectator Company's Handy Guide.

the United States and the other as to the rates which should be charged for annuities sold to residents of the United States. Mr. Hunter's paper gives us some idea as to the mortality which has been experienced here, but the facts are small and it is a question as to how far we can depend upon them in making up rates. We are much indebted to him, all the same, for the information which he has brought before the Society. We have to depend, I think, on the experience in Britain particularly, and the various annuity experiences there have shown that the rate of mortality has become less at the older ages; each successive experince shows a lower rate of mortality. The census returns there show the same thing. The point I want to make on that is that any improvement in vitality, of course, results in a loss to the company selling annuities; whereas a similar gain in vitality means a gain as far as insurance is concerned. That is a point we have to keep in mind when we are deciding about the rates we shall charge for annuities. While this improvement in vitality has been going on the rate of interest has been declining and as those are the two factors which enter into the question I think we have to be very careful about the rates we shall charge. No doubt during the last few years the rate of interest has been improving and it may go on improving for some time, but I just want to call attention to that fact in passing.

Then we know that the effect of selection is great in the case of annuities, as in the case of insurance. In the case of insurance, of course, we make a gain and we calculate premium rates on an aggregate table or on an ultimate table because we know we are going to be at a heavy expense in getting that insurance and we know that we are going to make up some, or perhaps all, of that expense by favourable mortality. In the case of annuities there is not that feature at all. Any favourable mortality results in a loss to the company. It appears, therefore, that we should base our annuity rates on select tables.

Then in this connection I wish to call attention to a point in Mr. Hunter's paper that has been referred to by Mr. Hann. On page 117 of the *Transactions*, at the foot of the page, you will see the total Exposures, and if you add up the first five years you will find that in the case of Males they are over 71 per cent. of the total exposed, and in the case of Females on the two pages further on, the exposures under five years are rather less than 67 per cent. of the total exposures. I refer to that in connection with the comparisons Mr. Hunter makes of the actual deaths as compared with the expected by the Weeks-McClintock rates of mortality. The Weeks-McClintock Table is an Aggregate Table. Mr. Hunter pointed this out, but I want to call attention to it again. In view of the fact that the Males have had their annuities for a shorter period than the Females we would expect that the ratio of deaths would be smaller in the Males than in the Females, which is borne out by Mr. Hunter's figures. The Weeks-McClintock Table is a table for the calculation of Reserves and was not intended for the calculation of Premiums. Mr. McClintock stated this several times in the

discussion which followed the reading of that paper, and it is printed in one of the old numbers of the *Transactions*. He added, however, that the table might be used for the calculation of Premiums provided we loaded it at the older ages so as to make it equivalent to a Select Table. But that is another matter altogether.

The comparison which Mr. Hunter makes of the actual deaths with the expected, by the British Offices Select Table, I think shows conclusively that the mortality rate here is considerably lower than in Britain. There is, perhaps some self-selection, but I do not want to make a point of that.

There are several points to be kept in mind in deciding on the Scale of Considerations. Of course, we must first select a Mortality Table which will, as nearly as possible, indicate the mortality which those annuitants will experience. I mean that is our object. Then we must fix upon a rate of interest which we will be sure to earn in the future. Unfortunately we have no Mortality Table, I think, at the present time which can be trusted. I think that on that account we should load up the rates derived from any Table and perhaps more especially at the older ages.

The matter of expense comes in. It is a simple matter. A single commission is paid when the annuity is purchased and that is an easy matter. But then the renewal expenses have to be taken into account. We do get a considerable amount of profit from interest from our annuities at the present time, but we cannot count upon that, and that is a variable quantity at any rate. It is larger in the earlier years when the annuitant is young, and may vanish or practically so, when he gets old. I think the more satisfactory way is to make some provision for a fixed charge during the lifetime of the annuitant. I am sorry that I have no suggestions of any importance to make, but I just wanted to call attention to those few points.

Mr. Hunter: In the tables showing the amount of annuity payments ceasing by death it was pointed out that the Company was relieved of a yearly payment of $24,000 in the first policy year. This was a remarkable case of a man who was trying to beat the Company by taking out annuities every year and by converting the payments made by the Company into additional annuities. It happened that he had just made a large payment for an annuity a few months before his death, and was arranging to pay still more to us. It was one of the earliest problems presented to me in my connection with the New York Life, as to whether it was possible for a man to get the better of a company by following the course already outlined. This problem involves the maximum annuity which a company can afford to take on an individual risk.

There is one matter which I did not bring out forcibly in the paper as presented to the Fifth International Congress, namely, the probability that the low mortality was in part due to understatement of age. There was plenty of evidence to show that the companies had not been so strict as they should have been in their requirements as to evidence of age,

although I had no means of determining whether the question of age had a great or small effect on the resulting mortality. One good result of the published investigation, however, has been that some of the companies, have become much more particular with regard to evidence of age. It has been brought sharply to the attention of many of the managers that annuity business is not worth having unless it can be obtained at a proper rate, and that the proper rate cannot be calculated unless the correct age is ascertained. The actuary of one of the fortunate companies which made a slight profit on its annuity business, pointed out to the president of his company that the margin of profit was equal to the difference in consideration corresponding to a difference in age of one-half of a year on the average, thus bringing before the chief executive very forcibly the necessity for having the age correct. In European countries the greatest care is taken in determining the age, the annuity not being issued unless a certified copy of the official record of birth can be obtained, and while that is not possible in many cases in this country, it can be obtained in more cases than is generally supposed. Many of our annuitants, for example, were born in Ireland, and my experience has been that the parish priests in Ireland have a very satisfactory record of the date of baptism, which usually occurs within a few days after birth. Compulsory registration has also been in force in Ireland since 1864, and in England and Scotland since 1837 and 1855 respectively, and a certified extract of the date of birth can be obtained from the Registrar General in the capitals of each of these countries at a fee of 2s. 6d., plus a receipt stamp of 1d., the entire charge, including postage, being covered by 3s. In cases where the applicants were born in any of these countries, the consideration money could be accepted, in order to avoid any chance of losing the business, with the distinct understanding that a certified copy of the date of birth should be obtained.

Mr. Hann thinks that the low mortality is partly due to the short average duration of the annuities, and the same explanation has been made by other actuaries. I cannot agree with this point of view, however, as the experience on annuitants resident in the United States and Canada was measured by the British Offices' Select Annuity Experience, which takes into account the low mortality in the early annuity years. Mr. Hutcheson states my position in saying that the comparison of the actual with the expected deaths by the British Offices' Select Annuity Table shows conclusively that the mortality rate in the United States and Canada is considerably lower than in Britain. Mr. Henderson's figures also confirm this view as he states that the experience of his Company in Great Britain, both on male and female annuitants, closely follows the British Offices' Select Annuity Table, while the experience of his Company in the United States and Canada shows a percentage of actual to expected deaths of about 80 per cent., measured by the same Table.

(In accordance with the suggestion made before the adjournment of the last meeting on May 17, 1907 (Vol. X, Page 139), the subject of

making annual dividends or annual apportionment and distribution of surplus upon policies of life insurance, was taken up for discussion).

Mr. Ferguson: The discussion this morning is presumably limited to annual dividends but, believing that it is possible to find a comprehensive method of division equally applicable to tontine and annual dividends, I propose to extend my remarks to cover the general question of surplus distribution. My particular object, however, is to present a method for the calculation of annual dividends.

It may be premised that the natural position for a newly-organized company, unfettered by traditional or legislative restrictions, would be to defer allotment of surplus until the close of a period sufficiently long to produce an average of results. The conception of tontine dividends as accumulations of annual or quinquennial distributions would never present itself to such a company and it is safe to say that if such an idea has ever obtained, it has been founded on a perversion of a natural law of insurance or indeed of any business viz.—that profits must not be paid till they have been earned by the parties receiving them.

It requires, in my opinion, a considerable stretch of the imagination to see how a policy, which has been procured under present expense conditions, can be said to have earned a dividend within the first three years at least. The trend of actuarial and public opinion seems to lie in the direction of requiring each policy to stand on its own footing as regards expenses, and the restrictions to annual or triennial dividends, being directly inconsistent with this tendency, must be regarded as illogical in the extreme.

Annual dividends are, however, prescribed by law and are demanded by part of the public. We must therefore overcome our objections to what we believe to be an unscientific procedure and devise a means of meeting the popular and legislative demands.

In suggesting a method of annual distribution, I will forget the past for a moment and regard the tontine as the primary system of surplus allotment. Under this system there is a fixed period, at the end of which a dividend is payable, concerning which we are so certain that we are wont at times to give an estimate thereof. In ascertaining rates of annual dividend, I propose to make a close estimate of the surplus available at the end of a certain period and then by a process of discounting ascertain the amounts that may be paid in advance, distributing them over the intervening years in whatever way appears most desirable.

A ten-year period, being of sufficient length to permit the elimination of the loss from loading by the gain from mortality, seems a suitable one to choose, and for estimating the deferred surplus the method described by Mr. Weeks in the *Transactions* of the Society, Vol. IX, p. 93, is as nearly perfect as could be desired. It remains to discount the deferred surplus so ascertained by using the table combining rates of mortality, interest and withdrawals, which is involved in the preparation of the asset shares.

In deciding what portion of the deferred surplus should be paid in advance in a given year, any rule which commends itself to the management may

be adopted, but one may be suggested which, though still arbitrary, is not founded on caprice. Find by the suggested method of discounting the value at the date of issue of the estimated deferred dividend, and dividing such present value into ten equal parts, pay one of these parts with its proper accumulation in each of the ten years of duration.

This rule produces a scale of dividends which increases satisfactorily with the duration, and it should be found safe if the ten-year estimate has been made on a sufficiently conservative basis. It must be remembered that the rate of withdrawal may be increased by the payment of dividends in advance and therefore it will be wise to make a deduction from the estimated deferred surplus before discounting it.

Mr. Weeks in the paper referred to above, has given an example of the calculation of standard asset shares for the Life 20 Payments plan at age 40. The excess of the 10th year asset share over the American 3 per cent. reserve is $88.30 and for payment in advance we may take 75 per cent. of this as a very conservative estimate. Discounting this amount in accordance with the rule stated above, we would have as equivalent annual dividends: $3.60, $3.91, $4.35, $4.75, $5.11, $5.41, $5.68, $5.98, $6.28 and $6.62, for durations 1 to 10, respectively.

For deferred dividend policies, Mr. Weeks' method of allotment cannot be improved on. For matured tontines and for other policies more than 10 years in force, there are several good methods based on the contribution formula, which are perfectly satisfactory. The method I have described for younger non-tontine policies fits in well with the other two. It produces results which bear a fixed relation to the results on similar tontine policies and which by a steady increase may be made to merge imperceptibly into the dividends payable on older annual dividend policies. I would respectfully commend it to the consideration of members of the Society.

Mr. Henderson: Mr. President, I am rather gratified by Mr. Ferguson's paper as it tends to cover a point which was raised—which I raised, as you will recall, last spring in the discussion where I said I considered that enough consideration had not been given to the fact that a contingency reserve was accumulated and care was not taken to pay it back to the people from whom it was accumulated. Some of the subsequent speakers seemed to imply that I was afraid it was transferred from Endowment to Life or Life to Endowment. That was not my objection at all or my fear. My fear was that the contingency reserve was accumulated from the early policy-holders of the company and never repaid to them. Mr. Ferguson's investigation was much on the line of what I had nebulously in mind although mine did not extend to fixing any definite period, but rather looking forward to the future history of the group of policies being issued and estimating what dividends could be maintained and keeping concurrently what you might call a secret ledger dividend account which would be accumulated on the tontine plan and credited to each class of policies and the dividends adjusted annually or periodically as the case might be and as the experience showed they should be.

(The following was prepared as a written discussion in connection with the paper on "Mortality among Insured Lives Engaged in Certain Occupations," but the Society ordered it printed in the *Transactions* as a separate actuarial note.)

Note on An Approximate Method of Making Mortality Investigations.

By Arthur Hunter.

In connection with the mortality experience under hazardous occupations, published in the last number of our *Transactions*, several members have asked me to describe the methods employed in preparing the data and in producing the results contained in the paper. As it would have made the original paper too long to give the exposures for all groups at the various ages, and as, accordingly, my ultimate calculations had to be offered in abstract and accepted on faith, it seems proper that an explanation of the methods used should be given, to be taken as a part of the record in connection with the discussion of this paper. This explanation may in itself be of value to students and to those who have not had practical experience in mortality investigations.

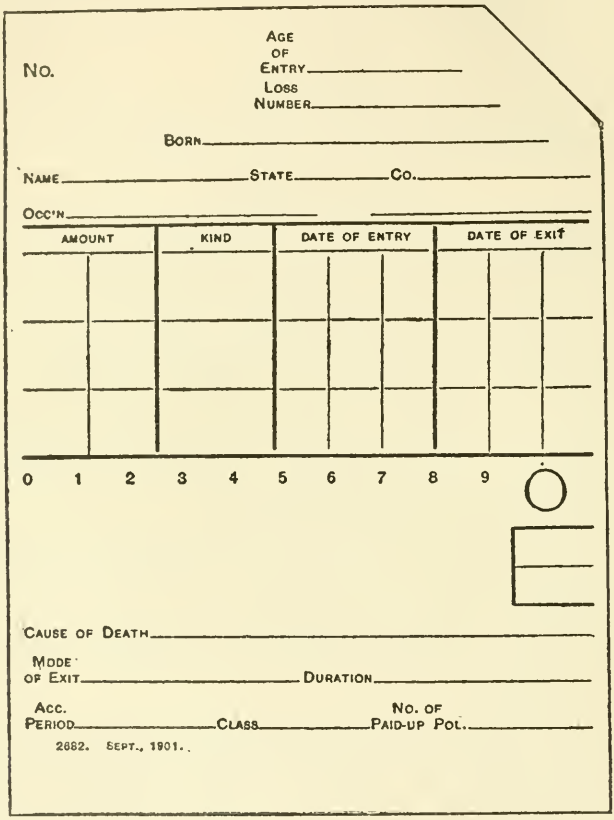
The problem which presented itself was,—

1st, to determine methods of procedure which could be carried on by a force of women clerks, and did not require the services of a staff of embryo actuaries or actuarial assistants; and

2nd, to establish a division for mortality investigations in which the work would be largely automatic, and which would require the minimum of expert supervision.

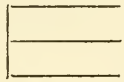
In order to carry out both these ideas it was necessary to make the rules as simple as possible, to have a plan which would be followed in all investigations, and to have a system of checks which would be more effective than would be necessary in the case of clerks with actuarial knowledge. The results have shown that women become very proficient in their work, turning out a large supply of it, and that supervision is largely confined to seeing that each clerk has a proper share of work, that no hitches arise, and that the accuracy and output of each clerk are properly determined. The final work of the actuary is to determine from his knowledge of the conditions underlying the data, and from his knowledge of the mortality in collateral groups whether the results are reasonable or not. He, of course, can put into effect a number of checks which can be done automatically before he takes in hand the final work, such as comparing (a) the average amount of the death loss with the average amount of the "paid-for" policies, (b) the average exposure by policies with the average exposure by amounts, and (c) the average amount of expected death losses with the average amount of the actual death losses.

The information necessary for mortality investigations described in the paper was entered on cards, a specimen of which is given below:



Before entering into a description of the manner in which the data were placed on the above card, it will not be amiss to give a short explanation of the form of the card itself. The card was prepared solely for mortality purposes, as experience has shown that it is not true economy to put the same card to too many uses, such as for mortality, valuation and record of dividends. We desired to make the card as complete as possible, so that we might investigate, if thought desirable, the mortality according to occupation, physical impairment or characteristics, nationality, lives, amounts of insurance, habitat, plans of insurance, dividend class and so forth. Accordingly, more information is provided for in this card than is usually required for an ordinary mortality investigation. The numbers from 0 to 9, which run along the middle of the card from left to right, denote, with certain figures added, unfavorable personal

or family history, physical impairment, the country of birth, race and other information, according to a schedule prepared for this purpose. It was not found feasible to use numbers to denote occupation, as the occupations are so diversified, and as generic titles are not satisfactory without full details. By the addition of one figure to the numbers on the cards provision is made for 100 numbers, which designate the facts we need. For example, if an insured had a mitral heart murmur, the impairment could be denoted by the number "36," and this would be entered on the card by adding "6" to the number "3" printed on the card. Letters of the alphabet are also used in the same manner as in the symbols of the Medical Information Bureau,—as, for example, "X" standing for "found by examination," and "A" for "within the past year." The M. I. B. symbols were not used in place of the Nylic symbols, because the latter were employed by the Company some time before the former symbolic system was adopted.

The circle on the extreme right hand of the card, after the number 9, is used to distinguish the sex of the insured. If the insured is a male, the circle is left blank; if a female, the letter "F" is inserted. The usual way of distinguishing men from women, by using a different colored card was not adopted because cards of different colors had been used for other distinctions, such as for the under-average business. In the space marked thus  on the right hand side of the card are placed the height and weight of the insured.

The Loss Number refers to the number of the Death Loss papers.

Provision is made after the word "Class" for designating the various dividend classes, such as Miners', Marine and Under Average.

All the data are placed on one side of the card. Our earlier cards had the data on both sides, and the disadvantages of that system resulted in our going to the expense of rewriting thousands of cards in order to have all the data on one side. Such cards are more easily sorted and tabulated and there is less chance of error. It might also be mentioned that a thin card is advantageous, and that a cheap grade of paper cardboard is not economical.

Material from which Information is Obtained.—The age at entry, date of birth, name of insured, state and country in which the insured lives, occupation, amount and kind of policy, and date of entry are obtained from the register cards. The register card of the New York Life has taken the place of the policy register, and gives a complete history of the policy from its issue to its close. It is a question whether it is better to write the cards from the register cards after the policies have been paid for or to write them from the application papers at the time the policy is written. If the latter plan is adopted it necessitates the writing of cards for all policies whether they were paid for or not, and while this increases the amount of work, it furnishes the data for

investigations into the proportion of "Not Taken" business under various classes and in different communities. Both systems have been tried in our office, the latter being now in use.

The symbols regarding physical impairments, etc., are not used and the heights and weights are not inserted except for under-average risks, and in other special cases. Such information is taken from the application papers, by specially trained clerks before the investigation is made.

In order to re-check the data which are transferred from the application papers to the mortality card, the latter is compared with the valuation sheets of the Massachusetts Insurance Department. These valuation sheets show the number, date and kind of policy, age of insured and amount of insurance. Thus an independent check is made of the most essential data, as the valuation sheets are written direct from the application papers, without reference to the register card and not by the clerk who wrote the register card or the clerk who checked the data transferred from the application papers.

Duration of Insurance.—The date and cause of exit are obtained from the register card, the deaths being checked from a register of death claims in the Actuarial Department. That register is checked month by month with the account of the Policy Claims Department, and, accordingly, is strictly accurate. To again check the most important part of the data in the investigation, viz., the deaths, the cards are compared with those of the Policy Claims Department, and the date of death and cause of death are entered therefrom.

The investigation being made throughout by policy years, the exact policy year in which death takes place is ascertained by observing the exact dates of entry and of death. In the exposed to risk the dead are considered as exposed to the end of the policy year in which death takes place, deaths in the first policy year being taken as of Duration 1; deaths in the second policy year as of Duration 2 and so on. The duration taken for the lapsed and surrendered policies is the "Mean" duration, obtained by subtracting the calendar year of entry from the calendar year of exit. The existing are traced to the anniversaries of the policies. Treated in this way, the duration for all policies is an integer, and all question of fractional duration is eliminated. It may be questioned whether the "Mean" duration for the surrender and lapsed policies is sufficiently accurate, but tests have been made which show without any doubt that for the purpose of these investigations the "Mean" duration is as reliable as the "Nearest" duration, and much less troublesome than the "Exact"; being indeed practically accurate. For example, in the class of bartenders, taking the entrants of all ages from 15 to 64, the exposures on the basis of the "Exact" duration, of the surrendered and lapsed policies, were 1,546 years, and by the "Mean" duration method, 1,541. The error in the exposure was greatest for the group age 25 to 29, in which case it was one and seven-tenths per cent. of the exposures.

All policies that became paid-up for reduced amount, or paid-up for the full amount for a term of years were treated as terminated policies as

of the date when the premium was due and unpaid. That was the treatment so far as these investigations were concerned, although the data were prepared for investigating at a future date their experience after becoming fractional paid-up policies or being put on term extension.

Mortality Investigation by Policies and by Amounts.—No attempt was made to eliminate duplicate policies on the same life, as it was found by experiments that the death loss by policies varied only slightly from that by lives. Although the experience by policies gave satisfactory results in these investigations by the New York Life, it does not follow that it will do so generally. In a small experience, it might happen that the few men who died carried a large number of small policies, and this event might seriously disturb the mortality experience. It is advisable, therefore, to scrutinize the death losses carefully in order to see whether there have been any deaths among men who carried a number of policies out of proportion to the number of policies exposed to risk. We do not advocate a mortality experience by policies, but have found the use of policies reliable for certain limited investigations and this method saves much clerical labor.

As a check on the approximate accuracy of the investigation by policies the data are also tabulated by amounts so that a comparison may be made between the ratio of actual to expected deaths by amounts and by policies. If there is a large difference between these two ratios, the data are always investigated to determine the cause.

Sorting of Cards.—The cards were sorted first according to general occupation, then subdivided into particular sub-occupations. For example, the cards on miners were sorted out from all the occupations, and then were divided into iron miners, quartz miners, etc. There were then subdivided into "Existing," "Lapsed," and "Dead." Each of these groups was then sorted according to age at entry, and further subdivided according to duration. The cards in each of these subdivisions were counted and entered on sheets prepared for this purpose, a sample of which is attached.

The data for each quinquennial group is tabulated as follows:

Policy Year	Ages 15-19					
	Existing	Lapsed	Dead	Exposed	Rate of Mort.	Expected Deaths
1					4.2	
2					4.4	
3					4.7	
4					4.9	
5					5.1	
6					5.3	
7					5.5	
8					5.7	
9					5.9	
10					6.1	
11					6.2	
12					6.3	
13					6.4	
14					6.5	
15					6.6	
16					6.8	
17					6.9	
18					7.0	
19					7.2	
20					7.4	
21					7.6	
22					7.8	
23					8.0	
24					8.3	
25					8.6	
26					8.9	
27					9.3	
28					9.7	
29					10.1	
30					10.6	

The aggregate experience—by policy years, by age at issue, and by attained ages is then tabulated in the following manner:

By Policy Year.

Years	Act.	Exp.	Ratio	Ages	Act.	Exp.	Ratio
1–5				15–24			
6–10				25–39			
11–15				40–49			
16–20				50–End			
21–25							
26–30							
ALL							

By Age at Issue.

Ages	Act.	Exp.	Ratio
15–19			
20–24			
25–29			
30–34			

By Attained Ages.

Ages	Act.	Exp.	Ratio	Ages	Act.	Exp.	Ratio
25–39				55–59			
40–44				60–64			
45–49				65–69			
50–54				70–74			

It will be noticed by inspection of the sample sheet that the cards are sorted and recorded according to groups of five ages, and not according to each age. This method was adopted only after several tests as to the effect which it would have upon the expected mortality compared with the mortality obtained by grouping the exposed to risk according to each age. No material difference was found in the results by the two methods. At the younger ages the expected mortality by the group age method is very slightly less than the expected mortality by each age: at ages from 30 to 44 the expected mortality is practically the same by either method, and beyond those ages the expected mortality by group ages is slightly in excess of that by individual ages. Careful tests showed that the expected mortality by group ages was about ½ of 1 per cent. greater than the expected mortality by each age.

The printed figures in the subjoined sheet are the rates of mortality per thousand on the Compound Progressive Mortality Table (New York Life Standard) at the central age of the group. The multiplication of the exposed to risk by the rates of mortality is done by means of a multiplication table such as Meech's or Cogsworth's, as we find the work is more rapidly done than by calculating machines, and is sufficiently accurate for our purpose. The tabulation of the data on these sheets and the ensuing work of calculation are done in duplicate by different clerks, the comparison of the results being made by the head of the division.

The advantages of having a sheet of the above kind are obvious: it is both a working sheet and a summary sheet, showing in compact form the mortality by policy years, by attained ages, and by ages at issue. The grouping of the mortality by attained ages is not strictly accurate, but it is rarely needed, and the approximate method gives a general idea of the ratio of the actual to the expected mortality, which is sufficient for most purposes and saves much labor.

In preparing mortality statistics a distinction should be made between absolute accuracy and necessary reasonable correctness, for the particular purpose of the investigation. Where the groups of data are not large the exact results would not be applied in practice, and hence approximate methods may be safely employed. For example, the ratio of actual to expected deaths in the case of bartenders was 178 per cent., but as the total deaths were 75, the conservative actuary would make provision in quoting premium rates for about 190 per cent. or 200 per cent. mortality. By grouping the exposures by quinquennial ages and by using the "Mean" duration for lapses, there may be a maximum error of 2 per cent., but that is not of importance if the exact figures are not used. I believe in the greatest accuracy when necessary, but not in expending money and labor in obtaining results which are to be used *approximately*, or with a substantial loading for contingencies.

Abstract from the Minutes of the Semi-Annual Meeting of the Actuarial Society of America, held in Toronto, Canada, on Thursday, October 10th, 1907.

FIRST SESSION.

King Edward Hotel,
Toronto, Canada, *October 10th*, 1907.

The meeting was called to order by the President, Mr. Wells, at a quarter before eleven o'clock, A. M.

FELLOWS PRESENT.

Alsop,	Graham, W. J.	Pierson,
Blackadar,	Henderson,	Plumley,
Bradshaw,	Hunter, A.	Rhodes,
Carpenter,	Hutcheson,	Richter,
Craig, J. D.	Ireland,	St. John,
Craig, J. M.	Kilgour,	Sanderson,
Crawford,	Lee,	Sheppard,
Dawson,	Macaulay,	Wegenast,
Dow,	Macdonald,	Welch,
Fackler, E. B.	Marshall, E. P.	Wells,
Ferguson,	Moir,	Wood, A. B.
File,	Nichols,	Wright, W. C.
Flynn,	Papps,	

ASSOCIATES ATTENDING.

Bain,	Hall, A. F.	Morris, W. O.
Blackadar, E. G.	Hall, J. B.	Muckle,
Bleil,	Macfarlane,	Parker,
Brown,	Mackenzie,	Portch,
Cathles,	Macorquodale,	Reid,
Dark,	McKechnie, J. B.	Somerville,
Dickenson,	Matheson,	Wood,
Fisher,	Moore,	Washburne, A. C.
Gibb,		

After the calling of the roll, a quorum of Fellows being present, the minutes of the Annual Meeting, held in New York on May 16 and 17, 1907, were approved as printed in the *Transactions*.

The Secretary then read an abstract of the proceedings of the Council since the Annual Meeting, reporting that the attention of the Council had been chiefly directed to the consideration of certain proposed amendments to the Constitution of the Society, and of a suggestion made to the Council by Dr. Oscar H. Rogers, President of the Medical Directors' Association, that the mortality under term policies be specially investigated, considering separately yearly renewable term, non-renewable term, renewable term, and convertible term. This last subject, the Secretary stated, had been referred to a special committee of the Council.

A proposed amendment to Article X of the Constitution by adding at the end of that article the following words:—

"the candidacy having been notified confidentially to every member of the Council at least twenty days before the meeting at which action is to be taken,"

was, upon recommendation by the Council, adopted.

The consideration of two other amendments which had been proposed respectively to Articles IV and VII of the Constitution, relating to the number of Vice-Presidents of the Society and the manner of their election, was upon recommendation of the Council, postponed until the Annual Meeting in May, 1908.

The Society then proceded to the reading of papers prepared for this meeting (pages 141-299).

Upon the completion of the presentation of papers of several of which abstracts only were read, the Society at a quarter-past twelve took a recess until two o'clock.

Afternoon Session, *October* 10, 1907.

After the Society had reassembled at two o'clock the discussion of papers presented at the previous meeting of the Society in May, 1907, was begun.

At four o'clock the Society adjourned to meet together at dinner at the Toronto Club in the evening, and to reassemble for business the following morning at half-past nine.

SECOND DAY.

FRIDAY, *October* 11, 1907.

The meeting was called to order by the President at half-past nine o'clock, and the discussion of papers read at the meeting in May was then resumed.

Acting upon the question as to the expediency of continuing the publication of advertisements in the *Transactions,* which had been referred to the Council at the last meeting, it was decided by the Council that it was expedient to continue the advertisements, and that one copy of the *Transactions* be sent to each advertising company for each $5. paid for advertising. The action of the Council was approved by the Society.

On motion of Mr. Ireland, seconded by Mr. St. John, the thanks of the Society were voted to the Ontario members, and especially to Mr. Macdonald, Chairman of the Committee of Arrangements, for the generous and successful provision made for the comfort and convenience of the Society and the entertainment of its members during this meeting.

At one o'clock the Society finally adjourned, to reassemble informally for luncheon at the Hunt Club, as guests of the Ontario members.

ARTHUR HUNTER,
Secretary.

Vol. X No. 39

ACTUARIAL SOCIETY OF AMERICA

"The work of science is to substitute facts for appearances and demonstrations for impressions."—RUSKIN.

TRANSACTIONS

May 21st and 22nd, 1908

NOTICE.

The Society is not responsible for statements made or opinions expressed in the articles, criticisms and discussions published in these *Transactions.*

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ACTUARIAL SOCIETY OF AMERICA.

May 21st, 1908.

THE COUNCIL.

Officers: JOHN K. GORE, President.
ARCHIBALD A. WELCH, Vice-President.
HENRY MOIR, Vice-President.
ARTHUR HUNTER, Secretary.
DAVID G. ALSOP, Treasurer.
CLAYTON C. HALL, Editor of the Transactions.

Ex-Presidents: DAVID PARKS FACKLER,
HOWELL W. St. JOHN,
EMORY McCLINTOCK,
THOMAS B. MACAULAY,
OSCAR B. IRELAND,
ISRAEL C. PIERSON,
RUFUS W. WEEKS,
DANIEL H. WELLS.

		Term Expires.
Elected:	WILLIAM C. MACDONALD,	May, 1909
	JOEL G. VAN CISE,	May, 1909
	WILLIAM A. MARSHALL,	May, 1909
	WILLIAM A. HUTCHESON,	May, 1910
	FRANK SANDERSON,	May, 1910
	EDWARD J. SARTELLE,	May, 1910
	JAMES M. CRAIG,	May, 1911
	FREDERICK H. JOHNSTON,	May, 1911
	PERCY C. H. PAPPS,	May, 1911

FELLOWS.

Thomas Gans Ackland,
David Griscom Alsop,
Jesse John Barker,
Amédée Bégault,
Alfred Kimball Blackadar,
Samuel Sweet Boyden,
Thomas Bradshaw,
John Jones Brinkerhoff,
David Carment,
Raymond Van Arsdale Carpenter,
James Chisholm,
Hubert Cillis,
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NOTATION.

ADOPTED OCTOBER 23D, 1890.

RESOLVED, that in the presentation of papers the Symbols of the Text-Book of the Institute of Actuaries, with the additions noted below, be followed when convenient, and that in any event the use of such symbols in other senses be avoided; that the letters denoting commutation columns, according to the system of Dr. Farr, customarily followed by American Actuaries, be printed in plain "full-face" type; and that otherwise there be entire liberty as regards notation.

$k_x = \frac{d_x}{l_{x+1}} = \frac{C_x}{D_{x+1}}$.

$u_x = \frac{l_x}{l_{x+1}}(1+i) = \frac{D_x}{D_{x+1}}$.

S = Sum insured.

K = Cost of insurance = $q(S - V)$.

Π = Single premium (alternative for A).

The following notation for the discounted value of the future profits from mortality was recommended for adoption May 21st, 1908.

G_x = the present value at date of issue of the assumed mortality gains on a Single Premium Whole Life Insurance of 1 issued at age x.

$G^2_{x\,\overline{n}|}$ = the value at the beginning of the second policy year of the assumed mortality gains on a Single Premium Endowment Insurance of 1 for n years issued at age x.

${}_ng_x$ = the present value at date of issue of the assumed mortality gains on a Limited Payment Life Insurance of 1 issued at age x by n annual payments.

$g^2_{x\,\overline{n}|}$ = the value at the beginning of the second policy year of the assumed mortality gains on an Annual Premium Term Insurance of 1 for n years issued at age x.

Vol. X. No. 39.

TRANSACTIONS

MAY 21ST AND 22ND, 1908.

The Rate of Sickness

With special reference to the Experience of the Travelers Insurance Company of Hartford on their Health Policies.

H. J. MESSENGER.

The first attempts to write health insurance in the United States were made about sixty years ago. During the decade, 1840–1850, several companies were organized for the purpose of engaging in this business. Among them were the Massachusetts Health Insurance Company of Boston, the Health Insurance Company of Philadelphia, the Spring Garden Health Insurance Company of Philadelphia County, the Eagle Life and Health Insurance Company of Jersey City and the Essex County Life Insurance Company of Massachusetts. Starting with the formation of these companies the period from that time up to about 1897 can be regarded as the first period of health insurance in the United States.

It was a period characterized throughout by failures. Those engaged in the work were not trained insurance men and had no conception of the necessity of a contingent liability reserve. None of the companies had the requisite financial backing and the only experience they had to go by was the experience of the English friendly societies, showing, with a fair degree of accuracy, the number of weeks of genuine, actual sickness in England among the middle

and industrial classes at the different ages. How great would be the expense of conducting these health companies and how much they would have to pay for claims that did not represent genuine sickness, they had to learn from experience. The result was that, although a few of the men connected with the earlier companies were persons of ability in a general way and were reliable and earnest, yet they soon withdrew, and their places were generally taken by incompetent and irresponsible parties and what little scientific basis had been brought into play by two or three of the earliest companies was soon thrown to the winds.

Not one of the companies that started during that early period has survived and not one has furnished any experience in regard to the rate of sickness that can be considered of any real value. A full account of the early efforts to establish health insurance in this country will be found in an article by the writer on "Health Insurance in the United States," published in Volume I of the "Proceedings of the Fourth International Congress of Actuaries."

Early in the last decade of the last century the stock accident companies of this country were engaged in a very sharp competition for accident business and each one in turn had added some new feature to the contract in order to get ahead of the others. Finally, about 1897 one of the companies offered in connection with the accident policy, and for a very small additional premium, what was called a special health policy paying weekly indemnity in case of disability as the result of contracting any one of a certain limited number of specified diseases. The contract as first issued covered fifteen diseases, as follows :—typhus, typhoid, or scarlet fever, smallpox, varioloid, diphtheria, measles, Asiatic cholera, erysipelas, appendicitis, diabetes, peritonitis, bronchitis, pleurisy and pneumonia—most of which will be readily recognized as being infectious or contagious and, therefore, to a large extent partaking of the nature of an accident. Other accident companies very soon took a similar course and in a few years many of the leading companies were well started in the experiment of health insurance.

The placing of this form of insurance on the market by the stock accident companies meant that the undertaking would have all the financial backing necessary, and that the business would be conducted by men who were familiar with business methods and that the new venture would be given a thorough trial under much more favorable conditions than had formerly existed. The important things lacking

were the knowledge of the kind of contract required by the public and the probable experience in regard to the rate of sickness. It was soon found that the policy was regarded as altogether too limited in its scope and the number of diseases included in the contract was in a short time increased by most of the companies to twenty-one and later on to over thirty besides covering certain amounts for surgical operations. About the same time some of the companies commenced issuing a health policy which, with very few limitations, practically covered all diseases ; and, of late years this policy has been gradually taking the place of those contracts limited to a few specified diseases.

The amount of health insurance written by the stock accident companies has been continually on the increase until it has now reached considerable proportions. The total premium receipts from all forms of health insurance and the corresponding claims paid by those accident companies reporting their health business separately to the insurance departments (as given below) are taken from the "Insurance Year Book" for 1907, and give the figures for each year from 1899 to 1906, inclusive. The very large increase for the year 1903 over 1902 is to a considerable extent due to the fact that several of the companies for the first time separated their health reports from their accident reports in 1903.

Total Health Premiums Received and Health Claims Paid From 1899 to 1906 Inclusive.

Year.	Premiums Received.	Losses Paid.
1899	$ 21,735	$ 3,815
1900	137,769	44,829
1901	257,139	120,142
1902	335,339	168,282
1903	1,261,161	517,617
1904	1,951,796	821,766
1905	2,164,411	847,359
1906	2,406,179	933,910
	$8,535,529	3,457,720

Coming now to the rate of sickness as determined by the experience of the companies doing health insurance in the United States, particularly the experience of the Travelers Insurance Company of Hartford, the columns of percentages from A. to Q., inclusive, give the number of weeks of sickness for each year of exposure, as follows :—

Column A. is the experience of the Travelers Insurance Company on its special health contract issue of March, 1899 to March, 1901—exposure of March, 1899 to January, 1906, and covering fifteen specified diseases, as follows :—typhus, typhoid or scarlet fever, smallpox, varioloid, diphtheria, measles, Asiatic cholera, erysipelas, appendicitis, diabetes, peritonitis, bronchitis, pleurisy and pneumonia.

Column B. is the experience of the Travelers Insurance Company on its special health contract issue of March, 1901 to November, 1902—exposure of March, 1901 to January, 1906, and covering twenty-one specified diseases, as follows :—typhus, typhoid or scarlet fever, smallpox, varioloid, diphtheria, measles, Asiatic cholera, erysipelas, appendicitis, diabetes, peritonitis, pleurisy, pneumonia, cerebral apoplexy, brain fever, epilepsy, hydrophobia, sunstroke, malignant pustule and mumps. In this second form bronchitis was struck out of the old contract and seven new diseases added.

Column C. is the experience of the Travelers Insurance Company on its special health contract issue of November, 1902 to January, 1905—exposure of November, 1902 to January, 1906, and covering thirty specified diseases, as follows :—typhoid fever, yellow fever, scarlet fever, typhus fever, scarlatina, smallpox, varioloid, diphtheria, measles, Asiatic cholera, cholera morbus, erysipelas, appendicitis, diabetes, peritonitis, tetanus, pleurisy, pneumonia, acute hydrocele, angina pectoris, cerebral apoplexy, brain fever, epilepsy, hydrophobia, sunstroke, cancer, malignant tumor, whooping cough, acute cerebral spinal meningitis and mumps.

Column D. is a combination of the experience of the Travelers Insurance Company on its three special health contracts, as given in columns A., B. and C.

Column E. is the combined special health experience of the Travelers Insurance Company issue of March, 1899 to January, 1906—exposure of March, 1899 to January, 1907. It is the same as the experience in Column D. with another year's experience added. In this added experience most of the contracts paid indemnity while "necessarily confined to the house" instead of while "continuously and wholly disabled."

Column F. is the experience of the Travelers Insurance Company on its instalment special health contract issue of October, 1899 to January, 1905—exposure of October, 1899 to January, 1906. The contract in this case was partly the same as that in column A. covering fifteen specified diseases and partly the same as that in column B.

covering twenty-one specified diseases ; and the policy was issued to the industrial classes—very largely to railroad employees.

Column G. is the experience of the Travelers Insurance Company on its instalment special health contract issue of October, 1899 to January, 1906—exposure of October, 1899 to January, 1907. It is the same as the experience in Column F. with another year's experience added and with a similar change in the wording of the contract from "continuously and wholly disabled" to "necessarily confined to the house."

Column H. is the experience of the Travelers Insurance Company on its general health policy issue of November, 1902 to January, 1905—exposure of November, 1902 to January, 1906. This contract covered practically all diseases.

Column I. is the experience of the Travelers Insurance Company on its general health policy issue of November, 1902 to January, 1906—exposure of February, 1902 to January, 1907. It is the same as the experience in Column H. with another year's experience added and with a similar change in the wording of the contract from "continuously and wholly disabled" to "necessarily confined to the house."

Column J. represents the combined experience on the general health contract of some thirteen accident companies represented in the International Association of Accident Underwriters and is based upon policies issued or renewed in 1903. As an experience it is in the same class as that given in Column H.

Column K. is the experience of the Manchester Unity, 1846–1848.

Column L. is the experience of the Manchester Unity, 1856–1860.

Column M. is the experience of the Manchester Unity, 1866–1870.

Column N. is the experience of the Ancient Order of Foresters, 1871–1875.

Column O. is the experience of the Friendly Societies, 1876–1880.

Column P. is the experience of the Manchester Unity, 1893–1897.

Column Q. is the experience of the Manchester Unity, 1893–1897, with the occupations more injurious to health, such as outdoor building trades, dock laborers, bargemen, railroad employees, sea-faring men, fishermen, quarry workers, iron and steel workers and miners excluded.

Column R. is the mortality rate according to the American Experience.

NUMBER OF WEEKS' SICKNESS FOR EACH YEAR OF EXPOSURE.

Age	A	B	C	D	E	F	G	H	I	Age	J
Under 25	.29	.29	.28	.29	.29	.32	.33	.55	.75	10-19	.33
25–29	.21	.24	.18	.21	.22	.32	.30	.68	.60		
30–34	.20	.21	.20	.20	.20	.27	.27	.68	.55	20-29	.37
35–39	.21	.21	.19	.20	.20	.33	.34	.70	.65		
40–44	.15	.22	.17	.18	.17	.33	.32	.60	.61	30-39	.39
45–49	.18	.23	.17	.18	.18	.31	.29	.65	.62		
50–54	.20	.22	.26	.23	.25	.51	.46	.93	.87	40-49	.38
55 and over	.25	.45	.28	.30	.35	.28	.25	.95	1.35		
All Ages	.20	.23	.20	.21	.21	.32	.32	.67	.64	50-59	.49
No. yrs. Exposure.	24034	16697	22872	63603	72776	23418	26552	6362	12373		10309

Age	K	L	M	N	O	P	Q	R
16–19	.39	.76	.54	1.04	.88	.92	.81	.007710
20–24	.68	.83	.75	.82	.85	.90	.82	.007906
25–29	.76	.82	.81	.85	.87	.95	.87	.008199
30–34	.84	.86	.93	.97	1.02	1.06	.97	.008617
35–39	.94	1.00	1.06	1.15	1.24	1.27	1.14	.009250
40–44	1.18	1.24	1.26	1.37	1.47	1.58	1.44	.010275
45–49	1.51	1.55	1.64	1.71	1.89	1.99	1.81	.012057
50–54	2.03	2.01	2.22	2.27	2.39	2.75	2.54	.015461
55–59	3.21	3.02	3.05	3.21	3.36	4.02	3.76	.021425

Studying these results the most noticeable feature is the remarkable uniformity in the rate of sickness according to the first five columns (A., B., C., D. and E.) in which A., B. and C. represent the experience on three slightly modified forms of the special health policy, D. is a combination of A., B, and C., and E. is practically the same as D. only with one additional year's experience.

For ages under twenty-five four of these columns give .29 of a week of sickness per year of exposure and the other column gives .28. For ages thirty to thirty-four four of the columns give .20 of a week of sickness per year of exposure and the other column gives .21. For ages thirty-five to thirty-nine two of the columns give .21, two give .20 and one gives .19. For such a small exposure and for a practically new business the uniformity is almost astonishing. The only instance where there is any great variation is for ages fifty-five and over, where the amount of exposure is very small.

There is another striking feature shown by these columns. The rate of sickness starts high at the younger ages (commencing at about .29 for ages under twenty-five) and gradually decreases until it reaches about .18 (for the two five-year periods forty to forty-four and forty-five to forty-nine) and then gradually increases with strong evidence of a rapid increase at the higher ages. For all ages combined the rate of sickness is .20 of a week for two of these columns, .21 for two of these columns and .23 for the remaining column.

The experience in Column F., which represents insurance on industrial classes (largely railroad men) under the special health contract and under Column G., which is practically the same thing with one year's experience added, shows a nearly uniform rate of sickness up to age fifty and then quite a decided increase except for ages above fifty-five, where the exposure is too small to be given much consideration. For all ages combined this industrial experience shows a rate of sickness about 50 per cent. greater than the experience in Columns A., B., C., D. and E.

Column H., which represents the Travelers' experience on the general health policy covering practically all diseases and Column I., which is practically the same thing with another year's experience added, shows a rate of sickness about three times that upon the regular special health policy and about twice that upon the industrial special health policy. It shows a tendency—although not so decided

a tendency—to start high and then decrease until towards age fifty and then a rapid increase.

Column J., which gives the combined experience of several accident companies on the general health policy, shows a nearly uniform rate of sickness up to age fifty and then a material increase.

The general conclusion drawn from these various experiences given in Columns A. to J., inclusive, is that there is no more and may be less sickness at ages thirty-five to fifty than at ages eighteen to twenty-five. Special attention is called to this point because it had been previously supposed by many that the rate of sickness, like the rate of mortality, gradually increases with increasing age—at least from age eighteen up.

In studying the results in Columns K. to Q., inclusive, giving the various experiences of the English friendly societies, we see there is a more or less gradual and uniform increase in the rate of sickness from age twenty on, and an increase considerably greater than the increase in the mortality rate for the corresponding ages as given in Column R., which represents the American experience mortality rate.

Three things should be borne in mind in the interpretation of these results, with special reference to the fact that this experience gives a much lower rate of sickness than any of the English friendly societies and the fact that the sickness rate shows no tendency to increase until nearly age fifty.

First,—This Manchester Unity's experience, even after the elimination of most of those occupations which are apt to cause the highest rate of sickness, is made up mostly of the middle industrial classes, while a large part of the Travelers' experience included in the figures for Columns A. to I. (except F. and G.) represents the professional, clerical, commercial and other high-grade classes.

Second,—This Manchester Unity's experience represents a very old experience—so old that in the experience for 1893 to 1897 nineteen of the members have reached ages between ninety-five and one hundred.

Third,—This Manchester Unity's experience includes disability from accident which is not included in the Travelers' experience.

On the other hand, the Travelers' experience dates from the close of 1899, and the greater part of it is much more recent than that date. The combined experience on the general health policy, as shown in Column J., which is much more favorable than that of the

Travelers, as shown in Column I., is much newer business than that of the Travelers as it represents policies written or renewed in 1903, while the Travelers' experience comes down several years later. This means that in the case of the Manchester Unity in the great mass of the experience the benefit of selection has entirely run out while the Travelers' experience—and possibly to a still greater extent the combined experience of the accident companies — represents risks which are still benefiting very largely from recent selection. It should be borne in mind that all of these health policies issued by the accident companies in this country contain a clause giving the company the right to cancel the policy at any time and return the unearned premium ; also renewing the policy from year to year is voluntarily on the part of the company ; and, practically, the Travelers has declined to renew from 1 per cent. to 2 per cent. of the health policies when the year has expired. One can easily see what an immense difference it would make in the mortality experience of an American life company if the company had the right to pick out and cancel each year 1 per cent. of the worst risks on the books.

As compared with the benefit of selection as shown in the first year or two of a life insurance experience the evidence of benefit of selection as shown in the Travelers' experience on its general health policy in Column I. is not excessive. It is quite probable that if the Travelers' general health business should ever become as old as that of the Manchester Unity its rate of sickness would approximate that of the Manchester Unity from 1893 to 1897, making allowance for the fact that the health policy does not cover disability from accident ; and, if it does not approximate this rate it would probably be due, first, to the fact that the company has the right of cancellation and, second, to the fact that aside from this right to cancel the contract the business is not so persistent.

Coming now to some special points developed in the course of the investigation and confining ourselves to the experience on the regular special health policies, as given in Column E., on the special health instalment policies, as given in Column G., and on the general health policies, as given in Column I., we obtain the following tables as the result of sub-dividing this experience according to certain occupations, according to certain geographical sub-divisions and according to the distribution by population.

Special Health Policies.

Occupation	Years of Exposure	Weeks of Sickness
1. Professional and Clerical	49,529	.19
2. Industrial (indoors)	11,457	.24
3. Industrial (outdoors)	6,328	.29
4. Commercial Travelers	3,676	.26
5. Liquor Business	1,786	.15
Total	72,776	.21
Geographical Section		
1. North	51,743	.20
2. South	13,987	.26
3. West	7,046	.23
Total	72,776	.21
Population		
1. 25,000 and Over	31,859	.18
2. Under 25,000	40,917	.23
Total	72,776	.21

Special Health Instalment Policies.

Occupation	Years of Exposure	Weeks of Sickness
1. Professional and Clerical	1,337	.20
2. Industrial (indoors)	13,038	.28
3. Industrial (outdoors)	12,177	.37
Total	26,552	.32
Geographical Section		
1. North	20,611	.30
2. South	3,270	.43
3. West	2,671	.31
Total	26,552	.32
Population		
1. 25,000 and Over	7,850	.31
2. Under 25,000	18,702	.32
Total	26,552	.32

General Health Policies.

Occupation	Years of Exposure	Weeks of Sickness
1. Professional and Clerical	10,883	.62
2. Industrial (indoors)	489	.56
3. Industrial (outdoors)	248	.88
4. Commercial Travelers	558	.89
5. Liquor Business	195	.77
Total	12,373	.64
Geographical Section		
1. North	11,543	.63
2. South	650	.78
3. West	180	.38
Total	12,373	.64
Population		
1. 25,000 and Over	7,368	.62
2. Under 25,000	5,005	.67
Total	12,373	.64

Studying the figures in these special tables it is seen that the professional and clerical occupations have a very low rate of sickness according to the special health and the special health instalment experience. In the general health experience the rate of sickness for the professional and clerical occupations is also comparatively favorable, although not so favorable as for the industrial indoor classes. Comparing the indoor and outdoor occupations, the indoor occupations have a very much lower rate of sickness on all three policy forms. This is contrary to the often expressed opinion that it is much healthier to work outdoors in the pure, fresh air than to follow an indoor occupation. Commercial travelers show a rate of sickness above the average, while we have the somewhat astonishing result that men engaged in the liquor business make the most favorable showing of all, except on the general health policy where the total exposure for this occupation is only one hundred and ninety-five years (not enough to be given serious consideration).

The geographical divisions of this experience show that there is very much less sickness in the north than in the south, with the west not as favorable as the north, except in case of the general health policy where the showing for the west on a very small exposure (only

one hundred and eighty years) is exceedingly favorable. This variation in the rate of sickness in the different sections of the country—with the north giving invariably better results than the south—is confirmed by the Travelers' experience in other lines of business. The Travelers' life experience shows the highest mortality rate in the south, the lowest in the north and an average rate for the west ; and, a similar statement can be made in regard to the loss ratios of the Travelers' business in its accident and liability lines.

In regard to population, cities of 25,000 and over, according to this experience, have less sickness than the smaller places on all three forms, although the difference is very slight on the special health instalment policy. This result is contrary to the quite general impression that the large cities are not as healthy as the small cities and country towns. Nearly all experience in regard to the mortality rate would seem to indicate that the large cities are not as healthy as the smaller places.

In the various tables, representing the experience of the Travelers Insurance Company on its health business, as published in this paper, indemnity claims in general have been limited to twenty-six weeks in order to make the experience uniform, notwithstanding the fact that part of the time some of the contracts gave indemnity for a longer period. In the special health experience, however, six claims for permanent disability have been counted (each as one hundred weeks of sickness), and, similarly, one claim in the special health instalment experience. In a certain portion of this experience contracts give indemnity for partial as well as for total disability and two hundred and ninety-four weeks of indemnity under partial disability in this experience has been counted in as one hundred and forty-seven full weeks of sickness. Some of the general health contracts provide for certain surgical operations at the expense of the company ; and, in this experience on the general health policies twenty-nine weeks of sickness are included as the equivalent of the surgical operations. As far as reported to the company there has been a death rate on the special health experience of 1.51 per thousand, on the special health instalment experience 1.66 per thousand and on the general health experience 2.91 per thousand.

(Presented by Mr. Walter S. Nichols.)

An Inquiry into the Nature of the Fundamental Principles of a Contract of Indemnity.

ALBERT W. WHITNEY.

Fore Word, by Mr. Nichols.

The problem here discussed by Professor Whitney is one whose importance can only be appreciated by those who are familiar with the complex questions arising in connection with fire insurance adjustments. The proportions in which several policies involved in a loss shall contribute to its settlement involve no difficulty so long as they all cover the various items in similar proportions. But when the items included under one policy differ from those embraced in another, the ratios in which their several contributions shall be made become a matter of dispute. Such questions have, from time immemorial, been disposed of by the courts as pure legal propositions concerning the construction of the fire insurance contract and by fire insurance adjusters on the basis of various artificial rules. In both cases they have been dealt with on principles so inconsistent among themselves that they cannot be harmonized and every such adjustment has been virtually a law to itself. Numerous works and memoirs have been written on the subject of such adjustments by fire underwriters, none of which are in accord with the rest. The difficulty has been to find a mathematical basis for apportionment which will apply alike to all cases, while in harmony with the legal construction of the contract, as defined by the courts.

While the solution of the problem must depend on the construction of the contract as a question of law, such construction in turn, if correct, must rest on mathematical assumptions regarding the character of the equities involved whose proper application will lead to consistent results.

The late William C. Whiting, who was a member of this Society, recognized this fact and employed his well known mathematical genius in an attempt to establish such a basis.

Professor Whitney, in the following paper, approaches this problem from a novel standpoint and deals with it as a problem of fluent quantities, laying down certain fundamental assumptions, which through the application of the calculus, apparently furnish a consistent theoretical solution to the problem in its broadest generality. Special cases frequently arise in which one or more of the policies may be exhausted before complete indemnity has been furnished. These simply require the repeated application of the same principles until either complete indemnity has been secured or all the policies are exhausted. These he has not attempted to discuss. The mathematical principles involved may be likened somewhat to those met with in the life office, in the application of the contribution plan to the various sources from or to which the receipts and expenditures flow; or to the assumption of the gradual exhaustion of vitality analogous to the exhaustion of a receiver by means of an air pump, by which Benjamin Gompertz deduced his original law of mortality. I believe it is the first time that the fundamental conceptions of the calculus have been applied to this class of questions.

This paper arises from an attempt to solve a classic problem in fire insurance known as "The Apportionment of Losses under Non-Concurrent Policies." In order to solve this problem it is necessary to define, with great precision, the exact nature of the fundamental provisions of the insurance contract. The ground for supposing that this subject may be of interest to students of life insurance is the fact that the fundamental principles of indemnity are, for all kinds of insurance, the same. So many elements enter into the problem of furnishing indemnity in the case of fire that a statement of the principles of the fire insurance contract will possibly be sufficiently general to cover all insurance; it will certainly include the more ordinary forms of life insurance.

The life insurance problem in at least three respects is only a special case of the more general problem. In the first place in

life insurance the loss takes place all in an instant; a fire has a period of duration, a death has not. In making this statement I had in mind the ordinary single-payment type of policy. It occurs to me that a reversionary annuity, particularly an instantaneous reversionary annuity, is, as regards duration, an almost exact counterpart of a fire insurance policy. The first contingency in the case of the fire insurance is with regard to the question of whether the fire will *start*; the second contingency is with regard to the *duration* of the fire, hence the amount of loss; this corresponds to the length of life of the beneficiary after the death of the insured.[1]

In the second place in life insurance (perhaps I should be more cautious by saying, in the more ordinary forms of life insurance,) it is not necessary to consider the possibility of two losses occurring cotemporaneously.

But probably the really most fundamental difference between the case of fire and of life is the fact that in case of fire the loss is physically appraisable and has an upper limit, namely the value of the property, which, however, has no absolutely necessary relation to the amount of covering insurance. But no attempt is made after the loss has occurred to appraise the value of a life and the value of the life is construed to be at least equal to the amount of insurance carried.

This comparison has perhaps been already carried too far, although as a matter of fact the nature of the contingencies in the two cases might be compared in much greater detail. I will state the problem. The problem in its simplest concrete form is this: books and furniture in a certain house are covered by a blanket policy of $3,000 in Company A; books alone are covered by an additional policy of $2,000 in Company B; a fire occurs doing damage of $500 to furniture and $1,500 to books; ascertain the contribution of each company to make good the loss. The policies are non-concurrent in the sense that they do not cover the same items. It is understood that far more complicated cases of non-concurrence might and do occur when there are a greater number

[1] It is interesting to note that in the problem of determining coinsurance rates in fire insurance exactly these two probabilities occur and have been referred to as the Ignition Hazard and the Damage Hazard.—Report of the Coinsurance Committee of the Board of Fire Underwriters of the Pacific on Coinsurance Rates.

of items and a greater number of policies. Neither the cases of concurrent policies nor of non-overlapping policies furnishes any difficulty; each is contained however in the general problem as a trivial case.

This problem calls immediately for a far more searching inquiry into the nature of the indemnity contract than is necessary in ordinary cases. I have thrown such a statement of the contract into the form of axioms—axioms which depend for their truth (provided this is the proper interpretation of the contract) upon the mutual agreement of the parties concerned. Axiomatic statements furthermore prepare our minds for necessary logical conclusions.

Let me interject a word here to explain that this is in no sense an actuarial problem. It does not concern itself with *how* indemnity is furnished; there need be from this point of view no mass of the insured and no law of averages. Sufficient it is to know that the insurer binds himself to furnish indemnity up to a certain amount. This amount, for the sake of being graphic, we may refer to as the "indemnity fund;" ordinarily it would be known as the "insurance in force."

Definition: Insurance is a plan for furnishing indemnity; the indemnity accrues continuously as the loss takes place; accrued indemnity flows from an "indemnity fund" (or funds), viz.: the "insurance in force" and attaches at the point at which loss is taking place.

Axiom I: When an indemnity fund (or funds) protects a property upon which loss is taking place, or more than one property upon which losses are simultaneously taking place, the total flow to each property during an instant is equal to the loss upon that property during the instant in question.

Axiom II: When a property is protected by more than one indemnity fund the flow from each fund during a given instant is proportional to the size of the fund at that instant.

Axiom III: The flow to each item of loss continues either until the entire loss upon that item is made good or until all the indemnity funds contributory thereto are exhausted.

Naturally the first question that arises with regard to such a system of axioms is as to whether it agrees with the interpretation of the contract in ordinary cases.

It will be noticed that these axioms make use of the phraseology of differentials (the phraseology of rates might equally well have

been used with perhaps some sacrifice of vividness). These same axioms could be framed for ordinary cases in the phraseology of finite intervals. What I mean is of course this (take an example): C covers an item with $1,000 of insurance, D covers the same item with $2,000 of insurance; there are no other items; there is no other insurance. There has been a loss of $1,500; ordinary usage prescribes a payment of $500 from C and $1,000 from D, viz., a sharing of the loss in the ratio of the amount covering, viz., 1:2. Now axiom II prescribes that the flow from the two funds during an instant shall be in the ratio of the funds at the instant in question. But, if the two policies contribute to exactly the same loss and start in the ratio of 1:2, it is evident that they will continue to contribute in this ratio, resulting in contributions of respectively $500 and $1,000 as by ordinary usage. Since then this ratio of flow in the ordinary case does not change from instant to instant the axioms may be reworded, substituting for "size of the fund at that instant" "size of the fund at the beginning of the loss," viz., the insurance in force, that is, the face of the policy, and instead of the flow during an instant we may equally well use the flow during a finite interval of time. That is, in ordinary usage, there is no necessity of viewing the process of furnishing indemnity as a flow. It is rather a casting up of the necessary indemnity after the loss is an accomplished fact. But we may go one step further in our test of these axioms by the rules of ordinary usage.

After a loss has occurred which does not exhaust the insurance in force the risk stands covered not by the original amount of insurance in force but by what remains of this after the loss has been paid; this is the "size of the indemnity fund at the instant in question." If now a second fire occurs this new indemnity fund is operative. Axiom II is the pushing of this principle to its limit, that is, a fire is considered as a succession of infinitesimal fires. In ordinary cases this point of view is made unnecessary by the fact that after any interval of time the indemnity funds, while changing in absolute value, continue to stand in the same ratio to each other. But now we come to the point: it is exactly this fact, that in a case of non-concurrency the indemnity funds do *not* continue to stand in the same ratio, that necessitates the conception of flow during an *instant* and, to anticipate, it is perfectly evident

that the problem must express itself not in a system of ordinary equations but in a system of differential equations.

That, in a case of non-concurrence, the indemnity funds do not remain in the same ratio may be realized by remembering that Company A, in addition to contributing to the loss on books in conjunction with Company B, their contributions being initially in the ratio 3:2, is further being drained by its single-handed contribution to the loss on furniture.

So much for the axioms. But in addition to these a postulate is necessary in order to make the problem determinate. I will state the postulate and then consider possible alternatives.

Postulate: The rate at which loss takes place on each of one or more properties which have been destroyed by a single fire is proportional to the final amount of loss sustained, that is, the rate at which values are destroyed remains constant while the loss occurs.

This is unquestionably in general *not true,* but this is a curious case where fiction is not stranger than truth but more just. The operation of the postulate, in the concrete case under consideration, demands, since the final losses on books and furniture stand in the ratio of 3:1, that we shall consider the loss at any instant to have stood at the same ratio, that is, not only were $500 of furniture being destroyed while $1,500 of books were being destroyed, but also $1 of furniture was being destroyed while $3 of books were being destroyed at *every stage of the fire.*

An alternative postulate would be that the furniture burned first, followed by the books. The application of the axioms would then require a prior adjustment of the furniture claim followed by a flow for the loss of the books under, however, the operation of the indemnity funds as already modified by their contributions to the first claim. In detail: A would pay the furniture-loss of $500; A and B would then contribute to the book-loss of $1,500 in the ratio of 3000-500:2000, *i. e.,* 5:4. A would therefore pay $833.33 and B $666.67. A's entire contribution would therefore be $1,333.33 and B's 666.67.

If the books burned first and the furniture second A's contribution would be $1,400 and B's $600. In both cases the insured is fully indemnified but the companies share the loss differently. We may however easily advance a case where these two postulates

would lead to settlements which would differ as regards the entire amount of indemnity received by the insured.

Suppose the loss on books to have been \$4,000 and on furniture \$3,000. Under the postulate that the books burned first the book-loss of \$4,000 would be shared in the ratio of 3 : 2, that is A pays \$2,400 and B \$1,600. This leaves A with an indemnity fund of \$600 which will now be exhausted by the furniture-loss of \$3,000. The insured will therefore obtain in all only \$4,600 on his \$7,000 loss (A paying \$3,000 and B paying \$1,600), in spite of the fact that B still holds an indemnity fund of \$400.

So much must make clear the necessity for, at any rate, some postulate. But these are not the only postulates that might be made; there are an infinite number of ways in which we might suppose the fire to have taken place. But obviously the fire is to be looked at as a whole and, in our ignorance of the exact manner in which it took place and even in spite of possible knowledge as to exactly how it took place, the only fair procedure is to predicate of the fire at each stage that which we know to be true of it as a whole. This is expressed in the postulate first given.

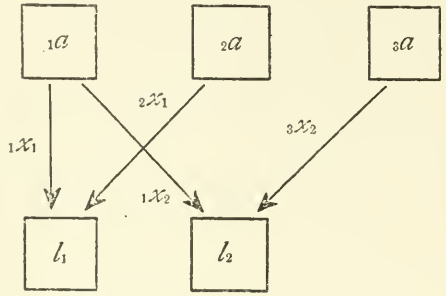
Before dismissing this part of the subject however I might say that the disposition of the courts has been to insist that under non-concurrent policies the insured is to be given the benefit of the most favorable construction possible. This is equivalent to saying: the manner of the fire shall be construed to have been such-wise as to produce the greatest possible indemnity. To determine *just what* this manner should be would be, I fear, in any complicated case, extremely difficult, but until the exact conditions for the maximum were known the problem would not be soluble.

I will now proceed to derive the differential equations for the general problem of non-concurrence. Let the indemnity funds be:

$$ {}_1a,\ {}_2a \ldots\ {}_ma. $$

Let the losses be l_1 on item I, l_2 on item II . . . . l_n on item n. Let each fund cover one or any number of items. If, however, each of the funds covers the same items this will reduce to a case of concurrent policies and will be trivial from our point of view, and if two of the funds cover the same items these two funds may be united.

Let ${}_ix_j$ be the amount of indemnity that up to the instant in question has flowed from fund ${}_ia$ to loss l_j; let ${}_ix$ be the amount of indemnity that up to the instant in question has flowed to all the items to which this fund is contributory; let x_j be the amount of indemnity that up to the instant in question has flowed to loss l_j from all contributory funds; i and j being such as each particular problem may admit. This may be given for a special case the following diagrammatic representation:



In this case:

$$\begin{aligned} {}_1x_1 + {}_2x_1 &= x_1 \\ {}_1x_2 + {}_3x_2 &= x_2 \end{aligned}$$

Also:

$$\begin{aligned} {}_1x_1 + {}_1x_2 &= {}_1x \\ {}_2x_1 &= {}_2x \\ {}_3x_2 &= {}_3x \end{aligned}$$

The equations for the general case are:

$$\begin{aligned} d_1x_1 = ({}_1a - {}_1x)_1c_1l_1d\lambda_1,\ d_2x_1 &= ({}_2a - {}_2x)_2c_1l_1d\lambda_1 \ldots . \ d_mx_1 \\ &= ({}_ma - {}_mx)_mc_1l_1d\lambda_1. \\ d_1x_2 = ({}_1a - {}_1x)_1c_2l_2d\lambda_2,\ d_2x_2 &= ({}_2a - {}_2x)_2c_2l_2d\lambda_2 \ldots . \ d_mx_2 \\ &= ({}_ma - {}_mx)_mc_2l_2d\lambda_2. \\ \vdots \qquad\qquad & \vdots \qquad\qquad \vdots \\ d_1x_n = ({}_1a - {}_1x)_1c_nl_nd\lambda_n,\ d_2x_n &= ({}_2a - {}_2x)_2c_nl_2d\lambda_2 \ldots . \ d_mx_n \\ &= ({}_ma - {}_mx)_mc_nl_nd\lambda_n. \end{aligned} \qquad [1]$$

$d\lambda_1,\ d\lambda_2 \ldots . \ d\lambda_n$ are to be determined.

The factors of the form ${}_ic_j$ are each either 0 or 1. ${}_ic_j = 0$ indicates that the indemnity fund ${}_ia$ is not contributory to the loss l_j. In the array:

$$\begin{matrix} {}_1c_1 & {}_2c_1 & \cdots & {}_mc_1 \\ {}_1c_2 & {}_2c_2 & \cdots & {}_mc_2 \\ \vdots & \vdots & & \vdots \\ {}_1c_n & {}_2c_n & \cdots & {}_mc_n \end{matrix} \qquad [2]$$

we may assume no two columns to be the same, for if two columns were the same the two corresponding policies would be concurrent and the two corresponding indemnity funds could be united and treated as one.

These equations satisfy axiom II ; for instance the equations in the first row express the content of this axiom with regard to item I. [It must be remembered that the c's are only existence-factors.]

Adding rows we have :

$$\begin{aligned} dx_1 &= \{({}_1a - {}_1x){}_1c_1 + ({}_2a - {}_2x){}_2c_1 + \cdots \\ &\qquad + ({}_ma - {}_mx){}_mc_1\} l_1 d\lambda_1 = Z_1 l_1 d\lambda_1 \\ dx_2 &= \{({}_1a - {}_1x){}_1c_2 + ({}_2a - {}_2x){}_2c_2 + \cdots \\ &\qquad + ({}_ma - {}_mx){}_mc_2\} l_2 d\lambda_2 = Z_2 l_2 d\lambda_2 \\ &\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ dx_n &= \{({}_1a - {}_1x){}_1c_n + ({}_2a - {}_2x){}_2c_n + \cdots \\ &\qquad + ({}_ma - {}_mx){}_mc_n\} l_n d\lambda_n = Z_n l_n d\lambda_n \end{aligned} \qquad [3]$$

Z_1, Z_2 . . . . Z_n being abbreviations for the bracketed sums ; not all the Z_j's can be the same.

Now according to the postulate :

$$dx_1 : dx_2 : \cdots : dx_n = l_1 : l_2 : \cdots : l_n.$$

Combining this with axiom I, if $d\lambda$ is the part of a final loss of \$1 destroyed during the instant in question, we have :

$$\begin{aligned} dx_1 &= Z_1 l_1 d\lambda_1 = l_1 d\lambda \\ dx_2 &= Z_2 l_2 d\lambda_2 = l_2 d\lambda \\ &\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ dx_n &= Z_n l_n d\lambda_n = l_n d\lambda \end{aligned} \qquad [4]$$

That is, if : $\qquad d\lambda_1 = \dfrac{d\lambda}{Z_1}, \quad d\lambda_2 = \dfrac{d\lambda}{Z_2} \cdots d\lambda_n = \dfrac{d\lambda}{Z_n},$

both axiom I and the postulate will be satisfied. The equations therefore become:

$$d_1x_1 = ({}_1a - {}_1x)_1c_1 \frac{l_1}{Z_1} d\lambda, \quad d_2x_1 = ({}_2a - {}_2x)_2c_1 \frac{l_1}{Z_1} d\lambda \ldots . \ d_mx_1 = ({}_ma - {}_mx)_mc_1 \frac{l_1}{Z_1} d\lambda$$

$$d_1x_2 = ({}_1a - {}_1x)_1c_2 \frac{l_2}{Z_2} d\lambda, \quad d_2x_2 = ({}_2a - {}_2x)_2c_2 \frac{l_2}{Z_2} d\lambda \ldots . \ d_mx_2 = ({}_ma - {}_mx)_mc_2 \frac{l_2}{Z_2} d\lambda$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$d_1x_n = ({}_1a - {}_1x)_1c_n \frac{l_n}{Z_n} d\lambda, \quad d_2x_n = ({}_na - {}_nx)_2c_n \frac{l_n}{Z_n} d\lambda \ldots . \ d_mx_n = ({}_ma - {}_mx)_mc_n \frac{l_n}{Z_n} d\lambda \qquad [5]$$

Adding columns we have:

$$d_1x = ({}_1a - {}_1x)\left\{ {}_1c_1 \frac{l_1}{Z_1} + {}_1c_2 \frac{l_2}{Z_2} + \ldots . \ {}_1c_n \frac{l_n}{Z_n} \right\} d\lambda = ({}_1a - {}_1x)_1Z d\lambda$$

$$d_2x = ({}_2a - {}_2x)\left\{ {}_2c_1 \frac{l_1}{Z_1} + {}_2c_2 \frac{l_2}{Z_2} + \ldots . \ {}_2c_n \frac{l_n}{Z_n} \right\} d\lambda = ({}_2a - {}_2x)_2Z d\lambda$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$d_mx = ({}_ma - {}_mx)\left\{ {}_mc_1 \frac{l_1}{Z_1} + {}_mc_2 \frac{l_2}{Z_2} + \ldots . \ {}_mc_n \frac{l_n}{Z_n} \right\} d\lambda = ({}_ma - {}_mx)_mZ d\lambda \qquad [6]$$

${}_1Z$, ${}_2Z \ldots . \ {}_mZ$ being abbreviations for the bracketed expressions.

${}_ic_j$ figures both in the numerator of the j^{th} term of ${}_iZ$ and also in the i^{th} term of Z_j so that ${}_ic_j = 0$ causes the j^{th} term of ${}_iZ$ and also the i^{th} term of Z_j to be lacking. This becomes more evident by observing that the Z_j's are formed from the array of c's by summation of rows and the ${}_iZ$'s by summation of columns. This peculiarity of the equations leads at once to one integral. But first we may well introduce the following abbreviations: let: ${}_ia - {}_ix = {}_iz$, $\lambda = 1 - \mu$. Then:

$$\begin{aligned} d_1z &= {}_1z_1 Z d\mu \\ d_2z &= {}_2z_2 Z d\mu \\ &\vdots \\ d_mz &= {}_mz_m Z d\mu \end{aligned} \qquad [7]$$

By adding :

$d({}_1z + {}_2z + \ldots\ldots {}_mz) = (l_1 + l_2 + \ldots\ldots l_n)d\mu$ for it may easily be seen that $\sum_1^m {}_iz_iZ = \sum_1^n l_j$.

Integrating and determining the constant by observing that when :

$\lambda = 0$: $\mu = 1$, ${}_ix = 0$, ${}_iz = {}_ia$, we have :

$$\sum_1^m {}_iz = \mu \sum_1^n l_j + \sum_1^m {}_ia - \sum_1^n l_j \text{ or } \sum_1^m {}_ix = \lambda \sum_1^n l_j. \qquad [8]$$

This is only an expression of the application of axiom I to the problem as a whole, and indeed might have been obtained more simply from equations [4].

It is impossible in this paper to discuss the general solution of this system of equations. It will be worth while however to give the solution in the special case already used as an example. In this case :

$$\begin{aligned} {}_1a &= 3000 \\ {}_2a &= 2000 \\ l_1 &= 1500 \\ l_2 &= 500 \end{aligned}$$

The array is :

$$\begin{aligned} {}_1c_1 &= 1 \quad {}_2c_1 = 1 \\ {}_1c_2 &= 1 \quad {}_2c_2 = 0 \end{aligned}$$

$$\begin{aligned} \therefore Z_1 &= {}_1z + {}_2z \\ Z_2 &= {}_1z \\ {}_1Z &= \frac{l_1}{{}_1z + {}_2z} + \frac{l_2}{{}_1z} \\ {}_2Z &= \frac{l_1}{{}_1z + {}_2z} \end{aligned}$$

The equations are therefore :

$$\begin{aligned} d_1z &= {}_1z \left\{ \frac{l_1}{{}_1z + {}_2z} + \frac{l_2}{{}_1z} \right\} d\mu \\ d_2z &= {}_2z \left\{ \frac{l_1}{{}_1z + {}_2z} \right\} d\mu. \end{aligned} \qquad [7']$$

adding we get the one integral : ${}_1z + {}_2z = {}_1a + {}_2a - \lambda(l_1 + l_2)$,

or $\frac{{}_1z + {}_2z}{{}_1a + {}_2a} = 1 - \lambda \frac{l_1 + l_2}{{}_1a + {}_2a}$. $\qquad [8']$

The other integral follows easily.

The complete solution is :

$$\frac{{}_1z + {}_2z}{{}_1a + {}_2a} = 1 - \lambda \frac{l_1 + l_2}{{}_1a + {}_2a}, \quad \frac{{}_2z}{{}_2a} = \left(1 - \lambda \frac{l_1 + l_2}{{}_1a + {}_2a}\right)^{\frac{l_1}{l_1 + l_2}} \qquad [9']$$

Substituting the values of the a's and l's we have :

$$_1z + {}_2z = 5000(1 - 2/5\lambda), \quad {}_2z = 2000(1 - 2/5\lambda)^{3/4}.$$

$\lambda = 1$ is the formal condition for complete indemnity. It follows from the integration of equations [4] that *formally* the indemnity is complete for all losses at the same time ; this however may occur by means of some or all of the ${}_iz$'s becoming negative, that is by the over-drawing of the indemnity funds. This is an innocent enough act from a mathematical point of view, but is of course not to be contemplated ; when a fund is exhausted that particular source of indemnity ceases to exist.

In the example above when $\lambda = 1$ we get :

$$_1z = 1636.54$$
$$_2z = 1363.46$$

As this overdraws neither of the indemnity funds this is the proper solution.

In the second case cited the a's have the same value as before but $l_1 = 4000$, $l_2 = 3000$.

This gives :

$$_1z + {}_2z = 5000(1 - 7/5\lambda), \quad {}_2z = 2000(1 - 7/5\lambda)^{4/7}.$$

$\lambda = 1$ is inadmissible.

$\lambda = 5/7$ exhausts the fund ${}_2a$ but only after the fund ${}_1a$ has become exhausted.

$\lambda = .63008$ is the true boundary condition.

When $\lambda = .63008$ the fund ${}_1a$ is exhausted.

$2520.32 has been paid at this time toward loss l_1 and $1890.24 has been paid toward loss l_2. There is still unpaid $1479.68 on loss l_1 and $1109.76 on loss l_2. There still remains $589.44 in the fund ${}_2a$. This now goes entirely toward the remaining loss on item I. The total payment on item I is therefore $3109.76 and on item II $1890.24. This is altogether a total of $5000, as it should be, for both the funds are exhausted. On the hypothesis that the loss on item I occurred first we have already seen that the corresponding payments would be $4000 and $600 and on the hypothesis that the loss on item II occurred first that the corresponding payments would be $2000 and $3000.

Mortality in Semi-Tropical and Tropical Countries: Rates of Premium Charged, and Valuation Basis for these Countries.

BY ARTHUR HUNTER.

Among the most interesting problems which the actuary is endeavoring to solve are the relationships of race and of climate to longevity. Unfortunately, there are great difficulties surrounding the solution of these problems because data can rarely be obtained which are accurate, of sufficient extent and of a proper degree of homogeneity to lead to reliable conclusions. We cannot, for example, determine what the effects of the climate of the United States and Canada have been on Irishmen and Scandinavians resident in these countries from the fact that a mortality of 25% in excess of the normal on the former, and 5% less than the normal on the latter has been shown by the Specialized Mortality Investigation, because the environment of the two races is dissimilar. The Scandinavians in that investigation were largely farmers, living an open-air life, while the Irishmen were generally employed in in-door occupations and lived in cities. The factors of occupation and environment have a large bearing on mortality, and must accordingly be carefully considered in any comparison between two races living in the same country, or between groups of the same race living in different climates. Taking another example of the difficulty of obtaining proper standards of comparison, we could not say that the mortality in France was the same as in the United States if an American Insurance Company experienced 100% of its standard "Select" mortality table in both of these countries, because the insured in the former country are drawn from people in better financial circumstances, on the average, than in the latter, there being few mechanics,

artisans and small shopkeepers insured by the American companies in France.

In a study of the mortality in the Tropics, the factor of occupation is not of as much moment as in the United States, less than 2½% of insured lives in the Tropics being engaged in occupations that are usually treated as under-average. Practically the only applicants for insurance engaged in occupations involving hazard are those employed in the manufacture or sale of alcoholic liquors. The conditions surrounding the business are different from those in the United States, Canada or Britain. In the American Tropics a large percentage of the storekeepers sell wines and spirits in one form or another, and nearly all restaurants and hotels serve wine with meals. Instead of increasing the difficulty of the problem of dealing with the liquor trade the general use of alcohol, usually in the form of light wines, greatly decreases it, because the Latin races, who are the principal inhabitants of the American Tropics, do not drink to excess to the same extent as the Anglo-Saxon or the Teuton in his native country. The Latin is accustomed to take wine with his meals as a matter of course, and while everyone drinks more or less, there is little drinking to the point of intoxication. Those who have studied the liquor problem in Italy and Spain will doubtless bear me out in the statement that drunkenness is almost unknown, and that the consumption of wine appears to have little or no effect in reducing the longevity of the population. The same condition applies to the Asiatic Tropics, the excessive use of alcoholic beverages being little known among Japanese, Chinese or East Indians.

While the question of occupation among insured lives in the Tropics can largely be ignored, there are other disturbing elements which prevent our determining the effect of race and of climate on mortality such as the different standards of morals and the different sanitary conditions which prevail in different countries and in different sections of the same country. These matters will be dealt with in their proper place; suffice it to say at the present time, that the effect of climate should not be considered as the sole cause of the high mortality prevailing in the Tropics. So far as the effect of *race* on mortality in Tropical climates is concerned no attempt has been made to draw any definite conclusions from our data, partly because of the paucity of the statis-

tics on foreigners resident in the tropics, and partly because different climates have a different effect on foreigners so that it is inadvisable to group the data. We should not, for example, add the statistics on foreigners living in Japan to the data on unacclimated foreigners living in Rio Janeiro. The former are as good risks as, if not better than, the native Japanese of the same social standing, while the latter, if not acclimated, are distinctly worse risks than the Brazilians in a corresponding walk of life—the companies showing such a belief by charging an acclimation extra as high as $50. per $1,000. insurance. It is practically impossible to differentiate between the effect of climate and race on longevity in the present statistics, and we must, accordingly, content ourselves with seeing merely the aggregate effects of Tropical residence.

Allied to the subject of mortality among foreigners, resident in Tropical countries, is that of acclimation. The vast bulk of the business is on the lives of natives of the various countries, so that the question of acclimation is a minor one. Acclimation extras were charged to foreigners in nearly all Tropical countries at one time, during the first three years of their residence, but these extras have been dispensed with in many countries, such as Cuba, where the sanitary conditions have greatly improved, and where yellow fever now rarely occurs. Extra premiums for non-acclimation are still imposed in certain parts of Central America and in one or two of the South American countries, although there is no unanimity of opinion regarding the reasons for imposing these extras, some authorities believing that it is to protect the companies against the ravages of yellow fever, and others, to cover the extra hazard incident to a foreigner becoming adapted to the new conditions.

Another factor which has some weight in determining the mortality of the various companies in the United States, Canada and Britain is the proportion of Endowment insurance to the total insurance in force. In Tropical countries it is questionable whether the selection exercised by the insured in his choice of the form of policy has any appreciable effect on the mortality. The plan of insurance partly depends on the tradition in each community as to the best plan of insurance, and partly on the relative safety of investments in each country. For example, notwithstanding the much lower percentage of commission paid on Endow-

ment insurance than on Ordinary Life policies, it is rare indeed in Japan, that any form of policy is sold to natives on the Life or Limited Payment plans, over 99% of our business being on the 15 and 20 Year Endowment plans. The same thing is true regarding native Chinese. In countries like Argentine, the percentage of 20 Year Endowment policies sold in 1907 out of the total business paid for in that year was 60%. In other countries, such as Brazil and Chili, marked changes have occured in the distribution of the business by plan, the policies issued twenty years ago being almost entirely on the Whole Life plan, whereas, at the present time, the business is principally on the Endowment plans.

Subjects to be Treated.

While the main subject of this paper is the mortality in the Tropics, it is necessary for a proper understanding of it to deal with the rates of premium charged. The collateral subject of the proper bases of reserves in both Semi-Tropical and Tropical countries is also of interest. The following, therefore, will be the subjects dealt with:

1. Rates of premium charged throughout the world by the New York Life.

   (The same rates are generally used by the Mutual and the Equitable in countries in which they do business, and similar rates by several Canadian Companies.)

2. Rates of mortality which the Semi-Tropical and Tropical premiums will safely bear.

3. Relative mortality in the principal Semi-Tropical and Tropical countries.

4. Comparison of the mortality experience of the New York Life in Semi-Tropical and Tropical countries with the Compound Progressive Table, the Jones' Tropical Table and the American Table.

5. Graduation of the New York Life's Tropical and Semi-Tropical experience.

6. Comparison of New York Life's mortality experience with that of other companies doing business in the Tropics.

7. Improvement in Mortality in Tropical Countries.

8. Reserve basis for policies issued in Tropical and Semi-Tropical countries.

1. RATES OF PREMIUM CHARGED THROUGHOUT THE WORLD.

In order to give those interested a knowledge of the basis of the tables presented in this paper, and in the hope that the information may be of assistance to the officers of companies who occasionally accept applicants going to a foreign country, a statement is supplied of the scale of premium charged in all the principal countries, throughout the world, although the data in the present experience of the New York Life cover only the countries in which *the Semi-Tropical and Tropical scale of premium is charged.* There are many countries, geographically large, which yet have very small importance from the insurance standpoint. For example, in many parts of Asia, such as Afghanistan and Persia, no business is done among natives, the only policies issued being to Consuls or other foreigners going to reside there temporarily. If business were actively pushed among the natives of some of these Asiatic countries a higher scale of premium might be charged. Again, a higher scale might be charged in Turkey and in Asia Minor if we insured the Turks, but there seems to be some religious objection among these people to insurance, as applications are not received from them. The countries have been divided in accordance with the following scales of premium:

(a) The Northern or Domestic scale.
(b) The Russian scale, i. e., higher than the Domestic, but lower than the Semi-Tropical scale.
(c) The Semi-Tropical scale.
(d) The Tropical scale.
(e) The Domestic scale, plus from $20. to $50. per $1,000. annually, which would give higher rates than the Tropical scale.

Northern or Domestic Scale.

North America.

United States of America, Canada, Bermuda and Bahama Islands.

Africa.

Madeira, St. Helena and Canary Islands. Egypt as far as the second cataract of the Nile. Cape Colony, Natal, Orange River Colony and British Bechuanaland.

Oceanica.

Australia, New Zealand, Fiji, and New Caledonia.

Europe.

All Europe with the exception of Russia, Servia and Poland.

Pacific Islands.

Hawaii.

Russian Scale.

Europe.

Russia, Servia, and Poland.

Asia.

Asiatic Russia, including Siberia and Russian Turkestan.

Semi-Tropical Scale.

North America.

Mexico—Northern and Central States as far south as, and including the State of Mexico, but excluding the State of Vera Cruz and part of Tamaulipas.

South America.

Chili, Argentine, Uruguay and the two Southerly States of Brazil.

West Indies.

Jamaica.

Asia.

Chinese Empire, north of 30 degrees North Latitude. Manchuria and Corea. Japan, exclusive of Formosa.

Tropical Scale.

North America.

Mexico, Southern States of, including Vera Cruz and part of Tamaulipas.

Central America.

Guatemala, Honduras and British Honduras, San Salvador, Costa Rica and Nicaragua.

South America.

Panama, Colombia, Venezuela, British, Dutch and French Guiana, Ecuador, Peru, Bolivia and Brazil, with the exception of the two most Southerly States of Brazil, (Semi-Tropical rates), also excepting the four States of Amazonas, Para, Matto Grosso and Goyas (business not accepted).

West Indies.

Cuba, Hayti, Porto Rico, Trinidad, Barbados, Martinique, Guadeloupe and Dutch and Danish West Indies.

Africa.

Egyptian Soudan, Islands of Madagascar, Mauritius, Cape Verde and Ascension.

Asia.

Chinese Empire, south of 30 degrees North latitude. Eastern Turkestan. India, including Burmah, Ceylon and Straits Settlements. East India Islands—Sumatra, Java, Borneo, Celebes and Philippines. Persia, Afghanistan, Baluchistan and Arabia. Formosa.

Oceanica.

Islands of New Guinea, Admiralty, New Britain, Solomon, Marshall, Caroline, Pelew, Ladrone, New Hebrides, Santa Cruz, Marquesas, Cook, Bismarck, Society, Tonga, Friendly, (if accepted at all.)

Higher than Tropical Scale.

Africa.

West and south-west Africa, including German south-west Africa, Central Africa, East and north-east Africa, as far south as the Delagoa Bay territory, and all Central Africa north of the Tropic of Capricorn—Domestic premiums plus $20. to $50. per $1,000. annually.

The general rule is to charge an annual extra of $50. per $1,000. west of 20 degrees East Longitude; $30. per $1,000. east of 20 degrees East Longitude and north of the Equator; $20. per $1,000. east of 20 degrees East Longitude and south of the Equator. An annual premium is charged in all cases, with no refund for a fractional part of a year's residence in northern climates, the policies being limited to 10 and 15 Year Endowments in the most hazardous sections of Africa.

Asia.

Annam and Siam—Northern rates plus $20. per $1,000. extra per annum.

Examples of the Different Scales of Premiums.

At this point it would probably be of value to the readers to have examples of the different scales of premium charged by the New York Life, which, as already mentioned, are generally similar to those charged by American and Canadian companies, transacting a foreign business. A synopsis is accordingly given of the premium rates per thousand at the Northern, Russian, Semi-Tropical and Tropical scales for the three principal plans of insurance, Whole Life, 20 Payment Life and 20 Year Endowment. Prior to the year 1908 the Semi-Tropical rates were a mean between the northern and the Tropical but in the beginning of that year the northern rates on the Endowment plans were slightly changed, the Semi-Tropical and Tropical remaining unchanged.

Synopsis of Premium Rates.

WHOLE LIFE.

Ages.	Northern.	Russian.	Semi-Tropical.	Tropical.
25	$21.49	$24.70	$26.68	$31.86
35	28.11	32.00	34.13	40.14
45	39.55	44.10	47.00	54.44
55	60.72	65.00	70.81	80.90

20-PAYMENT LIFE.

25	31.83	37.80	38.27	44.70
35	38.34	44.40	45.61	52.87
45	48.52	54.40	57.22	65.91
55	66.69	71.50	77.41	88.12

20-YEAR ENDOWMENT.

25	49.33	52.70	54.72	58.90
35	51.91	54.90	57.49	62.50
45	57.34	60.40	63.77	70.21
55	70.81	73.60	80.10	89.69

2. RATES OF MORTALITY WHICH THE SEMI-TROPICAL AND TROPICAL PREMIUMS WILL SAFELY STAND.

The first question which will appeal to the actuary in studying the above table of rates, is,—what rate of mortality will they stand? This question will be answered here only so far as the Semi-Tropical and Tropical scales are concerned, as the mortality tables submitted in this paper do not cover policies issued at the Russian scale. The first step is to determine the net premiums corresponding to the gross rates. These were obtained by assuming that the net premiums on the Semi-Tropical and Tropical policies had the same percentages of loading as under the corresponding Domestic policies. For example, on the Ordinary Life plan, the Domestic policies are loaded 25% of the gross premium, and in obtaining the net premium for the tropics therefore, 75% of the gross tropical annual premium has been taken. As the gross premiums under the Domestic scale prior to 1908 were calculated by adding to the net premiums one-sixth of the Ordinary Life net premium and one-sixth of the net premium for the contract, the percentage of loading varies by plan and by age on all plans except the Ordinary Life.

Having derived these net Semi-Tropical and Tropical premiums, a simple way to ascertain the mortality which they will stand is to

show their relationship to net premiums based on multiples of the American Mortality Table with 3% interest. The multiples of the American Table selected for the comparison have been 1½ times the American q_x for the Semi-Tropical, and twice the same q_x for the Tropical. These are known as the Sesqui-American and the Double American respectively, and are used by us with 3% interest for the valuation of Semi-Tropical and Tropical policies in accordance with instructions received from the Superintendent of Insurance for New York State.

Comparison of Net Premiums on Tropical and Semi-Tropical Policies with the Net Premiums by Double American 3% and Sesqui American 3%, respectively.

Age at entry.	Net Tropical Premium.	Double American 3% Net Premium.	Net Semi-Tropical Premium.	Sesqui American 3% Net Premium.
Ordinary Life.				
25	$23.90	$24.87	$20.01	$20.63
35	30.11	31.69	25.60	26.60
45	40.83	44.26	35.25	37.29
55	60.68	69.34	53.11	57.93
20-Payment Life.				
25	35.08	34.65	30.03	30.12
35	41.16	40.23	35.51	35.38
45	50.74	50.20	44.05	44.12
55	66.94	71.71	58.80	61.46
20-Year Endowment.				
25	47.80	46.10	44.41	43.53
35	49.99	48.08	45.99	44.99
45	55.00	54.12	49.95	49.45
55	68.60	72.70	61.27	63.23

It cannot be determined from the foregoing whether the net Tropical premiums on the entire business of the Company are equal to or greater than the net Double American 3% premiums, nor whether the net Semi-Tropical premiums are equal to or greater than the net Sesqui-American 3% premiums, as the distribution of business both by age and by plan must first be known. Nearly 90% of all policies in force in Tropical and Semi-Tropical countries are on the three plans, Ordinary Life, 20 Payment Life and 20 Year Endowment, the percentages to the total in force being about 35%, 10% and 45%, respectively. The average age at

entry is 38 years. Making due provision for the distribution by age and by plan, the net premiums received in the Tropics and Semi-Tropics are, on the average, slightly larger than the net premiums on the valuation bases, Double American 3% and Sesqui American 3%. It may be said in general, therefore, that the premiums charged by the New York Life in the aggregate, cover about double the American q_x under Tropical policies, and 1½ times the same q_x under Semi-Tropical policies, a larger ratio of mortality being covered on the Endowment forms than on the Ordinary Life plan. These facts, while particularly applicable to the New York Life are of general importance as the Tropical and Semi-Tropical rates of American and Canadian Companies are practically the same as those of the Company with which I am connected. As the policies issued under these scales of premiums are almost exclusively participating, the surplus earnings from interest and loading are additional safeguards in case the company experiences a larger mortality than is provided for in the net premiums.

3. RELATIVE MORTALITY IN THE PRINCIPAL TROPICAL AND SEMI-TROPICAL COUNTRIES.

The relative mortality which the premium rates will bear having been stated, it will doubtless prove of interest to give a synopsis of the mortality actually experienced by the New York Life in the principal Tropical and Semi-Tropical countries. As a multiple of the American Table of Mortality is the basis of the valuation, and as the net Tropical and Semi-Tropical premiums have been compared with multiples of that table, we shall use the American as one of the standards in the following comparisons, which are based upon the mortality experience according to amounts insured. In order, also that the relationship of the mortality experience in Tropical countries to that in the United States and Canada may be seen, the ratio of actual to expected deaths on the Compound Progressive Table is given. That table represents the experience of the New York Life Insurance Company on standard lives resident in the temperate zone. In the last column of the comparison appears the scale of premiums, "T" standing for Tropical, and "S.T.," for Semi-Tropical. The experience has been given by individual countries or by homogeneous groups of countries.

	Approx. Ratio of Actual to Expected Deaths by American Table.	Ratio of Actual to Expected Deaths by Compound Progressive Table.	Scale of Premium.
North America.			
Mexico,	130%	170%	S. T. & T.
West India Islands.			
British West Indies	160	200	T.
Danish West Indies	170	225	T.
Hayti & Santo Domingo	160	215	T.
Cuba & Porto Rico	165	220	T.
Central America.			
All Countries	135	170	T.
South America.			
Colombia, Venezuela, British & Dutch Guiana, Ecuador, Peru, Bolivia	150	200	T.
Argentine, Uruguay & Chili	120	160	S. T.
Brazil	150	195	T. (principally)
Asia.			
India & East India Islands	150	195	T.
Philippines	155	205	T.
Japan & China	125	165	S. T. (principally)

The foregoing table proves that the premiums charged have been ample as the Semi-Tropical scale covers 150% of the American q_x and the Tropical scale 200% of the same table.

Nature and Extent of Data in the Present Investigation.

While the scales of premium in use throughout the world have been given, it should be borne in mind that the basis of the present investigation is the business issued at the Semi-Tropical and at the Tropical scales. Such limiting of the data excludes practically all of the Tropical part of Africa, where the Domestic scale, plus an annual extra premium of from $20. to $50. per thousand is charged. The amount of business thus excluded is small, and is entirely on the lives of white men who go to these countries for a brief period of years only.

The experience covers the years from 1873 to 1906 and has been divided into two sections:

(a) That applicable to countries in which the Tropical scale of premium is charged;

(b) That applicable to countries in which the Semi-Tropical scale of premium is charged.

It should be noted that the experience is not based on a geographical distribution into the Semi-Tropics and the Tropics, but on a distribution by premium scale according as the Semi-Tropical or the Tropical rate is charged. For example, the northern part of Australia is geographically in the Tropics, but the Domestic rate of premium is charged there; similarly, there are certain parts of the East Coast of Mexico which are north of the Tropic of Cancer, yet where the full Tropical rate is charged.

It was first deemed advisable to prepare the data on the basis of amounts insured, in the hope that a properly graduated table could be obtained therefrom, although, as will be shown later, the experience was also tabulated on the basis of lives.

4. MEASUREMENT OF EXPERIENCE BY THREE STANDARD MORTALITY TABLES.

In order to determine, in the first place, whether any existing table in use in this country, or a multiple of such table, is a proper measure of the Tropical and the Semi-Tropical death rates, the expected deaths on amounts insured were obtained by three standard tables:

(a) The Select Mortality Table of the New York Life, known as the Compound Progressive Table.
(b) Jones' Tropical Table.
(c) American Mortality Table.

With regard to (a), the Compound Progressive Table as already intimated represents the New York Life's experience on Deferred Dividend policies issued on standard lives in temperate climates, and is given in the *Transactions,* Volume IX, page 100.

With regard to (b), the Jones Tropical Table was prepared by Mr. C. N. Jones in 1894, and was based on the New York Life's experience within the American Tropics, the data on policies issued at the Semi-Tropical and Tropical rates being combined. It is an ultimate table, based upon amounts insured, and is given in the *Transactions,* Volume III, page 299.

Tropical Experience, 1873–1906.

AMOUNTS INSURED

Policy Years.	Actual Death Losses.	Expected Death Losses by Compound Progressive Table.	Ratio of Actual to Expected by Compound Progressive Table.	Expected Death Losses by Jones' Tropical Table.	Ratio of Actual to Expected by Jones' Tropical Table.	Expected Death Losses by American Table.	Ratio of Actual to Expected by American Table.
1	$ 1,729,932	$ 935,292	185%	$ 2,546,377	68%	$ 1,582,103	109%
2	1,687,349	812,123	208	2,082,467	81	1,285,417	131
3	1,796,808	795,850	226	1,928,446	93	1,183,599	152
4	1,528,282	691,236	221	1,601,887	95	978,232	156
5	1,121,673	632,080	177	1,402,696	80	852,872	132
2–5	6,134,112	2,931,289	209	7,015,496	87	4,300,120	143
3–5	4,446,763	2,119,166	210	4,933,029	90	3,014,703	148
6–20	6,900,921	4,100,035	168	7,956,970	87	4,766,715	145
1–20	14,764,965	7,966,616	185	17,518,843	84	10,648,938	139
			SEMI-TROPICAL EXPERIENCE.				
1	$ 424,314	$ 275,404	154%	$ 751,291	56%	$ 467,679	91%
2	420,838	229,763	183	590,634	71	365,444	115
3	402,463	225,721	178	548,612	73	340,254	118
4	253,637	180,166	141	419,655	60	257,847	98
5	222,416	150,970	147	337,409	66	203,011	110
2–5	1,299,354	786,620	165	1,896,310	69	1,166,556	111
3–5	878,516	556,857	158	1,305,676	67	801,112	110
6–20	1,308,904	1,033,199	127	2,005,802	65	1,205,385	109
1–20	3,032,572	2,095,223	145	4,653,403	65	2,839,620	107

The expected and actual deaths by these three tables are given in the following schedule:

An inspection of the foregoing table shows that the percentage of actual to expected death losses in the third, fourth and fifth policy years is generally as high as in the policy years after the fifth. In order to have a bird's eye view of the effect of medical selection, the percentages of actual to expected death losses are given for the first two policy years individually, for the third to the fifth policy years inclusive, and for the sixth to the twentieth years inclusive.

Tropical Experience.

AMOUNTS INSURED.

Ratio of Actual to Expected Death Losses.

Policy Years.	By Compound Progressive Table.	By Jones' Tropical Table.	By American Table.
1st	185%	68%	109%
2nd	208	81	131
3–5	210	90	148
6–20	168	87	145
	SEMI-TROPICAL EXPERIENCE.		
1st	154%	56%	91%
2nd	183	71	115
3–5	158	67	110
6–20	127	65	109

From the foregoing and from the preceding table the following deductions may be drawn:

(a) That a multiple of the Compound Progressive Table does not represent, with sufficient accuracy, the New York Life's mortality experience in the Semi-Tropics or the Tropics; and that the duration of the effect of selection is shorter than in the United States.

(b) That the ultimate rate of mortality is attained in the second year in the case of Semi-Tropical policies, and in the third year in the case of Tropical policies.

(c) That a multiple of either the American or the Jones' Tropical Table is apparently a proper representation of the ultimate rate of mortality under Semi-Tropical and Tropical conditions.

(d) That the present experience is more favorable than that prepared by Mr. Jones, as the ratio for the combined Tropical and Semi-Tropical data, exclusive of the first two years, is 83% for the former, and 100% for the latter, of Jones' graduated rates of mortality.

With regard to (a), it has been known to us for some time that the effect of medical selection did not last for so long a period in the Tropics as in the temperate climes, and that, accordingly, a multiple of the Compound Progressive, which gives effect to medical selection for 11 years, was not a satisfactory basis for constructing a Semi-Tropical or Tropical mortality table. Several writers have pointed out that the value of medical selection is not so great in the Tropics as in temperate climates.

With regard to (b), the first thought that comes to one's mind is that the effect of medical selection has passed off in two years, but while this may be so, the above tables, are not in themselves, sufficient evidence.

With regard to (c), a series of investigations was made to determine the relationship of the Semi-Tropical and the Tropical experiences to Jones' Tropical and to the American Table at the various attained ages for the 2nd to the 20th policy years, inclusive. It was evident from these calculations, which are given in the following table, that the mortality curve in the Tropics and Semi-Tropics, based on *amounts insured,* differed from that of Jones' or the American.

Tropical Experience.

AMOUNTS INSURED.

2nd to 20th Years Inclusive.

Attained Ages.	Exposed to risk.	Actual Death Losses.	Expected death losses by Jones' Tropical Table.	Ratio of Actual to Expected by Jones' Tropical Table.	Expected Death losses by American Table.	Ratio of Actual to Expected by American Table.
16–34	$217,816,000	$2,377,585	$2,856,061	83%	$1,845,456	129%
35–44	365,827,000	4,717,313	5,877,813	80	3,568,077	132
45–54	202,687,000	4,546,207	4,614,801	99	2,663,378	171
55 & older	40,838,000	1,393,928	1,623,791	86	989,924	141

SEMI-TROPICAL EXPERIENCE.

16–34	$ 59,531,000	$ 536,026	$ 783,583	68%	$ 505,747	106%
35–34	104,955,000	870,448	1,675,645	52	1,023,582	85
45–54	49,147,000	933,228	1,109,669	84	640,427	146
55 & older	8,592,000	268,556	333,215	80	202,185	133

Taking the American Table as representing the ultimate mortality in the United States and Canada, the foregoing indicates that the ultimate rate of mortality in the Tropics is an increasing percentage of the mortality experienced in this country up to about age 50. The same phenomenon appears in an investigation based on lives insured in the Tropics instead of on amounts insured, the corresponding ratios of the American Table for the four age groups being 124%, 146%, 175% and 156% respectively, and as will subsequently be seen, the graduated rates of mortality derived from that investigation exhibit a similar relationship to the American. The same condition was noted by Messrs. Hardy and Rothery (*J. I. A.* Vol. 27), who showed that the ratio of the rates of mortality in Finlaison's East India table, and in the Barbados Mutual Table to the H^M rate, increased to a maximum at about age 45, and then gradually declined toward the end of life. As the H^M curve is similar to the American, this confirms the New York Life's experience to the effect that the incidence of mortality in the Tropics is different from that in Temperate climates. It is probable that the mortality curve in certain Tropical countries is not on the lines indicated by the New York Life's experience. There is some doubt, for example, whether the rates of mortality in Finlaison's East India Table correctly represent the present mortality in British India, as the experience of the Standard Insurance Company, published in the *Transactions of the Fourth International Congress,* shows a decreasing ratio to the H^M to about age 40, and an increasing ratio thereafter. On the other hand, the more recently published experience of the British Empire (*Transactions of the Fifth International Congress*) shows an increasing ratio to the H^M until about age 65, the statistics after that age being too meager to be reliable.

The foregoing figures caused us to discontinue the work of trying to find a multiple of Jones' or the American Table, which would serve as a basis for the Semi-Tropical and the Tropical mortality, founded on the New York Life data. Many experiments were made to obtain a satisfactory graduation of our data,—1st by the crude summation methods; 2nd by the Woolhouse formula, and 3rd by the Makeham method—the data being divided into two sections, (a) first policy year; (b) all policy years, excluding the first. The Semi-Tropical and the Tropical experiences were treated

in the same way although in the latter the ultimate rate of mortality was not attained until the third year, the difference between the mortality in the second and subsequent years being too small to make it worth while to treat them differently.

No satisfactory graduated mortality table could be obtained by any of the foregoing processes. For example, the graduation by the Woolhouse formula of both the Semi-Tropical and the Tropical data gave an irregular curve; while the graduation of the Tropical experience by the Makeham method resulted in a higher mortality at the younger ages of entry for the first policy year than for the second to the twentieth policy years inclusive. Furthermore, the graduated rates of mortality by that method for the second to the twentieth policy years were practically constant from ages 15 to 35, while at ages above 55 it increased exceedingly rapidly, the rate of mortality doubling between ages 54 and 56 and practically doubling for every advance of two ages thereafter, so that at attained age 64 the mortality was 886 per 1,000, the corresponding mortality for the first policy year being only 134 per 1,000. As these irregularities, and the short period of medical selection in the Semi-Tropical experience might have been largely due to the variation in the amounts of the death losses, it was deemed advisable to prepare the data on lives.

Mortality Investigation According to Lives.

The data by lives do not exactly correspond to those by amounts as the former include all policies issued at the Tropical and Semi-Tropical rates of premium, irrespective of whether they were deferred dividend or annual dividend policies; while the latter include only deferred dividend policies during the deferred dividend period. The data on lives, are, therefore, more extensive than on amounts insured, and cover a longer period of time as they include annual dividend policies and the experience of the Company after the termination of the deferred dividend period.

The same procedure was followed in dealing with the data for lives as for amounts. Duplicate policies were eliminated in the usual manner in dealing with the former. Under-average cases, whether issued with an extra premium or put in special dividend classes were excluded. The statistics on women were, however, included under amounts insured, but were excluded under lives

as their numbers were small and as we desired to get homogeneous data, i. e., data based on men who were accepted as first-class or standard risks.

In the investigation by lives, the actual deaths were compared with the expected deaths, calculated by the three following tables:

(a) The Select Mortality Table of the New York Life, known as the Compound Progressive Table.
(b) Jones' Tropical Table.
(c) American Mortality Table.

The actual and expected deaths by these three tables are now given:

Tropical Experience,

1873–1906.

LIVES.

Policy Years.	Actual Deaths.	Expected Deaths by Compound Progressive Table.	Ratio of Actual to Expected by Compound Progressive Table.	Expected Deaths by Jones' Tropical Table.	Ratio of Actual to Expected by Jones' Tropical Table.	Expected Deaths by American Table.	Ratio of Actual to Expected by American Table.
1	484	253	191%	690	70%	430	113%
2	435	203	214	522	83	323	135
3	412	186	222	452	91	279	148
4	343	162	212	376	91	231	149
5	290	146	199	325	89	198	146
2–5	1480	697	212	1675	88	1031	144
3–5	1045	494	211	1153	91	707	148
6–33	1871	1091	172	1996	94	1201	156
1–33	3835	2041	188	4361	88	2662	144

SEMI-TROPICAL EXPERIENCE.

1	157	120	131%	324	48%	201	78%
2	128	86	149	220	58	135	95
3	111	72	154	174	64	107	104
4	73	45	162	103	71	63	116
5	42	32	131	71	59	43	98
2–5	354	235	151	568	62	348	102
3–5	226	149	151	348	65	213	106
6–26	361	257	140	464	78	279	129
1–26	872	612	142	1356	64	828	105

In order to obtain a bird's eye view of the relationship of the mortality under the Tropical and Semi-Tropical experience to the three standards selected, a synopsis is supplied. In this synopsis there also appears the ratio of actual to expected deaths by *amounts insured,* in order to show wherein the difference lies. As the distribution of business at the various policy years is of value in determining the reliance to be placed on the various percentages, the exposed to risk has been furnished both by amounts insured and by lives. It will be noted that the Tropical experience is about 3½ times as large as the Semi-Tropical experience, and that the combined data are large, the exposed to risk being over $1,265,000,000.

Comparison of Tropical and Semi-Tropical Mortality with Three Standard Mortality Tables.

TROPICAL EXPERIENCE.

Exposed to Risk.			Ratio of Actual to Expected Deaths.					
			By Compound Progressive Table.		By Jones' Tropical Table.		By American Table.	
Policy Years.	Amounts Insured.	Lives.	Amounts Insured.	Lives.	Amounts Insured.	Lives.	Amounts Insured.	Lives.
1	$166.571,000	45,903	185%	191%	68%	70%	109%	113%
2	131,901,000	33,687	208	214	81	83	131	135
3–5	295,326,000	70,610	210	211	90	91	148	148
6 to End	399,941,000	98,079	168	172	87	94	145	156
SEMI-TROPICAL EXPERIENCE.								
1	$ 49,623,000	20,973	154%	131%	56%	48%	91%	78%
2	37,815,000	13,719	183	149	71	58	115	95
3–5	79,832,000	20,682	158	151	67	65	110	106
6 to End	$104,578,000	21,494	127	140	65	78	109	129

There is a reasonably close agreement between the mortality by amounts insured and by lives, which gives a feeling of confidence in the accuracy of the work, the data for the two investigations having been obtained from entirely different sources by different bodies of workers. It should be borne in mind that the data upon which the experience was based were not identical for lives and for amounts, and that, accordingly, the average amount of each

policy cannot be obtained by dividing the amount of insurance exposed to risk by the number of lives exposed to risk. Taking the data on lives to correspond exactly to those by amounts insured, we find that the average policy increases with the duration which is probably due to the greater persistency of the larger policies over those for smaller amounts, and to the comparatively large number of moderate-sized policies issued in recent years on the lives of Japanese and East Indians.

It is evident from the foregoing tables that the deductions which were made from a synopsis of the experience on amounts insured must be modified before becoming applicable to the experience on lives. In the Tropical experience the ratio of actual to expected *death losses* by the Jones' Table was greater for the third to the fifth year than for the sixth and succeeding policy years; and in the Semi-Tropical experience the mortality in the second to the fifth policy years inclusive was greater than for the sixth and succeeding policy years. On the other hand, the ratio of actual to expected deaths by the Jones' Table, in the Tropical experience *by lives* was an increasing one, being 70% for the 1st year, 83% for the 2nd, 91% for the 3rd to the 5th, and 94% for the 6th and succeeding policy years, while the corresponding ratios by the American Table were 113%, 135%, 148% and 156% the effect of selection being more clearly seen by lives than by amounts. The ultimate rate of mortality in the Tropics, based on the experience by lives, is practically attained by the third year, and in the graduation of the experience we have, accordingly, excluded the first two policy years. So far as the Semi-Tropical experience is concerned, the effect of medical selection as indicated by the experience *on lives* lasts longer than under the Tropical experience, and that is what might reasonably be expected. Taking the Jones' Table as a standard, the ratio of actual to expected deaths for the first year was 48%, for the second year 58%, for the third year to the fifth inclusive, 65% and for the sixth and succeeding policy years, 78%. The corresponding ratios by the American Table were 78%, 95%, 106% and 129%. The ultimate mortality rate in the Semi-Tropics, therefore, is probably not attained until at least the sixth policy year. As the ultimate rate is not attained on the average in the United States until about the tenth year, it is reasonable to expect that the ultimate rate in

the Semi-Tropical experience should be attained in the neighborhood of the fifth or sixth policy year.

Attention is drawn to the fact that the ratio of actual to expected deaths for the sixth and succeeding policy years is 156% of the American under the Tropical experience, and 129% of the American under the Semi-Tropical experience. That fact, and also the fact already mentioned that the period of medical selection practically is a mean between the periods for the Tropics and temperate climates, decided us not to attempt the actual graduation of the Semi-Tropical data, but to base our ultimate mortality table upon a mean between the Tropical and the American q_x the resulting q_x being graduated by the Makeham formula.

5. DETAILS OF GRADUATION OF TROPICAL EXPERIENCE, BASED ON LIVES.

It might be thought to be a waste of labor to graduate the Tropical experience because a percentage of the Jones' Tropical Table is fairly representative of the present New York Life Tropical experience. As, however, the Jones' Table includes both Semi-Tropical and Tropical business, and as the data in Mr. Jones' investigation were one-seventh of those in the present experience, it appeared to be worth the labor to make a graduation of our present data for lives. In the Tropical experience the mortality for the third, to the fifth years is so close to that for the sixth and succeeding policy years, that the data have been combined for all policy years after the second. So far as the first and second policy years are concerned, a percentage of the ultimate rate will be taken because no satisfactory graduation can be made which will correspond by ages to the mortality for the third and succeeding years. In any event, the mortality for the first two policy years is only of value in determining the expected mortality in other experiences, while the ultimate rate will probably be used as a basis for calculating premiums and reserves.

As the graduated table would doubtless be employed in the calculation of Joint Life premiums and reserves, a graduation by the Makeham formula is desirable, although graduations by the Woolhouse and by the Graphic methods were made in order to determine in a general way, the mortality curve. The usual method

of making four groups of ages from which to obtain the constants was followed in the Makeham graduation, but a satisfactory q_x could not be obtained, notwithstanding that various groupings were tried, ranging from 8 to 13 ages in each group. The rate of mortality thus obtained was very low at the young ages, and too high at the older ages. The difficulty in obtaining a satisfactory graduation by the usual process is due to lack of data below age 25 and above age 70, the dearth of statistics at the older ages resulting from the termination of the Endowment policies, which constitute the majority of those included in the investigation, and from the Life and Limited Payment policies being largely on the deferred dividend plans, under which a comparatively small percentage is continued beyond the deferred dividend period.

As a matter of scientific interest it may be mentioned that the grouping by ages resulted in a low logarithm of c, the constant being as low as .0215, whereas under the Makehamized American it is .04579609. Experiments were made with the same logarithm of c as in the Makehamized American, but a table could not be obtained which gave rates of mortality that would be satisfactory for practical work at the young and the old ages. It was necessary, therefore, to obtain the constants by a tentative process, which called for a large amount of work by Mr. Frank B. Lucas, head of the Mathematical Division in the New York Life, whose skill and patience finally produced a graduation which was satisfactory at all ages. In obtaining this graduation, which is based on lives, we kept in mind the experience on amounts insured, as well as the necessity of having a practicable table, so that the crude rates of mortality were not slavishly followed. A table is now given of the graduated rates of mortality, and the usual error and accumulated error column:

Tropical Experience.
All Policy Years Except 1st *and* 2nd.

Log. c = 04 Log. g = — .000635 Log. s = — .0039630 Log. k = 5.0619730

Attained Age.	Exposed to Risk.	q_x	Actual Deaths.	Expected Deaths.	Error.	Accumulated Error.
17	8	.00975	1	0	— 1	— 1
18	30	.00982	0	0	0	— 1
19	50	.00989	0	0	0	— 1
20	112	.00997	1	1	0	— 1
21	181	.01005	1	2	+ 1	0
22	309	.01015	3	3	0	0

Tropical Experience.—Continued.

Log. c = 04 Log. g = —.000635 Log. s = —.0039630 Log. k = 5.0619730

Attained Age.	Exposed to Risk.	q_x	Actual Deaths.	Expected Deaths.	Error.	Accumulated Error.
23	510	.01024	1	5	+ 4	+ 4
24	815	.01036	8	8	0	+ 4
25	1202	.01048	15	13	— 2	+ 2
26	1712	.01062	20	18	— 2	0
27	2648	.01076	27	28	+ 1	+ 1
28	3279	.01092	36	36	0	+ 1
29	3921	.01110	49	44	— 5	— 4
30	4550	.01131	54	51	— 3	— 7
31	5165	.01151	43	59	+16	+ 9
32	5915	.01175	61	70	+ 9	+18
33	6364	.01200	75	76	+ 1	+19
34	6761	.01228	75	83	+ 8	+27
35	7133	.01259	89	90	+ 1	+28
36	7427	.01293	111	96	—15	+13
37	7698	.01330	95	102	+ 7	+20
38	7648	.01371	101	105	+ 4	+24
39	7554	.01413	93	107	+14	+38
40	7548	.01464	122	111	—11	+27
41	7311	.01517	122	111	—11	+16
42	7221	.01574	99	114	+15	+31
43	6855	.01640	113	112	— 1	+30
44	6488	.01709	107	111	+ 4	+34
45	6056	.01786	91	108	+17	+51
46	5580	.01871	136	104	—32	+19
47	5164	.01963	124	101	—23	— 4
48	4725	.02064	105	98	— 7	—11
49	4309	.02175	76	94	+18	+ 7
50	3909	.02296	100	90	—10	— 3
51	3500	.02430	86	85	— 1	— 4
52	3157	.02575	84	81	— 3	— 7
53	2737	.02733	90	75	—15	—22
54	2322	.02908	61	68	+ 7	—15
55	2002	.03098	67	62	— 5	—20
56	1695	.03309	56	56	0	—20
57	1433	.03536	49	51	+ 2	—18
58	1184	.03786	37	45	+ 8	—10
59	968	.04059	40	39	— 1	—11
60	787	.04357	31	34	+ 3	— 8
61	624	.04686	31	29	— 2	—10
62	517	.05040	32	26	— 6	—16
63	380	.05430	15	21	+ 6	—10
64	301	.05856	16	18	+ 2	— 8
65	229	.06321	11	14	+ 3	— 5
66	175	.06826	16	12	— 4	— 9
67	136	.07377	10	10	0	— 9
68	106	.07978	11	8	— 3	—12
69	70	.08636	4	6	+ 2	—10
70	45	.09345	2	4	+ 2	— 8
71	38	.10122	1	4	+ 3	— 5
72	32	.10967	2	4	+ 2	— 3
73	25	.11880	1	3	+ 2	— 1
74	22	.12868	2	3	+ 1	0
75	15	.13945	3	2	— 1	— 1
76	12	.15109	1	2	+ 1	0
77	6	.16367	1	1	0	0
78	4	.17725	1	1	0	0
79	2	.19183	0	0	0	0
80	2	.20760	0	0	0	0
81	2	.22465	1	1	0	0
82	1	.24250	0	0	0	0
83	1	.26206	0	0	0	0
84	1	.28283	0	0	0	0

It will be noted from the above table that the logarithm of c is .04, which is found to be approximately the value of that constant in so many tables that it has almost come to be regarded by some actuaries as a reasonable and proper standard for all graduations.

The commutation columns are given at the end of this paper, together with the single and annual premiums for whole life insurance, also the single life annuity values. The question of the corresponding reserves will be treated in a later part of the paper.

Graduation of Semi-Tropical Experience.

With regard to the Semi-Tropical experience, it did not seem advisable to make a graduation direct from the data for reasons which have already been mentioned. The mean has been taken between the graduated Tropical rates of mortality and the American rates of mortality, the resulting table having been re-graduated by the Makeham formula. The satisfactory effect of the application of this graduated ultimate rate of mortality to the Semi-Tropical data for the sixth and succeeding years may be seen in the following table. As already intimated, the effect of selection seemed to continue during the first five policy years, and, accordingly, these policy years should be excluded in making up an ultimate table corresponding to the American, or to the new Tropical table, as in both of these cases the lower mortality due to medical selection is eliminated.

Semi-Tropical Experience—Excluding First Five Years.

Log. c =.042778165		Log. g =—.0003091362		Log. s =—.003680459		Log. k =5.05681604
Attained Age.	Exposed to Risk.	q_x	Actual Deaths.	Expected Deaths.	Error	Accumulated Error.
20	2	.00896	0	0	0	0
21	5	.00901	0	0	0	0
22	6	.00907	0	0	0	0
23	13	.00915	0	0	0	0
24	25	.00921	0	0	0	0
25	34	.00929	0	0	0	0
26	58	.00939	0	1	+1	+ 1
27	81	.00948	0	1	+1	+ 2
28	118	.00958	0	1	+1	+ 3

Semi-Tropical Experience—Continued.

Log. c = .042778165		Log. g = —.0003091362		Log. s = —.003680459		Log. k = 5.05681604	
Attained Age.	Exposed to Risk.	q_x	Actual Deaths.	Expected Deaths.	Error.	Accumulated Error.	
29	154	.00971	2	1	—1	+ 2	
30	252	.00984	2	2	0	+ 2	
31	322	.00998	4	3	—1	+ 1	
32	394	.01014	5	4	—1	0	
33	465	.01032	2	5	+3	+ 3	
34	553	.01052	3	6	+3	+ 6	
35	673	.01073	4	7	+3	+ 9	
36	774	.01097	2	8	+6	+15	
37	861	.01124	11	10	—1	+14	
38	930	.01152	11	11	0	+14	
39	964	.01184	11	11	0	+14	
40	1009	.01219	5	12	+7	+21	
41	1029	.01257	6	13	+7	+28	
42	1044	.01300	19	14	—5	+23	
43	1033	.01348	9	14	+5	+28	
44	999	.01399	11	14	+3	+31	
45	982	.01457	15	14	—1	+30	
46	937	.01520	14	14	0	+30	
47	865	.01590	22	14	—8	+22	
48	795	.01667	19	13	—6	+16	
49	726	.01752	11	13	+2	+18	
50	658	.01845	15	12	—3	+15	
51	607	.01947	17	12	—5	+10	
52	534	.02061	11	11	0	+10	
53	488	.02186	5	11	+6	+16	
54	471	.02325	11	11	0	+16	
55	424	.02477	10	11	+1	+17	
56	362	.02643	18	10	—8	+ 9	
57	322	.02829	11	9	—2	+ 7	
58	272	.03032	11	8	—3	+ 4	
59	226	.03255	11	7	—4	0	
60	199	.03501	8	7	—1	— 1	
61	163	.03772	5	6	+1	0	
62	137	.04071	7	6	—1	— 1	
63	121	.04399	4	5	+1	0	
64	98	.04759	2	5	+3	+ 3	
65	78	.05156	6	4	—2	+ 1	
66	61	.05590	4	3	—1	0	
67	42	.06069	3	3	0	0	
68	32	.06595	3	2	—1	— 1	
69	23	.07171	3	2	—1	— 2	
70	18	.07802	1	1	0	— 2	
71	15	.08493	2	1	—1	— 3	
72	11	.09251	2	1	—1	— 4	
73	8	.10077	0	1	+1	— 3	
74	7	.10987	1	1	0	— 3	
75	4	.11972	0	0	0	— 3	
76	3	.13053	1	0	—1	— 4	
77	2	.14226	0	0	0	— 4	
78	1	.15502	0	0	0	— 4	
79	1	.16893	0	0	0	— 4	
80	1	.18397	0	0	0	— 4	
81	1	.20014	0	0	0	— 4	
82	1	.21788	1	0	—1	— 5	

Rates of Mortality During Period of Medical Selection.

A table is now given showing the rates of mortality during the period in which medical selection is effective. As already explained, the approximate values of $q_{[x]+n}$ have been obtained by taking uniform percentages of the ultimate table. In the case of the Tropical, the percentages taken were 78% for the first year, and 93% for the second year, these percentages of the ultimate giving expected death losses which agreed closely with the actual. In the case of the Semi-Tropical, the percentages of the ultimate used were 64% for the first year, 75% for the second year, and 86% for the third, fourth and fifth policy years combined. These percentages of the ultimate q_x applied to the exposed to risk gave expected deaths which were equal to the actual deaths. It is fully recognized that a fixed percentage for all ages may not be strictly accurate, but the data are not large enough to enable us to determine the percentage by ages or by groups of ages. In making allowance for the effects of medical selection in the United States a percentage of the American is taken, increasing with advancing age, and the same may be proper in Tropical countries although the evidence is not sufficiently strong to enable us to come to any satisfactory conclusion.

Rates of Mortality in Early Policy Years.

	Tropical Policy Years.		Semi-Tropical Policy Years.		
Attained Age.	1st 78% of Ultimate Rate.	2nd 93% of Ultimate Rate.	1st 64% of Ultimate Rate.	2nd 75% of Ultimate Rate.	3rd–5th 86% of Ultimate Rate.
15	.00752	——	.00561	——	——
16	.00756	.00901	.00563	.00659	——
17	.00761	.00907	.00565	.00662	.00759
18	.00766	.00913	.00568	.00665	.00763
19	.00771	.00920	.00571	.00669	.00767
20	.00778	.00927	.00573	.00672	.00771
21	.00784	.00935	.00577	.00676	.00775
22	.00792	.00944	.00580	.00680	.00780
23	.00799	.00952	.00586	.00686	.00787
24	.00808	.00963	.00589	.00691	.00792
25	.00817	.00975	.00595	.00697	.00799
26	.00828	.00988	.00601	.00704	.00808
27	.00839	.01001	.00607	.00711	.00815
28	.00852	.01016	.00613	.00719	.00824
29	.00866	.01032	.00621	.00728	.00835

Rates of Mortality in Early Policy Years—Continued.

Attained Age.	Tropical Policy Years. 1st 78% of Ultimate Rate.	Tropical Policy Years. 2nd 93% of Ultimate Rate.	Semi-Tropical Policy Years. 1st 64% of Ultimate Rate.	Semi-Tropical Policy Years. 2nd 75% of Ultimate Rate.	Semi-Tropical Policy Years. 3rd-5th 86% of Ultimate Rate.
30	.00882	.01052	.00630	.00738	.00846
31	.00898	.01070	.00639	.00749	.00858
32	.00917	.01093	.00649	.00761	.00872
33	.00936	.01116	.00660	.00774	.00888
34	.00958	.01142	.00673	.00789	.00905
35	.00982	.01171	.00687	.00805	.00923
36	.01009	.01202	.00702	.00823	.00943
37	.01037	.01237	.00719	.00843	.00967
38	.01069	.01275	.00737	.00864	.00991
39	.01102	.01314	.00758	.00888	.01018
40	.01142	.01362	.00780	.00914	.01048
41	.01183	.01411	.00804	.00943	.01081
42	.01228	.01464	.00832	.00975	.01118
43	.01279	.01525	.00863	.01011	.01159
44	.01333	.01589	.00895	.01049	.01203
45	.01393	.01661	.00932	.01093	.01253
46	.01459	.01740	.00973	.01140	.01307
47	.01531	.01826	.01018	.01193	.01367
48	.01610	.01920	.01067	.01250	.01434
49	.01697	.02023	.01121	.01314	.01507
50	.01791	.02135	.01181	.01384	.01587
51	.01895	.02260	.01246	.01460	.01674
52	.02009	.02395	.01319	.01546	.01772
53	.02132	.02542	.01399	.01640	.01880
54	.02268	.02704	.01488	.01744	.02000
55	.02416	.02881	.01585	.01858	.02130
56	.02581	.03077	.01692	.01982	.02273
57	.02758	.03288	.01811	.02122	.02433
58	.02953	.03521	.01940	.02274	.02608
59	.03166	.03775	.02083	.02441	.02799
60	.03398	.04052	.02241	.02626	.03011
61	——	.04358	——	.02829	.03244
62	——	——	——	——	.03501
63	——	——	——	——	.03783
64	——	——	——	——	.04093

Comparison With Jones' Tropical Table.

A comparison between the Jones' Tropical tables and the present Tropical data would be of interest, as they are both based upon the experience of the New York Life Insurance Company. The former was based upon the entire experience of the Company under Deferred Dividend policies within the American Tropics, and did not take account of dissimilar climatic conditions or differences in general sanitary surroundings, while the latter comprised the data on all policies issued at the Semi-Tropical and Tropical rates throughout the world, and differentiated between

the business issued under unlike conditions so far as represented by the two scales of premium. For the purpose desired by Mr. Jones it was not necessary to make the subdivision of the data on the lines of the present investigation. The materials were handled by him in a very scholarly fashion, the graduation being ably done by Mr. Hugh W. Robertson. While the present investigation is not confined to the American Tropics, the bulk of the business was obtained there. The mortality in the Asiatic Tropics is not greatly different from that in the American Tropics,—for example, the mortality in British India was comparatively close to the total experience in countries in Central and South America, in which the Tropical scale of premium is charged.

Mr. Jones' ultimate rate of mortality was based on the combined exposures of *amounts insured* in the third and succeeding policy years. He states, "An examination showed no marked difference as between the mortality ratios for policy years after the second, while those for the first and second were strikingly below the general average." Mr. Jones does not, however, mention the percentage which the mortality for the first two years by amounts insured bore to his ultimate table, although he does give such information by lives. He applied his graduated ultimate rate of mortality, based on amounts insured after the second policy year, to the total experience on lives in the American Tropics. It will be interesting to make a comparison between the Jones' experience on lives and the present experience combining the Tropical and Semi-Tropical data, and also with the present experience for the Tropical data alone.

Comparison of Jones' Tropical with Present Tropical Experience.

By Lives.

Measured by the Jones' ultimate rate based on *Amounts Insured.*

Percentage of Actual to Expected Deaths.

Policy Year.	Jones' Tropical Experience by Lives.	Present Tropical and Semi-Tropical Experience by Lives.	Present Tropical Experience by Lives.
1	65%	63%	70%
2		76	83
3–5	88	85	91
6–End	100	91	94
1–End	85	82	88

The foregoing table indicates that the present experience by lives is more favorable in point of mortality than the Jones' experience. So far as amounts insured are concerned, the ratio of actual to expected deaths in the latter experience, after the *second* policy year, was 100% of his graduated rates, while it is 83% of the same mortality rates under the present combined experience.

Barbados Mutual Mortality Table.

There is a Tropical Mortality Table in existence with which it would be of interest to compare the Tropical experience of the New York Life, namely the Barbados Mortality Table. This table was based upon the experience of the Barbados Mutual Insurance Company in the Island of Barbados, and is published in the *Institute of Actuaries' Journal,* Volume 27. It excludes the first two years of insurance.

Tropical Experience of the New York Life.

Measured by the Barbados Mortality Table.

Lives.

Policy Year.	Actual Deaths.	Expected Deaths.	Ratio of Actual to Expected.
1	484	721	67%
2– 5	1480	1769	84
6–33	1871	2085	90
1–33	3835	4575	84

The experience of the New York Life in countries in which it charged its Tropical Scale has been distinctly more favorable than that of the Barbados Mutual Company in the West Indies. It is, however, only proper to mention that the New York Life experience is more modern (1873-1906) than the Barbados Mutual (1846-1882), and, accordingly, the more favorable experience of the former may be partly due to the improved sanitary conditions. The authors of the Barbados Mutual Table (Messrs. G. F. Hardy and Rothery) considered that the experience gave "an indication of the rate of mortality in a Tropical climate under the most favorable circumstances . . . . The figures being of the character of a standard of minimum Tropical mortality."

6. TROPICAL EXPERIENCE OF OTHER COMPANIES.

It will also be of interest to compare the mortality of the New York Life with that of other Companies on their Tropical business. Unfortunately, there do not appear to have been any mortality tables published which are applicable to many of the principal Tropical and Semi-Tropical countries, such as Argentine, Mexico, Brazil and Chili. The principal Tropical experiences published are those applicable to British India and the West Indies, the last mentioned relating almost exclusively to the islands belonging to Britain, such as Jamaica, Barbados, and Trinidad. The three companies whose experiences have been published are the Barbados Mutual (1846-1882), the Scottish Amicable (1846-1891) and the Standard Company of Edinburgh (1846-1900). While these experiences are given in the aggregate it is recognized that these Islands vary considerably from one another in point of mortality.

Mortality Experience in West Indies.

Measured by the H^M Mortality Table.

	Actual Deaths.	Expected Deaths.	Ratio of Actual to Expected.
Barbados Mutual (in Barbados)	385	262	147%
Barbados Mutual (other Islands)	168	100 Approx.	168
Scottish Amicable	240	165	145
Standard Company	993	603 Approx.	165

In order to obtain a comparison of the New York Life's experience with the foregoing companies, the expected death losses have been prepared on the basis of the H^M Table for the British West Indies, excluding the Bahama and Bermuda Islands. As our actuaries are not so familar with the H^M Table as the American, the expected deaths have also been calculated on the latter basis.

British West Indies—New York Life Experience, 1876-1906.

Measured by H^M Mortality Table.

	Actual Deaths.	Expected Deaths.	Ratio of Actual to Expected.
By Lives	110	96	115%
By Amounts	$337,900	$211,724	160%
Measured by American Table.			
By Lives	110	92	120%
By Amounts	$337,900	$201,285	168%

There is a marked difference between the ratio of actual to expected deaths by lives and by amounts insured, which is probably due to the comparatively small amount of data. The expected deaths are somewhat less by the American Table than by the H^M, which should be kept in mind in making any comparison on these bases in connection with other tables in this paper.

The experience of the New York Life in the British West Indies is more favorable *by lives* than that of the companies mentioned, while the ratio *by amounts* is higher than the ratio *by lives* in the Barbadoes Mutual (in Barbados) and the Scottish Amicable. These experiences indicate that there is a mortality in the British West Indies of approximately 150% of the American Table, which is nearly double the mortality experienced by good companies on first-class lives in temperate climates.

Several mortality experiences relative to British India have been published as follows:

Oriental Insurance Company, 1874 to 1891 (*Transactions of Third International Congress,* page 343).

Standard Insurance Company, 1846 to 1900 (*Transactions of the Fourth International Congress,* page 111).

British Empire Insurance Company, 1872 to 1902 (*Transactions of the Fifth International Congress,* page 61).

Indian Government Postal Insurance Scheme, 1884 to 1904 (*Transactions of the Fifth International Congress,* page 71).

A synopsis of these experiences, together with that of the New York Life, is now given:

Mortality Experience in British India.

Measured by the H^M Mortality Table.

	Actual Deaths.	Expected Deaths.	Ratio of Actual to Expected.
Oriental Company.....	779	642	121%
Standard Insurance Co..	1,432	978 (Approx.)	146
British Empire Insurance Co...	675	480	141
Indian Government Postal Insurance Scheme.....	615	400 (Approx.)	154
New York Life Insurance Co...	727	529	137

The low mortality of the Oriental was partly due to most of the lives having recently passed a medical examination, while the relatively high mortality in the Indian Government Postal Insurance Scheme was probably due to the risks being almost exclusively native lives. The mortality in India seems to depend partly upon whether the lives are Europeans, Eurasians, or Asiatics.

Another comparison we may make is that of the Standard Insurance Company with the New York Life in China, the former covering the years 1846 to 1900, and the latter the years 1886 to 1906.

Mortality Experience in China.

Measured by the H^M Mortality Table.

	Actual Deaths.	Expected Deaths.	Ratio of Actual to Expected.
Standard Insurance Company	96	64 (Approx.)	150%
New York Life.....	89	74	120

The ratio of actual to expected deaths in China would partly depend on the section of the country in which the business is issued, the Northern section being more healthful than the Southern part of China.

7. IMPROVEMENT IN MORTALITY.

An interesting question is whether or not the mortality is improving in the Tropical countries. The most conclusive figures on this point heretofore furnished have been by Mr. Spencer C. Thompson, in a paper before the Fourth International Congress. He showed that there had been a marked improvement in the experience of his Company (Standard Insurance Company), in their mortality in British India, in the West Indies and in China.

So far as the New York Life experience is concerned, there is a distinct improvement in the mortality in Tropical countries, as indicated by the comparisons with Jones' Tropical Table, but the statistics to prove that point for individual countries are not given because the conditions have changed in many of these countries and a detailed explanation would have to be given in each case to avoid misunderstanding, and this would unduly extend the paper. For example, there has been a marked improvement in the mortality in Mexico, due not solely to improved sanitary conditions, but to the increasing knowledge of the means of avoiding an undue proportion of fradulent claims, to the better education of medical men in Mexico and to the larger proportion of Americans who are now insured in the healthier sections of Mexico. To take another example, that of India, the mortality in the last decade could not be properly compared with that of previous years because the proportion of Europeans, Eurasians and Natives would not be the same, and because of the difficulty of allowing for the relative prevalence of bubonic plague. At the present time, about 95% of the applicants are Natives and Eurasians, which is a much larger proportion than in the previous decade. That proportion is a matter of moment, because the published experiences, with one exception, indicate that the Eurasian is a better risk than the Native, while in at least one experience he is a better risk than the European. Statistics which differentiate between Eurasian and Native should be accepted with caution. Dr. Olin M. Eakins, who has had charge of medical affairs in India for the New York Life Insurance Company for several years, informs me that there are as many degrees of Eurasians in India as of Mulattoes in the United States, and that they usually form communities by themselves, intermarrying with one another, so that there are several generations of Eurasians born in India. Among the Eurasians are some who are nearly Europeans, and others who are practically Asiatics, many of the former being probably included in the statistics of life insurance companies with the Europeans, as they are apt to call themselves by that name. The principal data upon the mortality of natives of India have been prepared by those who have not had an extended experience in India, and therefore there would be considerable difficulty in determining from the application papers which of the insured were really Eurasians. If the statistics showing a better mortality on Eurasians than on Asiatics are to be relied

upon, Dr. Eakins' opinion is that it is not due to any improvement in the Asiatic stock through the mixture of European blood, but probably to the Eurasians living in more sanitary surroundings and in better circumstances than the Asiatics, thereby making them less liable to the attacks of the various epidemics which cause a large number of the deaths among the Asiatics. It is the popular belief that the half-breed inherits the weaknesses of both races without the strength of either, and the statistics so far published do not prove that India is an exception.

One of the greatest factors in determining the mortality in the Tropics is the method of doing business and the question of moral hazard. Special care and skill must be exercised in the selection of medical examiners and agents who are trustworthy, and there must be careful supervision of the selection of risks. There might be great improvements in the sanitary conditions of a country, but a marked improvement might not occur in the mortality if the business methods had not also improved. Two countries, for example, which were similar in climate and sanitary conditions, might show dissimilar rates of mortality, the higher mortality being due to the untrustworthiness of medical examiners' reports, although the Home Office officials had endeavored to do business in both countries under similar strict conditions.

Looking at the problem from the standpoint of the improved conditions, we would normally expect an improvement in mortality during the last decade. The advance in sanitary measures, the better knowledge of how to live in the Tropics, a more moderate use of stimulants on the part of foreigners resident in the Tropics, the knoweldge of the part which insects and vermin (especially rats) play in carrying infection, should result in improving the mortality. Sanitary systems have been put into effect in large cities in the Tropics, where, until recently there was no effective drainage. These must assist in lowering the mortality so far as the population is concerned, but might not have the same effect upon insured lives, as the improved sanitary conditions might be offset by incompetent or dishonest medical examiners. However good the sanitary conditions may be, however honest the doctors, applicants and agents, a higher mortality must be expected in the Tropics and in the Semi-Tropics than in temperate climates. The white race has not become thoroughly acclimated, in my judgment, to the Tropical zone, and the inhabitants of the Tropics do not have

the vitality of the white man in the temperate zone. But the question may be asked,—Why should not people live as long in the mountainous parts of Tropical countries, such as Mexico, where there are plateaus six thousand feet above sea level, and where the climate is delightful, as they do in the temperate zone? The answer seems to be that man has not evolved physically so as to be fully adapted to an altitude of six thousand feet or more above sea level.

If one were to believe the opinions of many men from Tropical countries, there are scores of places in the Tropics which are "the healthiest in the world." The resident of the Tropics, in most instances, has no means of determining how healthful his country or city is compared with others, save from misleading population statistics, and he cannot conceive why a warm equable climate is not much more conducive to longevity than the changeable climate of such places as New York. Even the General Manager of the Barbados Mutual Insurance Company, after the reading of the report on the mortality of his Company before the Institute of Actuaries, in which it was shown that the mortality was nearly twice as high, relatively, as in England, expressed the opinion "that Barbados was the most healthful place in the world."

8. VALUATION OF POLICIES ISSUED AT SEMI-TROPICAL AND TROPICAL RATES.

Owing to the lack of published valuation tables based upon policies issued at the Semi-Tropical and at the Tropical scales of premium, the Superintendent of Insurance for New York State adopted the Sesqui-American Table for the former and the Double American for the latter. The Sesqui-American was prepared by increasing the American q_x by 50% at all ages, and the Double-American by increasing the American q_x by 100% at all ages, with some necessary adjustments towards the end of the tables. This method of increasing the q_x by a fixed percentage for all ages cannot be defended on a purely scientific basis, but the resulting valuation tables are fairly satisfactory from a practical standpoint.

Comparisons are now submitted at decennial ages of the reserves on the graduated Tropical rates of mortality and 3% interest, the Double American 3%, the Jones' Tropical 3%, and the American 3%. The three principal plans only are given—Ordinary Life, 20 Payment Life and 20 Year Endowment—because they represent nearly 90% of the business in the Tropics.

Comparison of Terminal Reserves, on Tropical 3%, Double American 3%, Jones' Tropical 3% and American 3%.

Ordinary Life.

End of Policy Year.	Tropical 3%	Double American 3%	Jones' Tropical 3%	American 3%	Tropical 3%	Double American 3%	Jones' Tropical 3%	American 3%
	Age 25.				Age 35.			
1	$ 10.68	$ 9.64	$ 10.05	$ 8.60	$ 15.67	$ 15.02	$ 14.88	$ 12.88
5	56.48	51.54	53.29	45.76	81.73	79.68	77.88	68.16
10	120.71	112.21	114.40	98.94	171.30	171.23	163.99	146.01
15	192.58	182.95	183.37	160.36	267.07	273.66	256.95	233.28
20	271.33	264.23	259.63	230.50	366.53	382.36	354.38	327.58
	Age 45.				Age 55.			
1	22.60	23.79	21.65	19.61	31.65	35.57	30.57	28.87
5	115.57	123.64	111.19	102.20	157.42	176.79	152.70	145.61
10	235.58	254.80	227.73	212.62	309.14	344.87	301.37	290.50
15	355.90	386.59	345.66	327.27	448.94	494.76	439.73	427.64
20	471.89	511.90	460.47	441.35	571.62	619.18	562.36	551.19

20-Payment Life.

	Age 25.				Age 35.			
1	20.29	19.88	19.93	17.81	24.66	23.97	24.06	22.00
5	108.66	107.82	107.15	95.49	130.91	129.18	128.29	117.52
10	237.24	239.82	235.26	208.95	282.63	284.74	278.55	255.78
15	389.66	401.94	388.72	343.86	459.87	472.79	455.99	418.33
20	571.79	603.06	574.07	504.59	672.69	704.19	671.01	609.92
	Age 45.				Age 55.			
1	29.76	30.05	28.99	27.62	35.54	38.11	34.66	34.24
5	155.33	158.74	152.01	145.86	179.96	191.71	176.42	175.61
10	328.72	338.40	323.52	311.52	366.81	384.85	361.79	362.37
15	527.27	544.79	522.02	500.15	573.16	589.73	568.74	567.10
20	773.86	806.20	770.18	723.24	859.78	887.33	856.04	824.93

20-Year Endowment.

	Age 25.				Age 35.			
1	33.97	31.87	33.30	34.45	34.29	32.20	33.62	34.59
5	183.02	173.73	180.12	185.39	183.50	174.66	180.65	185.71
10	403.30	389.27	398.98	407.79	401.66	389.05	397.55	407.45
15	670.52	658.65	666.88	675.54	666.01	655.77	662.76	674.00
	Age 45.				Age 55.			
1	35.12	34.18	34.43	35.48	37.44	39.17	36.67	37.67
5	185.10	181.86	182.31	188.73	190.93	197.98	188.02	194.80
10	398.47	393.44	394.63	408.62	394.88	401.65	391.41	408.33
15	655.59	648.95	652.88	669.88	633.61	629.62	631.98	656.29

The life reserves under the new Tropical table are greater than on the American, especially on the Limited Payment plans, the excess being most noticable when the policy becomes paid-up. Under Endowment Insurance policies the Tropical reserves are not generally greater than on the American Table, conforming in this respect to the well-known fact that the mortality makes little difference in the reserves under Endowment insurance for periods of twenty years or less, provided the net premiums used in the valuation are based on the valuation table. The new Tropical tables generally give larger reserves than the Double American 3% at the younger ages, and smaller reserves at the older ages, the aggregate valuation of a company's business probably being approximately the same under both tables.

The next table is a comparison of the reserves under the new Semi-Tropical 3% table with the Sesqui-American 3%, and also the American 3%.

Comparison of Terminal Reserves by Semi-Tropical 3%, Sesqui-American 3% and American 3%

ORDINARY LIFE.

End of Policy Year.	Age 25			Age 35		
	Semi-Tropical 3%	Sesqui-American 3%	American 3%	Semi-Tropical 3%	Sesqui-American 3%	American 3%
1	$ 9.68	$ 9.25	$ 8.60	$ 14.40	$ 14.17	$ 12.88
5	51.48	49.38	45.76	75.65	75.07	68.16
10	110.81	107.15	98.94	159.99	160.97	146.01
15	178.08	174.17	160.36	251.83	257.12	233.28
20	253.07	250.88	230.50	349.05	359.90	327.58
	Age 45			Age 55		
1	21.21	22.02	19.61	30.50	32.76	28.87
5	109.33	114.61	102.20	152.95	163.61	145.61
10	225.07	237.05	212.62	303.29	322.13	290.50
15	343.60	361.87	327.27	444.38	467.00	427.64
20	460.10	482.82	441.35	570.07	591.48	551.19

20-PAYMENT LIFE.

	Age 25			Age 35		
1	19.16	19.15	17.81	23.53	23.33	22.00
5	102.76	103.33	95.49	125.24	125.20	117.52
10	224.71	227.93	208.95	271.23	274.18	255.78
15	369.41	378.55	343.86	441.93	451.76	418.33
20	541.79	561.44	504.59	644.96	665.44	609.92

20-Payment Life—*Continued.*

End of Policy Year.	Age 45 Semi-Tropical 3%	Age 45 Sesqui-American 3%	Age 45 American 3%	Age 55 Semi-Tropical 3%	Age 55 Sesqui-American 3%	Age 55 American 3%
1	28.82	29.18	27.62	34.98	36.47	34.24
5	151.18	154.16	145.86	178.53	185.05	175.61
10	321.51	328.94	311.52	366.65	376.45	362.37
15	516.36	528.82	500.15	573.86	583.31	567.10
20	752.61	773.19	723.24	847.33	863.32	824.93

20-Year Endowment.

	Age 25			Age 35		
1	34.16	33.13	34.45	34.44	33.36	34.59
5	184.10	179.55	185.39	184.70	180.09	185.71
10	405.52	398.55	407.79	404.69	398.13	407.45
15	673.04	667.22	675.54	670.05	664.87	674.00
	Age 45			**Age 55**		
1	35.24	34.77	35.48	37.50	38.35	37.67
5	186.57	185.06	188.73	192.69	195.84	194.80
10	403.00	400.69	408.62	401.57	403.77	408.33
15	662.48	659.24	669.88	645.16	641.79	656.29

As was anticipated, the reserve on the Ordinary Life and 20 Payment Life plans on the Semi-Tropical 3% is about a mean between the Tropical American 3% and the American 3%. On the 20 Year Endowment, however, the same effect is noted as under the Tropical 3%, and the Double American 3%, namely that the reserves are generally less than the American 3%, so that a company carrying the American 3% reserve on its Endowment policies (exclusive of those for long periods) issued at the Semi-Tropical and the Tropical rates of premium would be carrying a slightly higher reserve in the aggregate than called for by the Tropical, the Double American, the Semi-Tropical or the Sesqui-American tables.

Conclusion.

I cannot too strongly point out to the members that the excess of the Tropical mortality over the American mortality does not represent the additional mortality in the Tropics, because the American does not represent the experience among insured lives in temperate climates. This can be seen very readily from the first table presented, which showed that the Tropical mortality by

amounts was 139% of the American and 185% of the Compound Progressive, while the Semi-Tropical mortality was 107% of the former and 145% of the latter. As the Compound Progressive Table represents the mortality of the New York Life in temperate climates, its mortality in the Tropics was double, and in the Semi-Tropics one and a half times that experienced in the United States and Canada. The rates of premium were sufficient to cover the mortality at double the American q_x in the Tropics and at one and a half times the American q_x in the Semi-Tropics; and therefore there was as large a ratio of margin between the mortality experienced and that on which the reserves are based as in the temperate zone. Even had the mortality been greater instead of less than anticipated, the loss would not have fallen on policyholders in the United States, because the Company had the right to use interest and loading profits to meet excessive mortality, the deferred dividend policies providing that the actual mortality in the Tropics was to be taken into account in distributing the dividends to Tropical policies.

There are many obstacles in the way of doing a successful business in Tropical countries, such as a different language, a different currency (sometimes three currencies in one country), and a different standard of morals. In addition thereto, the people in some of the Tropical countries know very little about their family history, while their ages are frequently given incorrectly. The effect of some impairments is also different in the Tropics from what it is in temperate climates. For example, overweight is probably more of an impairment in the Tropics than here, while underweight is rather an advantage than an impairment. It should also be remembered that in the early years of business in a Tropical country the officers at the Home Office and the Managers of the Company in the foreign field have not a thorough knowledge of the conditions which need to be guarded against,—a knowledge which can be obtained only through experience. It is accordingly more difficult to do a profitable business in the Tropics than at home, though that it can be done has been proved by the experience of the New York Life Insurance Company.

SEMI-TROPICAL MORTALITY TABLE.

Age	l_x	d_x	p_x	q_x	μ_x
15	100059	877	.99124	.00876	.00878
16	99182	872	.99121	.00879	.00881
17	98310	868	.99117	.00883	.00885
18	97442	864	.99113	.00887	.00889
19	96578	861	.99108	.00892	.00893
20	95717	858	.99104	.00896	.00898
21	94859	855	.99099	.00901	.00903
22	94004	853	.99093	.00907	.00909
23	93151	852	.99085	.00915	.00915
24	92299	850	.99079	.00921	.00922
25	91449	850	.99071	.00929	.00930
26	90599	851	.99061	.00939	.00938
27	89748	851	.99052	.00948	.00948
28	88897	852	.99042	.00958	.00958
29	88045	855	.99029	.00971	.00969
30	87190	858	.99016	.00984	.00982
31	86332	862	.99002	.00998	.00996
32	85470	867	.98986	.01014	.01011
33	84603	873	.98968	.01032	.01028
34	83730	881	.98948	.01052	.01047
35	82849	889	.98927	.01073	.01068
36	81960	899	.98903	.01097	.01091
37	81061	911	.98876	.01124	.01116
38	80150	923	.98848	.01152	.01144
39	79227	938	.98816	.01184	.01174
40	78289	954	.98781	.01219	.01208
41	77335	972	.98743	.01257	.01245
42	76363	993	.98700	.01300	.01286
43	75370	1016	.98652	.01348	.01332
44	74354	1040	.98601	.01399	.01382
45	73314	1068	.98543	.01457	.01437
46	72246	1098	.98480	.01520	.01498
47	71148	1131	.98410	.01590	.01566
48	70017	1167	.98333	.01667	.01640
49	68850	1206	.98248	.01752	.01722
50	67644	1248	.98155	.01845	.01813
51	66396	1293	.98053	.01947	.01913
52	65103	1342	.97939	.02061	.02023
53	63761	1394	.97814	.02186	.02145
54	62367	1450	.97675	.02325	.02279
55	60917	1509	.97523	.02477	.02427
56	59408	1570	.97357	.02643	.02591
57	57838	1636	.97171	.02829	.02771
58	56202	1704	.96968	.03032	.02970
59	54498	1774	.96745	.03255	.03190
60	52724	1846	.96499	.03501	.03433
61	50878	1919	.96228	.03772	.03700
62	48959	1993	.95929	.04071	.03996
63	46966	2066	.95601	.04399	.04321
64	44900	2137	.95241	.04759	.04681
65	42763	2205	.94844	.05156	.05078
66	40558	2267	.94410	.05590	.05516
67	38291	2324	.93931	.06069	.05999
68	35967	2372	.93405	.06595	.06532
69	33595	2409	.92829	.07171	.07121

SEMI-TROPICAL MORTALITY TABLE—*Continued.*

Age	l_x	d_x	p_x	q_x	μ_x
70	31186	2433	.92198	.07802	.07770
71	28753	2442	.91507	.08493	.08487
72	26311	2434	.90749	.09251	.09277
73	23877	2406	.89923	.10077	.10150
74	21471	2359	.89013	.10987	.11113
75	19112	2288	.88028	.11972	.12176
76	16824	2196	.86947	.13053	.13348
77	14628	2081	.85774	.14226	.14642
78	12547	1945	.84498	.15502	.16070
79	10602	1791	.83107	.16893	.17646
80	8811	1621	.81603	.18397	.19385
81	7190	1439	.79986	.20014	.21304
82	5751	1253	.78212	.21788	.23421
83	4498	1065	.76323	.23677	.25758
84	3433	883	.74279	.25721	.28337
85	2550	712	.72078	.27922	.31182
86	1838	556	.69750	.30250	.34322
87	1282	420	.67239	.32761	.37787
88	862	305	.64617	.35383	.41611
89	557	213	.61759	.38241	.45831
90	344	141	.59012	.40988	.50487
91	203	90	.55665	.44335	.55626
92	113	54	.52212	.47788	.61296
93	59	30	.49153	.50847	.67554
94	29	16	.44828	.55172	.74459
95	13	8	.38462	.61538	.82078
96	5	4	.20000	.80000	.90487
97	1	1	.00000	1.00000	.99766

SEMI-TROPICAL MORTALITY TABLE. INTEREST 3%.

Age	D_x	N_x	S_x	C_x	M_x	R_x
15	64224	1463408	27381772	546.51	21600.60	665879.01
16	61806	1399184	25918364	527.58	21054.09	644278.41
17	59480	1337378	24519180	509.86	20526.51	623224.32
18	57237	1277898	23181802	492.72	20016.65	602697.81
19	55077	1220661	21903904	476.72	19523.93	582681.16
20	52997	1165584	20683243	461.22	19047.21	563157.23
21	50992	1112587	19517659	446.22	18585.99	544110.02
22	49060	1061595	18405072	432.21	18139.77	525524.03
23	47199	1012535	17343477	419.13	17707.56	507384.26
24	45406	965336	16330942	405.97	17288.43	489676.70
25	43677	919930	15365606	394.14	16882.46	472388.27
26	42010	876253	14445676	383.11	16488.32	455505.81
27	40403	834243	13569423	371.95	16105.21	439017.49
28	38855	793840	12735180	361.54	15733.26	422912.28
29	37361	754985	11941340	352.25	15371.72	407179.02
30	35921	717624	11186355	343.19	15019.47	391807.30
31	34532	681703	10468731	334.75	14676.28	376787.83
32	33191	647171	9787028	326.88	14341.53	362111.55
33	31898	613980	9139857	319.55	14014.65	347770.02
34	30649	582082	8525877	313.10	13695.10	333755.37
35	29444	551433	7943795	306.73	13382.00	320060.27
36	28279	521989	7392362	301.15	13075.27	306678.27
37	27154	493710	6870373	296.29	12774.12	293603.00

SEMI-TROPICAL MORTALITY TABLE—*Continued.*

Age	D_x	N_x	S_x	C_x	M_x	R_x
38	26067	466556	6376663	291.44	12477.83	280828.88
39	25016	440489	5910107	287.55	12186.39	268351.05
40	24000	415473	5469618	283.94	11898.84	256164.66
41	23017	391473	5054145	280.87	11614.90	244265.82
42	22066	368456	4662672	278.58	11334.03	232650.92
43	21145	346390	4294216	276.73	11055.45	221316.89
44	20252	325245	3947826	275.01	10778.72	210261.44
45	19387	304993	3622581	274.20	10503.71	199482.72
46	18548	285606	3317588	273.68	10229.51	188979.01
47	17734	267058	3031982	273.70	9955.83	178749.50
48	16944	249324	2764924	274.19	9682.13	168793.67
49	16176	232380	2515600	275.10	9407.94	159111.54
50	15430	216204	2283220	276.38	9132.84	149703.60
51	14704	200774	2067016	278.01	8856.46	140570.76
52	13998	186070	1866242	280.14	8578.45	131714.30
53	13310	172072	1680172	282.52	8298.31	123135.85
54	12640	158762	1508100	285.31	8015.79	114837.54
55	11986	146122	1349338	288.28	7730.48	106821.75
56	11349	134136	1203216	291.19	7442.20	99091.27
57	10727	122787	1069080	294.59	7151.01	91649.07
58	10120	112060	946293	297.90	6856.42	84498.06
59	9527.5	101939.7	834232.8	301.11	6558.52	77641.64
60	8949.1	92412.2	732293.1	304.20	6257.41	71083.12
61	8384.2	83463.1	639880.9	307.02	5953.21	64825.71
62	7832.9	75078.9	556417.8	309.57	5646.19	58872.50
63	7295.1	67246.0	481338.9	311.57	5336.62	53226.31
64	6771.3	59950.9	414092.9	312.88	5025.05	47889.69
65	6261.1	53179.6	354142.0	313.44	4712.17	42864.64
66	5765.3	46918.5	300962.4	312.87	4398.73	38152.47
67	5284.6	41153.2	254043.9	311.39	4085.86	33753.74
68	4819.1	35868.6	212890.7	308.56	3774.47	29667.88
69	4370.2	31049.5	177022.1	304.25	3465.91	25893.41
70	3938.7	26679.3	145972.6	298.33	3161.66	22427.50
71	3525.7	22740.6	119293.3	290.72	2863.33	19265.84
72	3132.3	19214.9	96552.7	281.32	2572.61	16402.51
73	2759.7	16082.6	77337.8	269.99	2291.29	13829.90
74	2409.4	13322.9	61255.2	257.00	2021.30	11538.61
75	2082.2	10913.5	47932.3	242.01	1764.30	9517.31
76	1779.5	8831.3	37018.8	225.51	1522.29	7753.01
77	1502.2	7051.8	28187.5	207.48	1296.78	6230.72
78	1251.0	5549.6	21135.7	188.27	1089.30	4933.94
79	1026.2	4298.6	15586.1	168.31	901.03	3844.64
80	828.04	3272.37	11287.51	147.90	732.72	2943.61
81	656.01	2444.33	8015.14	127.47	584.82	2210.89
82	509.44	1788.32	5570.81	107.76	457.35	1626.07
83	386.84	1278.88	3782.49	88.924	349.589	1168.716
84	286.64	892.04	2503.61	71.581	260.665	819.127
85	206.72	605.40	1611.57	56.038	189.084	558.462
86	144.66	398.68	1006.17	42.484	133.046	369.378
87	97.960	254.024	607.490	31.158	90.562	236.332
88	63.948	156.064	353.466	21.968	59.404	145.770
89	40.119	92.116	197.402	14.895	37.436	86.366
90	24.055	51.997	105.286	9.5726	22.5408	48.9303
91	13.782	27.942	53.289	5.9323	12.9682	26.3895
92	7.4483	14.1595	25.3467	3.4557	7.0359	13.4213
93	3.7756	6.7112	11.1872	1.8639	3.5802	6.3854
94	1.8018	2.9356	4.4760	.96512	1.71626	2.80523
95	.78415	1.13383	1.54037	.46851	.75114	1.08897
96	.29282	.34968	.40654	.22743	.28263	.33783
97	.056858	.056858	.056858	.055201	.055201	.055201

SEMI-TROPICAL MORTALITY TABLE. INTEREST 3½%

Age	D_x	N_x	S_x	C_x	M_x	R_x
15	59724	1252064	22181598	505.77	17383.84	501963.82
16	57198	1192340	20929534	485.88	16878.07	484579.98
17	54778	1135142	19737194	467.29	16392.19	467701.91
18	52459	1080364	18602052	449.41	15924.90	451309.72
19	50235	1027905	17521688	432.70	15475.49	435384.82
20	48104	977670	16493783	416.62	15042.79	419909.33
21	46061	929566	15516113	401.13	14626.17	404866.54
22	44102	883505	14586547	386.65	14225.04	390240.37
23	42224	839403	13703042	373.14	13838.39	376015.33
24	40423	797179	12863639	359.67	13465.25	362176.94
25	38696	756756	12066460	347.51	13105.58	348711.69
26	37040	718060	11309704	336.16	12758.07	335606.11
27	35451	681020	10591644	324.79	12421.91	322848.04
28	33928	645569	9910624	314.17	12097.12	310426.13
29	32466	611641	9265055	304.62	11782.95	298329.01
30	31064	579175	8653414	295.35	11478.33	286546.06
31	29718	548111	8074239	286.69	11182.98	275067.73
32	28426	518393	7526128	278.61	10896.29	263884.75
33	27187	489967	7007735	271.04	10617.68	252988.46
34	25996	462780	6517768	264.28	10346.64	242370.78
35	24853	436784	6054988	257.66	10082.36	232024.14
36	23755	411931	5618204	251.75	9824.70	221941.78
37	22700	388176	5206273	246.48	9572.95	212117.08
38	21686	365476	4818097	241.28	9326.47	202544.13
39	20711	343790	4452621	236.91	9085.19	193217.66
40	19774	323079	4108831	232.81	8848.28	184132.47
41	18873	303305	3785752	229.18	8615.47	175284.19
42	18005	284432	3482447	226.21	8386.29	166668.72
43	17170	266427	3198015	223.62	8160.08	158282.43
44	16365	249257	2931588	221.16	7936.46	150122.35
45	15591	232892	2682331	219.44	7715.30	142185.89
46	14844	217301	2449439	217.97	7495.86	134470.59
47	14124	202457	2232138	216.93	7277.89	126974.73
48	13430	188333	2029681	216.27	7060.96	119696.84
49	12759	174903	1841348	215.94	6844.69	112635.88
50	12112	162144	1666445	215.90	6628.75	105791.19
51	11486	150032	1504301	216.12	6412.85	99162.44
52	10882	138546	1354269	216.73	6196.73	92749.59
53	10297	127664	1215723	217.51	5980.00	86552.86
54	9731.3	117366.9	1088059.1	218.60	5762.49	80572.86
55	9183.7	107635.6	970692.2	219.80	5543.89	74810.37
56	8653.3	98451.9	863056.6	220.95	5324.09	69266.48
57	8139.7	89798.6	764604.7	222.45	5103.14	63942.39
58	7642.1	81658.9	674806.1	223.87	4880.69	58839.25
59	7159.8	74016.8	593147.2	225.18	4656.82	53958.56
60	6692.5	66857.0	519130.4	226.40	4431.64	49301.74
61	6239.8	60164.5	452273.4	227.39	4205.24	44870.10
62	5801.4	53924.7	392108.9	228.18	3977.85	40664.86
63	5377.0	48123.3	338184.2	228.53	3749.67	36687.01
64	4966.7	42746.3	290060.9	228.39	3521.14	32937.34
65	4570.4	37779.6	247314.6	227.69	3292.75	29416.20
66	4188.1	33209.2	209535.0	226.18	3065.06	26123.45
67	3820.3	29021.1	176325.8	224.03	2838.88	23058.39
68	3467.0	25200.8	147304.7	220.92	2614.85	20219.51
69	3128.9	21733.8	122103.9	216.78	2393.93	17604.66

SEMI-TROPICAL MORTALITY TABLE—*Continued.*

Age	D_x	N_x	S_x	C_x	M_x	R_x
70	2806.3	18604.9	100370.1	211.53	2177.15	15210.73
71	2499.9	15798.6	81765.2	205.14	1965.62	13033.58
72	2210.2	13298.7	65966.6	197.55	1760.48	11067.96
73	1937.9	11088.5	52667.9	188.67	1562.93	9307.48
74	1683.7	9150.6	41579.4	178.73	1374.26	7744.55
75	1448.0	7466.9	32428.8	167.49	1195.53	6370.29
76	1231.6	6018.9	24961.9	155.32	1028.04	5174.76
77	1034.6	4787.3	18943.0	142.21	872.72	4146.72
78	857.41	3752.70	14155.67	128.42	730.51	3274.00
79	700.00	2895.29	10402.97	114.25	602.09	2543.49
80	562.08	2195.29	7507.68	99.910	487.839	1941.400
81	443.16	1633.21	5312.39	85.694	387.929	1453.561
82	342.48	1190.05	3679.18	72.094	302.235	1065.632
83	258.80	847.57	2489.13	59.205	230.141	763.397
84	190.84	588.77	1641.56	47.427	170.936	533.256
85	136.97	397.93	1052.79	36.950	123.509	362.320
86	95.385	260.960	654.855	27.878	86.559	238.811
87	64.281	165.575	393.895	20.347	58.681	152.252
88	41.760	101.294	228.320	14.276	38.334	93.571
89	26.072	59.534	127.026	9.6327	24.0582	55.2374
90	15.557	33.462	67.492	6.1610	14.4255	31.1792
91	8.8701	17.9047	34.0303	3.7995	8.2645	16.7537
92	4.7706	9.0346	16.1256	2.2026	4.4650	8.4892
93	2.4066	4.2640	7.0910	1.1823	2.2624	4.0242
94	1.1429	1.8574	2.8270	.60924	1.08008	1.76178
95	.49501	.71451	.96956	.29432	.47084	.68170
96	.18395	.21950	.25505	.14218	.17652	.21086
97	.035546	.035546	.035546	.034344	.034344	.034344

SEMI-TROPICAL MORTALITY TABLE. INTEREST 4%.

Age	D_x	N_x	S_x	C_x	M_x	R_x
15	55560	1077029	18082997	468.24	14135.09	381526.19
16	52954	1021469	17005968	447.66	13666.85	367391.10
17	50470	968515	15984499	428.47	13219.19	353724.25
18	48101	918045	15015984	410.09	12790.72	340505.06
19	45841	869944	14097939	392.94	12380.63	327714.34
20	43684	824103	13227995	376.52	11987.69	315333.71
21	41627	780419	12403892	360.78	11611.17	303346.02
22	39666	738792	11623473	346.08	11250.39	291734.85
23	37794	699126	10884681	332.38	10904.31	280484.46
24	36008	661332	10185555	318.85	10571.93	269580.15
25	34304	625324	9524223	306.58	10253.08	259008.22
26	32678	591020	8898899	295.14	9946.50	248755.14
27	31126	558342	8307879	283.79	9651.36	238808.64
28	29646	527216	7749537	273.19	9367.57	229157.28
29	28231	497570	7222321	263.61	9094.38	219789.71
30	26883	469339	6724751	254.37	8830.77	210695.33
31	25594	442456	6255412	245.72	8576.40	201864.56
32	24364	416862	5812956	237.64	8330.68	193288.16
33	23189	392498	5396094	230.08	8093.04	184957.48
34	22067	369309	5003596	223.26	7862.96	176864.44
35	20995	347242	4634287	216.62	7639.70	169001.48
36	19971	326247	4287045	210.63	7423.08	161361.78
37	18992	306276	3960798	205.23	7212.45	153938.70

SEMI-TROPICAL MORTALITY TABLE—*Continued.*

Age	D_x	N_x	S_x	C_x	M_x	R_x
38	18056	287284	3654522	199.94	7007.22	146726.25
39	17162	269228	3367238	195.38	6807.28	139719.03
40	16307	252066	3098010	191.06	6611.90	132911.75
41	15489	235759	2845944	187.18	6420.84	126299.85
42	14706	220270	2610185	183.87	6233.66	119879.01
43	13956	205564	2389915	180.89	6049.79	113645.35
44	13238	191608	2184351	178.05	5868.90	107595.56
45	12551	178370	1992743	175.81	5690.85	101726.66
46	11893	165819	1814373	173.79	5515.04	96035.81
47	11261	153926	1648554	172.13	5341.25	90520.77
48	10656	142665	1494628	170.78	5169.12	85179.52
49	10076	132009	1351963	169.70	4998.34	80010.40
50	9518.3	121933.0	1219953.5	168.85	4828.64	75012.06
51	8983.4	112414.7	1098020.5	168.22	4659.79	70183.42
52	8469.7	103431.3	985605.8	167.87	4491.57	65523.63
53	7976.1	94961.6	882174.5	167.67	4323.70	61032.06
54	7501.5	86985.5	787212.9	167.70	4156.03	56708.36
55	7045.5	79484.0	700227.4	167.81	3988.33	52552.33
56	6606.5	72438.5	620743.4	167.88	3820.52	48564.00
57	6184.6	65832.0	548304.9	168.21	3652.64	44743.48
58	5778.6	59647.4	482472.9	168.46	3484.43	41090.84
59	5387.8	53868.8	422825.5	168.64	3315.97	37606.41
60	5012.0	48481.0	368956.7	168.73	3147.33	34290.44
61	4650.5	43469.0	320475.7	168.66	2978.60	31143.11
62	4302.9	38818.5	277006.7	168.43	2809.94	28164.51
63	3969.0	34515.6	238188.2	167.88	2641.51	25354.57
64	3648.5	30546.6	203672.6	166.97	2473.63	22713.06
65	3341.2	26898.1	173126.0	165.66	2306.66	20239.43
66	3047.1	23556.9	146227.9	163.76	2141.00	17932.77
67	2766.1	20509.8	122671.0	161.42	1977.24	15791.77
68	2498.2	17743.7	102161.2	158.42	1815.82	13814.53
69	2243.7	15245.5	84417.5	154.71	1657.40	11998.71
70	2002.8	13001.8	69172.0	150.23	1502.69	10341.31
71	1775.5	10999.0	56170.2	144.99	1352.46	8838.62
72	1562.2	9223.5	45171.2	138.96	1207.47	7486.16
73	1363.2	7661.3	35947.7	132.08	1068.51	6278.69
74	1178.6	6298.1	28286.4	124.52	936.43	5210.18
75	1008.8	5119.5	21988.3	116.13	811.91	4273.75
76	853.89	4110.66	16868.84	107.17	695.78	3461.84
77	713.87	3256.77	12758.18	97.649	588.608	2766.055
78	588.76	2542.90	9501.41	87.759	490.959	2177.447
79	478.37	1954.14	6958.51	77.702	403.200	1686.488
80	382.26	1475.77	5004.37	67.621	325.498	1283.288
81	299.94	1093.51	3528.60	57.720	257.877	957.790
82	230.68	793.57	2435.09	48.326	200.157	699.913
83	173.48	562.89	1641.52	39.496	151.831	499.756
84	127.31	389.41	1078.63	31.487	112.335	347.925
85	90.931	262.101	689.223	24.412	80.848	235.590
86	63.020	171.170	427.122	18.330	56.436	154.742
87	42.266	108.150	255.952	13.314	38.106	98.306
88	27.326	65.884	147.802	9.2967	24.7919	60.2000
89	16.978	38.558	81.918	6.2428	15.4952	35.4081
90	10.082	21.580	43.360	3.9737	9.2524	19.9129
91	5.7210	11.4981	21.7797	2.4388	5.2787	10.6605
92	3.0620	5.7771	10.2816	1.4070	2.8399	5.3818
93	1.5373	2.7151	4.5045	.75161	1.43286	2.54185
94	.72656	1.17781	1.78942	.38543	.68125	1.10899
95	.31316	.45125	.61161	.18531	.29582	.42774
96	.11582	.13809	.16036	.089090	.110506	.131922
97	.022273	.022273	.022273	.021416	.021416	.021416

SEMI-TROPICAL MORTALITY TABLE. Interest 3%.

Age	a_x	A_x	P_x	Age	a_x	A_x	P_x
15	21.78600	.336332	.014760	57	10.44654	.666637	.058239
16	21.63832	.340648	.015047	58	10.07312	.677512	.061185
17	21.48450	.345099	.015348	59	9.69952	.688378	.064337
18	21.32643	.349715	.015664	60	9.32642	.699222	.067712
19	21.16281	.354484	.015995	61	8.95481	.710051	.071327
20	20.99340	.359402	.016341	62	8.58507	.720830	.075203
21	20.81885	.364488	.016705	63	8.21797	.731535	.079360
22	20.63871	.369747	.017087	64	7.85368	.742110	.083819
23	20.45247	.375168	.017488	65	7.49365	.752611	.088609
24	20.26010	.380752	.017909	66	7.13808	.762966	.093753
25	20.06212	.386530	.018352	67	6.78738	.773164	.099284
26	19.85820	.392486	.018817	68	6.44301	.783231	.105230
27	19.64805	.398614	.019305	69	6.10482	.793078	.111625
28	19.43083	.404922	.019819	70	5.77363	.802717	.118506
29	19.20784	.411438	.020360	71	5.44995	.812131	.125913
30	18.97784	.418125	.020929	72	5.13444	.821317	.133886
31	18.74120	.425005	.021529	73	4.82766	.830268	.142470
32	18.49839	.432091	.022160	74	4.52955	.838923	.151716
33	18.24823	.439358	.022826	75	4.24133	.847325	.161662
34	17.99188	.446837	.023528	76	3.96280	.855459	.172374
35	17.72820	.454490	.024268	77	3.69432	.863254	.183893
36	17.45854	.462367	.025049	78	3.43613	.870743	.196284
37	17.18185	.470432	.025874	79	3.18885	.878026	.209610
38	16.89834	.478683	.026745	80	2.95195	.884885	.223911
39	16.60829	.487144	.027666	81	2.72606	.891480	.239256
40	16.31138	.495785	.028639	82	2.51036	.897750	.255743
41	16.00799	.504623	.029670	83	2.30597	.903704	.273356
42	15.69791	.513642	.030761	84	2.11206	.909381	.292212
43	15.38165	.522840	.031916	85	1.92860	.914687	.312329
44	15.05990	.532230	.033140	86	1.75598	.919715	.333716
45	14.73183	.541791	.034439	87	1.59314	.924479	.356510
46	14.39821	.551516	.035817	88	1.44048	.928942	.380639
47	14.05910	.561398	.037280	89	1.29607	.933124	.406401
48	13.71459	.571419	.038834	90	1.16159	.937053	.433502
49	13.36573	.581599	.040485	91	1.02743	.940952	.464111
50	13.01192	.591889	.042242	92	.90104	.944632	.496903
51	12.65438	.602316	.044112	93	.77752	.948247	.533466
52	12.29261	.612834	.046103	94	.62926	.952525	.584637
53	11.92802	.623464	.048226	95	.44594	.957903	.662480
54	11.56028	.634161	.050489	96	.19418	.965200	.808246
55	11.19106	.644959	.052904	97		.970874	.970874
56	10.81919	.655758	.055482				

SEMI-TROPICAL MORTALITY TABLE. Interest 3½%.

Age	a_x	A_x	P_x	Age	a_x	A_x	P_x
15	19.96417	.291070	.013884	22	19.03322	.322549	.016101
16	19.84583	.295081	.014155	23	18.87976	.327738	.016486
17	19.72259	.299248	.014441	24	18.72093	.333109	.016891
18	19.59445	.303569	.014740	25	18.55644	.338680	.017318
19	19.46193	.308062	.015055	26	18.38607	.344440	.017767
20	19.32409	.312714	.015386	27	18.21018	.350397	.018240
21	19.18119	.317539	.015734	28	18.02762	.356553	.018739

SEMI-TROPICAL MORTALITY TABLE—*Continued.*

Age	a_x	A_x	P_x	Age	a_x	A_x	P_x
29	17.83943	.362932	.019264	64	7.60658	.708950	.082373
30	17.64457	.369506	.019818	65	7.26615	.720451	.087157
31	17.44374	.376303	.020403	66	6.92942	.731850	.092296
32	17.23658	.383321	.021019	67	6.59655	.743104	.097821
33	17.02211	.390543	.021670	68	6.26876	.754211	.103761
34	16.80197	.398009	.022358	69	5.94615	.765103	.110148
35	16.57470	.405680	.023083	70	5.62969	.775808	.117020
36	16.34081	.413585	.023850	71	5.31969	.786279	.124417
37	16.10026	.421716	.024661	72	5.01697	.796525	.132380
38	15.85308	.430069	.025519	73	4.72192	.806507	.140951
39	15.59939	.438665	.026427	74	4.43482	.816214	.150183
40	15.33858	.447470	.027387	75	4.15670	.825642	.160111
41	15.07084	.456497	.028405	76	3.88706	.834719	.170802
42	14.79739	.465776	.029484	77	3.62720	.843534	.182299
43	14.51701	.475252	.030628	78	3.37679	.851996	.194663
44	14.23110	.484965	.031840	79	3.13613	.860129	.207955
45	13.93759	.494856	.033128	80	2.90565	.867917	.222221
46	13.63898	.504976	.034495	81	2.68537	.875370	.237525
47	13.33425	.515285	.035948	82	2.47480	.882489	.253968
48	13.02331	.525760	.037492	83	2.27500	.889262	.271530
49	12.70821	.536460	.039134	84	2.08515	.895703	.290327
50	12.38705	.547288	.040882	85	1.90523	.901723	.310379
51	12.06216	.558319	.042743	86	1.73586	.907470	.331695
52	11.73167	.569448	.044727	87	1.57580	.912883	.354407
53	11.39817	.580752	.046842	88	1.42562	.917960	.378443
54	11.06076	.592160	.049098	89	1.28345	.922760	.404109
55	10.72029	.603666	.051506	90	1.15093	.927267	.431101
56	10.37738	.615267	.054078	91	1.01855	.931726	.461583
57	10.03218	.626944	.056829	92	.89381	.935941	.494211
58	9.68540	.638658	.059769	93	.77179	.940081	.530582
59	9.33783	.650412	.062916	94	.62516	.945035	.581501
60	8.98984	.662180	.066285	95	.44343	.951173	.658969
61	8.64206	.673938	.069896	96	.19326	.959609	.804191
62	8.29512	.685671	.073767	97		.966184	.966184
63	7.94984	.697354	.077918				

SEMI-TROPICAL MORTALITY TABLE. INTEREST 4%.

Age	a_x	A_x	P_x	Age	a_x	A_x	P_x
15	18.38497	.254411	.013124	30	16.45858	.328489	.018815
16	18.28974	.258089	.013380	31	16.28749	.335094	.019384
17	18.18991	.261922	.013649	32	16.10975	.341926	.019984
18	18.08578	.265914	.013933	33	15.92604	.349003	.020619
19	17.97742	.270078	.014232	34	15.73580	.356322	.021291
20	17.86510	.274418	.014546	35	15.53927	.363882	.022001
21	17.74790	.278934	.014878	36	15.33604	.371693	.022753
22	17.62532	.283628	.015228	37	15.12658	.379763	.023549
23	17.49833	.288520	.015597	38	14.91072	.388083	.024391
24	17.36625	.293599	.015986	39	14.68745	.396648	.025284
25	17.22889	.298889	.016396	40	14.45753	.405464	.026231
26	17.08617	.304379	.016829	41	14.22106	.414542	.027235
27	16.93812	.310074	.017286	42	13.97824	.423885	.028300
28	16.78371	.315981	.017768	43	13.72944	.433490	.029430
29	16.62495	.322142	.018278	44	13.47409	.443337	.030630

SEMI-TROPICAL MORTALITY TABLE—*Continued.*

Age	a_x	A_x	P_x	Age	a_x	A_x	P_x
45	13.21162	.453418	.031905	72	4.90417	.772929	.130912
46	12.94257	.463722	.033259	73	4.62009	.783825	.139468
47	12.66895	.474314	.034700	74	4.34371	.794527	.148685
48	12.38823	.485090	.036233	75	4.07484	.804828	.158592
49	12.10133	.496064	.037864	76	3.81404	.814836	.169262
50	11.81038	.507301	.039601	77	3.56213	.824531	.180734
51	11.51360	.518711	.041452	78	3.31908	.833886	.193071
52	11.21192	.530310	.043426	79	3.08500	.842862	.206331
53	10.90577	.542082	.045531	80	2.86064	.851509	.220561
54	10.59575	.554027	.047778	81	2.64576	.859762	.235825
55	10.28153	.566082	.050178	82	2.44013	.867683	.252223
56	9.96473	.578297	.052742	83	2.24470	.875208	.269735
57	9.64450	.590602	.055484	84	2.05875	.882374	.288475
58	9.32212	.602989	.058417	85	1.88242	.889114	.308461
59	8.99829	.615459	.061556	86	1.71612	.895525	.329707
60	8.67298	.627959	.064919	87	1.55879	.901576	.352344
61	8.34717	.640490	.068522	88	1.41104	.907264	.376296
62	8.02147	.653034	.072387	89	1.27106	.912663	.401867
63	7.69630	.665535	.076531	90	1.14045	.917715	.428749
64	7.37237	.677985	.080979	91	1.00981	.922688	.459093
65	7.05043	.690369	.085755	92	.88671	.927466	.491579
66	6.73092	.702635	.090886	93	.76615	.932063	.527737
67	6.41470	.714811	.096405	94	.62108	.937638	.578404
68	6.10259	.726851	.102336	95	.44096	.944629	.655557
69	5.79480	.738691	.108714	96	.19228	.954118	.800246
70	5.49181	.750295	.115576	97		.961538	.961538
71	5.19487	.761735	.122962				

TROPICAL MORTALITY TABLE.

Age	l_x	d_x	p_x	q_x	μ_x
15	100000	964	.99036	.00964	.00966
16	99036	960	.99031	.00969	.00971
17	98076	956	.99025	.00975	.00977
18	97120	954	.99018	.00982	.00983
19	96166	951	.99011	.00989	.00990
20	95215	949	.99003	.00997	.00997
21	94266	947	.98995	.01005	.01006
22	93319	947	.98985	.01015	.01015
23	92372	946	.98976	.01024	.01025
24	91426	947	.98964	.01036	.01035
25	90479	948	.98952	.01048	.01047
26	89531	951	.98938	.01062	.01060
27	88580	953	.98924	.01076	.01074
28	87627	957	.98908	.01092	.01090
29	86670	962	.98890	.01110	.01107
30	85708	969	.98869	.01131	.01126
31	84739	975	.98849	.01151	.01147
32	83764	984	.98825	.01175	.01169
33	82780	993	.98800	.01200	.01194
34	81787	1004	.98772	.01228	.01221
35	80783	1017	.98741	.01259	.01251
36	79766	1031	.98707	.01293	.01283
37	78735	1047	.98670	.01330	.01319

TROPICAL MORTALITY TABLE—*Continued.*

Age	l_x	d_x	p_x	q_x	μ_x
38	77688	1065	.98629	.01371	.01358
39	76623	1083	.98587	.01413	.01401
40	75540	1106	.98536	.01464	.01449
41	74434	1129	.98483	.01517	.01500
42	73305	1154	.98426	.01574	.01557
43	72151	1183	.98360	.01640	.01619
44	70968	1213	.98291	.01709	.01687
45	69755	1246	.98214	.01786	.01762
46	68509	1282	.98129	.01871	.01844
47	67227	1320	.98037	.01963	.01934
48	65907	1360	.97936	.02064	.02033
49	64547	1404	.97825	.02175	.02141
50	63143	1450	.97704	.02296	.02259
51	61693	1499	.97570	.02430	.02389
52	60194	1550	.97425	.02575	.02532
53	58644	1603	.97267	.02733	.02688
54	57041	1659	.97092	.02908	.02859
55	55382	1716	.96902	.03098	.03047
56	53666	1776	.96691	.03309	.03253
57	51890	1835	.96464	.03536	.03478
58	50055	1895	.96214	.03786	.03726
59	48160	1955	.95941	.04059	.03997
60	46205	2013	.95643	.04357	.04295
61	44192	2071	.95314	.04686	.04621
62	42121	2123	.94960	.05040	.04979
63	39998	2172	.94570	.05430	.05372
64	37826	2215	.94144	.05856	.05802
65	35611	2251	.93679	.06321	.06274
66	33360	2277	.93174	.06826	.06791
67	31083	2293	.92623	.07377	.07358
68	28790	2297	.92022	.07978	.07980
69	26493	2288	.91364	.08636	.08662
70	24205	2262	.90655	.09345	.09409
71	21943	2221	.89878	.10122	.10229
72	19722	2163	.89033	.10967	.11128
73	17559	2086	.88120	.11880	.12113
74	15473	1991	.87132	.12868	.13194
75	13482	1880	.86055	.13945	.14379
76	11602	1753	.84891	.15109	.15678
77	9849	1612	.83633	.16367	.17103
78	8237	1460	.82275	.17725	.18665
79	6777	1300	.80817	.19183	.20377
80	5477	1137	.79240	.20760	.22255
81	4340	975	.77535	.22465	.24315
82	3365	816	.75750	.24250	.26572
83	2549	668	.73794	.26206	.29048
84	1881	532	.71717	.28283	.31762
85	1349	411	.69533	.30467	.34739
86	938	308	.67164	.32836	.38002
87	630	222	.64762	.35238	.41581
88	408	155	.62010	.37990	.45504
89	253	103	.59289	.40711	.49806
90	150	65	.56667	.43333	.54523
91	85	39	.54118	.45882	.59696
92	46	23	.50000	.50000	.65367
93	23	12	.47826	.52174	.71585
94	11	6	.45455	.54545	.78404
95	5	3	.40000	.60000	.85880
96	2	2	.00000	1.00000	.94078

TROPICAL MORTALITY TABLE. INTEREST 3%.

Age	D_x	N_x	S_x	C_x	M_x	R_x
15	64186	1412885	25459328	600.73	23034.19	671351.25
16	61715	1348699	24046443	580.82	22433.46	648317.06
17	59338	1286984	22697744	561.55	21852.64	625883.60
18	57048	1227646	21410760	544.05	21291.09	604030.96
19	54842	1170598	20183114	526.55	20747.04	582739.87
20	52719	1115756	19012516	510.14	20220.49	561992.83
21	50673	1063037	17896760	494.23	19710.35	541772.34
22	48702	1012364	16833723	479.83	19216.12	522061.99
23	46804	963662	15821359	465.37	18736.29	502845.87
24	44976	916858	14857697	452.29	18270.92	484109.58
25	43214	871882	13940839	439.58	17818.63	465838.66
26	41515	828668	13068957	428.12	17379.05	448020.03
27	39878	787153	12240289	416.53	16950.93	430640.98
28	38300	747275	11453136	406.10	16534.40	413690.05
29	36778	708975	10705861	396.33	16128.30	397155.65
30	35310	672197	9996886	387.59	15731.97	381027.35
31	33895	636887	9324689	378.63	15344.38	365295.38
32	32529	602992	8687802	371.00	14965.75	349951.00
33	31210	570463	8084810	363.48	14594.75	334985.25
34	29937	539253	7514347	356.80	14231.27	320390.50
35	28709	509316	6975094	350.90	13874.47	306159.23
36	27522	480607	6465778	345.37	13523.57	292284.76
37	26375	453085	5985171	340.52	13178.20	278761.19
38	25266	426710	5532086	336.28	12837.68	265582.99
39	24194	401444	5105376	332.00	12501.40	252745.31
40	23157	377250	4703932	329.18	12169.40	240243.91
41	22153	354093	4326682	326.23	11840.22	228074.51
42	21182	331940	3972589	323.75	11513.99	216234.29
43	20241	310758	3640649	322.21	11190.24	204720.30
44	19329	290517	3329891	320.76	10868.03	193530.06
45	18446	271188	3039374	319.90	10547.27	182662.03
46	17589	252742	2768186	319.55	10227.37	172114.76
47	16757	235153	2515444	319.43	9907.82	161887.39
48	15949	218396	2280291	319.54	9588.39	151979.57
49	15166	202447	2061895	320.27	9268.85	142391.18
50	14404	187281	1859448	321.12	8948.58	133122.33
51	13663	172877	1672167	322.30	8627.46	124173.75
52	12942	159214	1499290	323.56	8305.16	115546.29
53	12242	146272	1340076	324.88	7981.60	107241.13
54	11561	134030	1193804	326.44	7656.72	99259.53
55	10897	122469	1059774	327.82	7330.28	91602.81
56	10252	111572	937305	329.40	7002.46	84272.53
57	9624.1	101320.4	825732.8	330.43	6673.06	77270.07
58	9013.4	91696.3	724412.4	331.29	6342.63	70597.01
59	8419.6	82682.9	632716.1	331.83	6011.34	64254.38
60	7842.6	74263.3	550033.2	331.72	5679.51	58243.04
61	7282.3	66420.7	475769.9	331.34	5347.79	52563.53
62	6738.9	59138.4	409349.2	329.76	5016.45	47215.74
63	6212.8	52399.5	350210.8	327.55	4686.69	42199.29
64	5704.4	46186.7	297811.3	324.30	4359.14	37512.60
65	5213.9	40482.3	251624.6	319.98	4034.84	33153.46
66	4742.1	35268.4	211142.3	314.25	3714.86	29118.62
67	4289.7	30526.3	175873.9	307.23	3400.61	25403.76
68	3857.5	26236.6	145347.6	298.81	3093.38	22003.15
69	3446.4	22379.1	119111.0	288.97	2794.57	18909.77

TROPICAL MORTALITY TABLE—*Continued.*

Age	D_x	N_x	S_x	C_x	M_x	R_x
70	3057.0	18932.7	96731.9	277.36	2505.60	16115.20
71	2690.7	15875.7	77799.2	264.41	2228.24	13609.60
72	2347.8	13185.0	61923.5	250.00	1963.83	11381.36
73	2029.5	10837.2	48738.5	234.08	1713.83	9417.53
74	1736.3	8807.7	37901.3	216.91	1479.75	7703.70
75	1468.8	7071.4	29093.6	198.85	1262.84	6223.95
76	1227.2	5602.6	22022.2	180.02	1063.99	4961.11
77	1011.4	4375.4	16419.6	160.72	883.97	3897.12
78	821.24	3363.96	12044.17	141.32	723.25	3013.15
79	655.99	2542.72	8680.21	122.17	581.93	2289.90
80	514.71	1886.73	6137.49	103.74	459.76	1707.97
81	395.98	1372.02	4250.76	86.367	356.018	1248.206
82	298.08	976.04	2878.74	70.178	269.651	892.188
83	219.22	677.96	1902.70	55.776	199.473	622.537
84	157.06	458.74	1224.74	43.127	143.697	423.064
85	109.36	301.68	766.00	32.347	100.570	279.367
86	73.824	192.320	464.323	23.535	68.223	178.797
87	48.139	118.496	272.003	16.469	44.688	110.574
88	30.268	70.357	153.507	11.164	28.219	65.886
89	18.223	40.089	83.150	7.2026	17.0549	37.6668
90	10.489	21.866	43.061	4.4129	9.8523	20.6119
91	5.7707	11.3769	21.1952	2.5706	5.4394	10.7596
92	3.0321	5.6062	9.8183	1.4719	2.8688	5.3202
93	1.4719	2.5741	4.2121	.74556	1.39689	2.45135
94	.68342	1.10215	1.63801	.36192	.65133	1.05446
95	.30160	.41873	.53586	.17569	.28941	.40313
96	.11713	.11713	.11713	.11372	.11372	.11372

TROPICAL MORTALITY TABLE.

INTEREST 3½%.

Age	D_x	N_x	S_x	C_x	M_x	R_x
15	59688	1212680	20732315	555.94	18680.44	511590.40
16	57114	1152992	19519635	534.91	18124.50	492909.96
17	54647	1095878	18366643	514.67	17589.59	474785.46
18	52285	1041231	17270765	496.23	17074.92	457195.87
19	50021	988946	16229534	477.94	16578.69	440120.95
20	47852	938925	15240588	460.81	16100.75	423542.26
21	45773	891073	14301663	444.28	15639.94	407441.51
22	43780	845300	13410590	429.26	15195.66	391801.57
23	41871	801520	12565290	414.30	14766.40	376605.91
24	40041	759649	11763770	400.72	14352.10	361839.51
25	38286	719608	11004121	387.58	13951.38	347487.41
26	36603	681322	10284513	375.66	13563.80	333536.03
27	34990	644719	9603191	363.71	13188.14	319972.23
28	33443	609729	8958472	352.89	12824.43	306784.09
29	31960	576286	8348743	342.74	12471.54	293959.66
30	30536	544326	7772457	333.56	12128.80	281488.12
31	29170	513790	7228131	324.27	11795.24	269359.32
32	27859	484620	6714341	316.21	11470.97	257564.08
33	26601	456761	6229721	308.30	11154.76	246093.11
34	25393	430160	5772960	301.18	10846.46	234938.35
35	24233	404767	5342800	294.76	10545.28	224091.89
36	23119	380534	4938033	288.72	10250.52	213546.61
37	22049	357415	4557499	283.28	9961.80	203296.09
38	21019	335366	4200084	278.41	9678.52	193334.29

TROPICAL MORTALITY TABLE—*Continued.*

Age	D_x	N_x	S_x	C_x	M_x	R_x
39	20030	314347	3864718	273.54	9400.11	183655.77
40	19080	294317	3550371	269.90	9126.57	174255.66
41	18164	275237	3256054	266.20	8856.67	165129.09
42	17284	257073	2980817	262.89	8590.47	156272.42
43	16437	239789	2723744	260.38	8327.58	147681.95
44	15620	223352	2483955	257.95	8067.20	139354.37
45	14834	207732	2260603	256.01	7809.25	131287.17
46	14076	192898	2052871	254.50	7553.24	123477.92
47	13346	178822	1859973	253.18	7298.74	115924.68
48	12641	165476	1681151	252.03	7045.56	108625.94
49	11962	152835	1515675	251.39	6793.53	101580.38
50	11306	140873	1362840	250.85	6542.14	94786.85
51	10673	129567	1221967	250.55	6291.29	88244.71
52	10061	118894	1092400	250.32	6040.74	81953.42
53	9470.7	108833.2	973505.8	250.12	5790.42	75912.68
54	8900.4	99362.5	864672.6	250.11	5540.30	70122.26
55	8349.3	90462.1	765310.1	249.95	5290.19	64581.96
56	7817.0	82112.8	674848.0	249.94	5040.24	59291.77
57	7302.6	74295.8	592735.2	249.52	4790.30	54251.53
58	6806.3	66993.2	518439.4	248.96	4540.78	49461.23
59	6327.2	60186.9	451446.2	248.16	4291.82	44920.45
60	5865.0	53859.7	391259.3	246.88	4043.66	40628.63
61	5419.8	47994.7	337399.6	245.40	3796.78	36584.97
62	4991.1	42574.9	289404.9	243.06	3551.38	32788.19
63	4579.3	37583.8	246830.0	240.26	3308.32	29236.81
64	4184.2	33004.5	209246.2	236.73	3068.06	25928.49
65	3805.9	28820.3	176241.7	232.44	2831.33	22860.43
66	3444.8	25014.4	147421.4	227.17	2598.89	20029.10
67	3101.1	21569.6	122407.0	221.03	2371.72	17430.21
68	2775.2	18468.5	100837.4	213.93	2150.69	15058.49
69	2467.5	15693.3	82368.9	205.89	1936.76	12907.80
70	2178.2	13225.8	66675.6	196.67	1730.87	10971.04
71	1907.8	11047.6	53449.8	186.57	1534.20	9240.17
72	1656.7	9139.8	42402.2	175.55	1347.63	7705.97
73	1425.1	7483.1	33262.4	163.58	1172.08	6358.34
74	1213.3	6058.0	25779.3	150.85	1008.50	5186.26
75	1021.5	4844.7	19721.3	137.62	857.65	4177.76
76	849.30	3823.16	14876.62	123.99	720.03	3320.11
77	696.59	2973.86	11053.46	110.16	596.04	2600.08
78	562.89	2277.27	8079.60	96.396	485.876	2004.039
79	447.46	1714.38	5802.33	82.930	389.480	1518.163
80	349.39	1266.92	4087.95	70.079	306.550	1128.683
81	267.50	917.53	2821.03	58.062	236.471	822.133
82	200.39	650.03	1903.50	46.950	178.409	585.662
83	146.66	449.64	1253.47	37.136	131.459	407.253
84	104.57	302.98	803.83	28.575	94.323	275.794
85	72.457	198.407	500.845	21.329	65.748	181.471
86	48.678	125.950	302.438	15.443	44.419	115.723
87	31.589	77.272	176.488	10.755	28.976	71.304
88	19.766	45.683	99.216	7.2550	18.2207	42.3279
89	11.842	25.917	53.533	4.6581	10.9657	24.1072
90	6.7836	14.0754	27.6156	2.8401	6.3076	13.1415
91	3.7141	7.2918	13.5402	1.6465	3.4675	6.8339
92	1.9420	3.5777	6.2484	.93817	1.82102	3.36635
93	.93817	1.63565	2.67068	.47292	.88285	1.54533
94	.43351	.69748	1.03503	.22847	.40993	.66248
95	.19039	.26397	.33755	.11037	.18146	.25255
96	.073580	.073580	.073580	.071092	.071092	.071092

TROPICAL MORTALITY TABLE. INTEREST 4%.

Age	D_x	N_x	S_x	C_x	M_x	R_x
15	55526	1046105	16983874	514.70	15291.79	392880.12
16	52876	990579	15937769	492.83	14777.09	377588.33
17	50349	937703	14947190	471.91	14284.26	362811.24
18	47941	887354	14009487	452.81	13812.35	348526.98
19	45645	839413	13122133	434.02	13359.54	334714.63
20	43455	793768	12282720	416.46	12925.52	321355.09
21	41368	750313	11488952	399.60	12509.06	308429.57
22	39377	708945	10738639	384.22	12109.46	295920.51
23	37477	669568	10029694	369.05	11725.24	283811.05
24	35667	632091	9360126	355.24	11356.19	272085.81
25	33941	596424	8728035	341.93	11000.95	260729.62
26	32292	562483	8131611	329.82	10659.02	249728.67
27	30721	530191	7569128	317.80	10329.20	239069.65
28	29222	499470	7038937	306.86	10011.40	228740.45
29	27791	470248	6539467	296.61	9704.54	218729.05
30	26425	442457	6069219	287.27	9407.93	209024.51
31	25122	416032	5626762	277.93	9120.66	199616.58
32	23878	390910	5210730	269.71	8842.73	190495.92
33	22690	367032	4819820	261.71	8573.02	181653.19
34	21555	344342	4452788	254.43	8311.31	173080.17
35	20472	322787	4108446	247.81	8056.88	164768.86
36	19437	302315	3785659	241.56	7809.07	156711.98
37	18448	282878	3483344	235.87	7567.51	148902.91
38	17502	264430	3200466	230.70	7331.64	141335.40
39	16598	246928	2936036	225.58	7100.94	134003.76
40	15734	230330	2689108	221.51	6875.36	126902.82
41	14907	214596	2458778	217.42	6653.85	120027.46
42	14117	199689	2244182	213.69	6436.43	113373.61
43	13360	185572	2044493	210.63	6222.74	106937.18
44	12635	172212	1858921	207.66	6012.11	100714.44
45	11942	159577	1686709	205.11	5804.45	94702.33
46	11278	147635	1527132	202.92	5599.34	88897.88
47	10641	136357	1379497	200.90	5396.42	83298.54
48	10031	125716	1243140	199.03	5195.52	77902.12
49	9446.0	115684.7	1117424.2	197.56	4996.49	72706.60
50	8885.1	106238.7	1001739.5	196.19	4798.93	67710.11
51	8347.2	97353.6	895500.8	195.02	4602.74	62911.18
52	7831.1	89006.4	798147.2	193.89	4407.72	58308.44
53	7335.8	81175.3	709140.8	192.81	4213.83	53900.72
54	6861.0	73839.5	627965.5	191.88	4021.02	49686.89
55	6405.3	66978.5	554126.0	190.83	3829.14	45665.87
56	5968.0	60573.2	487147.5	189.91	3638.31	41836.73
57	5548.6	54605.2	426574.3	188.67	3448.40	38198.42
58	5146.6	49056.6	371969.1	187.34	3259.73	34750.02
59	4761.2	43910.0	322912.5	185.84	3072.39	31490.29
60	4392.3	39148.8	279002.5	184.00	2886.55	28417.90
61	4039.3	34756.5	239853.7	182.02	2702.55	25531.35
62	3701.9	30717.2	205097.2	179.41	2520.53	22828.80
63	3380.2	27015.3	174380.0	176.49	2341.12	20308.27
64	3073.7	23635.1	147364.7	173.06	2164.63	17967.15
65	2782.3	20561.4	123729.6	169.11	1991.57	15802.52
66	2506.3	17779.1	103168.2	164.49	1822.46	13810.95
67	2245.4	15272.8	85389.1	159.27	1657.97	11988.49
68	1999.7	13027.4	70116.3	153.41	1498.70	10330.52

TROPICAL MORTALITY TABLE—*Continued.*

Age	D_x	N_x	S_x	C_x	M_x	R_x
69	1769.4	11027.7	57088.9	146.94	1345.29	8831.82
70	1554.5	9258.3	46061.2	139.68	1198.35	7486.53
71	1355.0	7703.8	36802.9	131.87	1058.67	6288.18
72	1171.0	6348.8	29099.1	123.49	926.80	5229.51
73	1002.5	5177.8	22750.3	114.51	803.31	4302.71
74	849.38	4175.25	17572.47	105.09	688.80	3499.40
75	711.62	3325.87	13397.22	95.418	583.712	2810.601
76	588.84	2614.25	10071.35	85.548	488.294	2226.889
77	480.64	2025.41	7457.10	75.643	402.746	1738.595
78	386.52	1544.77	5431.69	65.875	327.103	1335.849
79	305.78	1158.25	3886.92	56.399	261.228	1008.746
80	237.61	852.47	2728.67	47.431	204.829	747.518
81	181.05	614.86	1876.20	39.108	157.398	542.689
82	134.98	433.81	1261.34	31.472	118.290	385.291
83	98.311	298.829	827.528	24.773	86.818	267.001
84	69.757	200.518	528.699	18.971	62.045	180.183
85	48.104	130.761	328.181	14.092	43.074	118.138
86	32.161	82.657	197.420	10.154	28.982	75.064
87	20.770	50.496	114.763	7.0375	18.8279	46.0815
88	12.934	29.726	64.267	4.7245	11.7904	27.2536
89	7.7117	16.7917	34.5407	3.0188	7.0659	15.4632
90	4.3963	9.0800	17.7490	1.8318	4.0471	8.3973
91	2.3955	4.6837	8.6690	1.0568	2.2153	4.3502
92	1.2465	2.2882	3.9853	.59928	1.15850	2.13488
93	.59928	1.04165	1.69713	.30064	.55922	.97638
94	.27559	.44237	.65548	.14454	.25858	.41716
95	.12045	.16678	.21311	.069490	.114035	.158580
96	.046327	.046327	.046327	.044545	.044545	.044545

TROPICAL MORTALITY TABLE.

INTEREST 3%.

Age	a_x	A_x	P_x	Age	a_x	A_x	P_x
15	21.01235	.358866	.016303	36	16.46265	.491373	.028139
16	20.85367	.363501	.016633	37	16.17858	.499647	.029085
17	20.68904	.368274	.016980	38	15.68870	.508101	.030085
18	20.51953	.373214	.017343	39	15.59271	.516715	.031141
19	20.34492	.378306	.017723	40	15.29097	.525517	.032258
20	20.16421	.383552	.018123	41	14.98398	.534475	.033438
21	19.97837	.388971	.018542	42	14.67085	.543574	.034687
22	19.78691	.394565	.018981	43	14.35290	.552850	.036009
23	19.58931	.400314	.019443	44	14.03011	.562266	.037409
24	19.38549	.406237	.019928	45	13.70172	.571792	.038893
25	19.17592	.412335	.020437	46	13.36932	.581464	.040466
26	18.96069	.418621	.020972	47	13.03312	.591265	.042134
27	18.73903	.425070	.021534	48	12.69340	.601191	.043904
28	18.51110	.431708	.022126	49	12.34874	.611160	.045782
29	18.27715	.438531	.022749	50	12.00201	.621257	.047782
30	18.03702	.445539	.023404	51	11.65293	.631447	.049905
31	17.79000	.452703	.024093	52	11.30212	.641722	.052164
32	17.53706	.460074	.024819	53	10.94837	.651985	.054567
33	17.27821	.467631	.025584	54	10.59329	.662289	.057127
34	17.01293	.475374	.026391	55	10.23878	.672688	.059854
35	16.74064	.843279	.027241	56	9.88295	.683034	.062762

TROPICAL MORTALITY TABLE—*Continued.*

Age	a_x	A_x	P_x	Age	a_x	A_x	P_x
57	9.52778	.693370	.065861	77	3.32608	.874006	.202032
58	9.17333	.703689	.069170	78	3.09620	.880680	.215000
59	8.82029	.713970	.072704	79	2.87616	.887102	.228861
60	8.46922	.724187	.076478	80	2.66562	.893241	.243681
61	8.12084	.734355	.080514	81	2.46487	.899081	.259485
62	7.77568	.744402	.084826	82	2.27442	.904626	.276270
63	7.43412	.750360	.089442	83	2.09260	.909922	.294225
64	7.09668	.764172	.094381	84	1.92079	.914918	.313243
65	6.76430	.773862	.099669	85	1.75860	.919623	.333366
66	6.43730	.783379	.105331	86	1.60511	.924130	.354737
67	6.11619	.792738	.111399	87	1.46154	.928312	.377127
68	5.80145	.801913	.117903	88	1.32447	.932305	.401083
69	5.49347	.810866	.124874	89	1.19991	.935900	.425426
70	5.19323	.819627	.132342	90	1.08466	.939298	.450576
71	4.90021	.828127	.140355	91	.97149	.942589	.478109
72	4.61590	.836455	.148944	92	.84895	.946143	.511719
73	4.33984	.844459	.158143	93	.74883	.949039	.542671
74	4.07268	.852243	.168006	94	.61270	.953045	.590963
75	3.81441	.859777	.178584	95	.38836	.959582	.691161
76	3.56535	.867006	.189910	96		.970874	.970874

TROPICAL MORTALITY TABLE. INTEREST 3½%.

Age	a_x	A_x	P_x	Age	a_x	A_x	P_x
15	19.31698	.312968	.015404	46	12.70404	.536604	.039157
16	19.18755	.317339	.015720	47	12.39892	.546886	.040816
17	19.05376	.321877	.016051	48	12.09042	.557358	.042578
18	18.91453	.326574	.016399	49	11.77671	.567926	.044450
19	18.77062	.331435	.016764	50	11.46002	.578643	.046440
20	18.62144	.336470	.017148	51	11.13970	.589458	.048556
21	18.46722	.341685	.017552	52	10.81731	.600411	.050808
22	18.30790	.347091	.017977	53	10.49157	.611404	.053205
23	18.14260	.352664	.018423	54	10.16382	.622478	.055758
24	17.97178	.358435	.018893	55	9.83469	.633609	.058480
25	17.79559	.364399	.019387	56	9.50439	.644779	.061382
26	17.61383	.370565	.019908	57	9.17388	.655972	.064476
27	17.42581	.376912	.020456	58	8.84282	.667144	.067780
28	17.23189	.383471	.021033	59	8.51241	.678313	.071308
29	17.03148	.390223	.021641	60	8.18324	.689456	.075078
30	16.82571	.397197	.022282	61	7.85544	.700539	.079108
31	16.61364	.404362	.022957	62	7.53016	.711545	.083415
32	16.39546	.411751	.023670	63	7.20732	.722451	.088025
33	16.17082	.419336	.024421	64	6.88789	.733249	.092959
34	15.94010	.427144	.025215	65	6.57253	.743932	.098241
35	15.70313	.435162	.026053	66	6.26150	.754439	.103896
36	15.45979	.443381	.026937	67	5.95547	.764800	.109957
37	15.21003	.451803	.027872	68	5.65484	.774968	.116452
38	14.95537	.460465	.028860	69	5.36000	.784908	.123413
39	14.69381	.469302	.029904	70	5.07189	.794633	.130871
40	14.42542	.478332	.031009	71	4.79075	.804172	.138872
41	14.15288	.487595	.032178	72	4.51687	.813442	.147446
42	13.87347	.497019	.033416	73	4.25093	.822455	.156630
43	13.58837	.506636	.034729	74	3.99299	.831204	.166474
44	13.29910	.516466	.036119	75	3.74273	.839599	.177029
45	13.00378	.526443	.037593	76	3.50154	.847792	.188334

TROPICAL MORTALITY TABLE—*Continued.*

Age	a_x	A_x	P_x	Age	a_x	A_x	P_x
77	3.26917	.855654	.200426	87	1.44617	.917281	.374987
78	3.04567	.863181	.213359	88	1.31119	.921820	.398851
79	2.83136	.870424	.227184	89	1.18857	.926001	.423108
80	2.62609	.877386	.241965	90	1.07492	.929831	.448129
81	2.43002	.884004	.257726	91	.96328	.933604	.475534
82	2.24382	.890309	.274463	92	.84228	.937703	.508992
83	2.06587	.896352	.292365	93	.74345	.941034	.539755
84	1.89739	.902008	.311318	94	.60891	.945607	.587730
85	1.73827	.907407	.331379	95	.38647	.953096	.687427
86	1.58741	.912507	.352672	96		.966184	.966184

TROPICAL MORTALITY TABLE. INTEREST 4%.

Age	a_x	A_x	P_x	Age	a_x	A_x	P_x
15	17.83991	.275399	.014618	56	9.14966	.609636	.060065
16	17.73400	.279467	.014918	57	8.84126	.621490	.063151
17	17.62406	.283705	.015233	58	8.53185	.633375	.066448
18	17.50929	.288111	.015566	59	8.22246	.645297	.069970
19	17.39003	.292684	.015915	60	7.91305	.657184	.073733
20	17.26644	.297446	.016284	61	7.60458	.669064	.077757
21	17.13752	.302385	.016672	62	7.29768	.680875	.082056
22	17.00404	.307526	.017081	63	6.99222	.692598	.086659
23	16.86610	.312865	.017512	64	6.68946	.704242	.091585
24	16.72201	.318395	.017966	65	6.39007	.715800	.096860
25	16.57238	.324120	.018445	66	6.09376	.727152	.102506
26	16.41865	.330082	.018950	67	5.80182	.738385	.108557
27	16.25826	.336226	.019482	68	5.51468	.749462	.115042
28	16.09226	.342598	.020044	69	5.23245	.760309	.121992
29	15.92087	.349197	.020637	70	4.95581	.770891	.129435
30	15.74388	.356024	.021263	71	4.68546	.781306	.137422
31	15.56046	.363055	.021923	72	4.42169	.791460	.145980
32	15.37114	.370330	.022621	73	4.16489	.801307	.155145
33	15.17594	.377833	.023358	74	3.91564	.810944	.164972
34	14.97504	.385586	.024137	75	3.67366	.820258	.175507
35	14.76724	.393556	.024960	76	3.43966	.829247	.186782
36	14.55358	.401763	.025831	77	3.21399	.837937	.198847
37	14.33380	.410208	.026752	78	2.99661	.846277	.211749
38	14.10856	.418903	.027726	79	2.78785	.854300	.225537
39	13.87697	.427819	.028757	80	2.58769	.862039	.240277
40	13.63900	.436975	.029850	81	2.39608	.869362	.255990
41	13.39565	.446357	.031006	82	2.21388	.876352	.272677
42	13.14529	.455935	.032232	83	2.03963	.883095	.290527
43	12.89012	.465774	.033533	84	1.87452	.889445	.309424
44	12.62976	.475830	.034911	85	1.71830	.895435	.329410
45	12.36267	.486053	.036374	86	1.57010	.901154	.350630
46	12.09053	.496483	.037927	87	1.43120	.906495	.372859
47	11.81430	.507135	.039576	88	1.29828	.911582	.396636
48	11.53275	.517946	.041327	89	1.17743	.916257	.420797
49	11.24695	.528953	.043191	90	1.06537	.920570	.445716
50	10.95695	.540110	.045171	91	.95521	.924776	.472981
51	10.66302	.551411	.047279	92	.83570	.929402	.506293
52	10.36576	.562848	.049521	93	.73817	.933153	.536860
53	10.06564	.574420	.051910	94	.60517	.938278	.584533
54	9.76221	.586069	.054456	95	.38464	.946741	.683745
55	9.45673	.597808	.057170	96		,961538	.961538

(Presented by Mr. Frank Sanderson.)

Misstatements That Avoid the Policy.

J. M. LANGSTAFF.

Introduction.—It might be necessary to make some prefatory explanation in submitting a paper on the above subject, were it not that this society has never confined its interest to the strictly mathematical problems that concern the Actuary. The attention that has been given in a number of previous papers to various legal aspects of life insurance is itself sufficient to suggest the many points of contact between the Actuarial profession and the Law. The recodification of the Insurance Acts of Canada and many of the United States, consequent upon recent upheavals in the insurance world, has also served to emphasize the importance to the Actuary of possessing some acquaintance with the Common and Statute Law relating to his business. Moreover the Insurance Laws of the United States and Canada have been part of the Society's examination syllabus for some years, and it was thought that a paper like the present might be of some assistance to students.

The particular subject that I have chosen is of considerable practical importance, and, being self-contained, lends itself readily to treatment in an isolated paper. I had anticipated that some difficulty would be experienced in dealing with Canadian (*i. e.*, British) and American Law in the same paper. But on the whole the two systems are almost parallel. The most noticeable divergence is, perhaps, the tendency of American Courts to somewhat greater liberality in favour of maintaining the contract in force.

In writing this paper, I have aimed merely at stating main principles, and have tried to avoid being drawn aside from the main path of the argument and obscuring the subject by the discussion of minor points. With the same object in view, comparatively few of the hundreds of cases read and scanned have been cited, and those, the decisions bearing most directly on the points sought to be illustrated.

How policy may be avoided.—Leaving lapse out of consideration, there are three ways in which a policy may be avoided :

1. Breach of warranty.
2. Misrepresentation.
3. Concealment.

Warranty.

Express and implied.—Warranties are express or implied. Implied warranties, however, are almost if not altogether confined to marine insurance ; and for the purposes of the present paper may be passed over.

Definition.—An express warranty is a particular stipulation inserted on the face of the policy or clearly embodied therein as a part thereof by proper words of reference, whereby the insured agrees that certain facts are or shall be true, and upon the literal truth or exact fulfilment of which stipulation the validity of the contract depends.[1] By a warranty the insured guarantees the exact truth of his statements, upon penalty of forfeiting his right to recover under the policy. A warranty is a condition precedent and as such must be strictly complied with.

Unintentional breach of warranty fatal.—A breach of warranty is fatal, even though the insured acted in perfect good faith. The insured is estopped from claiming that a statement which he has declared shall be a warranty was made by mistake or in ignorance.[2]

Thus, in a leading American case[3] the application for insurance contained the following question, "Has the party or any of his family been afflicted with pulmonary complaints, consumption, or spitting of blood?" to which the applicant answered in the negative. By the terms of the policy, the answers in the application were made warranties. The applicant for insurance died a few months later of consumption, and the fact was established that, at the time of making application, he was in a decline. The Court held that there was a

[1] Joyce, section 1944.

[2] McGowan *vs.* Supreme Court, I. O. F., 107 Wis., 462; Commonwealth Mutual *vs.* Huntzinger, 98 Pa. St., 41; Foot *vs.* Ætna Life, 71 N. Y., 571; Prov. Savings *vs.* Llewellyn, 58 Fed. Rep., 940; Cazenove *vs.* British Equitable, 6 C. B. N. S., 437; Macdonald *vs.* Law Union, L. R., 9 Q. B., 328,

[3] Vose *vs.* Eagle Life, 6 Cush., 42.

breach of warranty and decided for the Defendants, saying, "It is immaterial that the insured did not suppose himself in an consumption ; the fact was so, and the statement was manifestly contrary to the fact . . . ."

Similarly in a well-known English case.[1] Here the Plaintiff had effected with the Defendants' office an insurance on the life of another person. In the policy was a proviso "that if the declaration under the hand of J. P. Macdonald delivered at the Defendants' office as the basis of the insurance is not in every respect true and if there has been any misrepresentation . . . . . then the insurance shall be void." The application was signed by the Plaintiff, and among the questions contained therein was the following : "Has the life been proposed for insurance at this or any other office or offices? If so, at what offices?" The Plaintiff answered "No." The evidence showed that the answer to this question was untrue in fact, but not untrue to the knowledge of the Plaintiff. The Court decided that the policy was void, Cockburn, C. J. remarking, "The terms would prima facie and naturally import, in the ordinary use of language, that the policy is vitiated if the representation, made as preliminary to the contract, was not in point of fact true. Whether untrue to the knowledge of the party proposing the life is to them (i. e. to the insurers) a matter of very little importance."

Warranty need not be of a material fact.—Whether the statement made is *ipso facto* material to the risk or not is of no consequence.[2] In the eyes of the Law, the mere fact that the statement has been formally made is evidence that in the opinion of the contracting parties it was of some weight. "It is competent to the contracting parties, if both agree to it and sufficiently express their intention so to agree, to make the actual existence of anything a condition precedent to the inception of any contract : and if they do so, the non-existence of that thing is a good defense. And it is not of

[1] Macdonald *vs.* Law Union, L. R., 9 Q. B., 328.

[2] Newcastle *vs.* Macmorran, 3 Dow., 255; Burritt *vs.* Saratoga Mutual, 5 Hill (N. Y.), 188; Supreme Lodge *vs.* McLaughlin, 108 Ill. App. 85; Weil *vs.* New York Life, 47 La. Ann., 1405; Miles *vs.* Conn. Mutual Life, 3 Gray (Mass.), 580; Dwight *vs.* Germania Life, 103 N. Y., 341; also, Lord Blackburn in Bettini *vs.* Gye, 1, Q. B. Div., 187, "Parties may think some matter, apparently of trivial importance, essential; and if they sufficiently express an intention to make the literal fulfillment of such a thing a condition precedent, it will be one."

any importance whether the existence of that thing was or was not material ; the parties would not have made it a part of the contract if they had not thought it material, and they have a right to determine for themselves what they shall deem material." [1]

Thus, in a leading British case,[2] the policy contained a condition that "in case any fraudulent or untrue statement was contained in any of the documents addressed to or deposited with the Company in relation to the within assurance, whether by the payee, the assured, or any referee or other person, then the policy should be void." In the suit on the policy after the death of the assured it was proved that several misstatements had been made. The Court held that the untruth of the answers avoided the policy, notwithstanding the Jury found that no material information had been withheld from the insurers and it was conceded that there was no intentional fraud.

It has even been held by a United States Court that if a married man represents himself as single there is a breach of the warranty.[3] And in another case, where the insured was married and stated that he was a widower, the correctness of his answers being warranted, the misstatement avoided the policy.[4]

A breach of warranty avoids a contract without regard to the fact whether such breach did or did not actually contribute to the loss. It is immaterial that the breach did not occasion or was not connected with the loss. The avoidance of the policy is based on other grounds, namely, that the warranty is a condition precedent upon which the validity of the contract depends. This principle is illustrated in a Canadian fire insurance case,[5] where the insured agreed to keep twelve pails full of water on each flat of the building during the continuance of the policy. He failed to do so and the property was burned, but it appeared that the loss was not in any way affected by his default, as the fire was not discovered until it was so far advanced as to make it impossible to enter the building. Held, nevertheless, that he could not recover.

[1] Lord Blackburn in Thomson *vs.* Weems, 9 Appl. Cases, 683.

[2] Cazenove *vs.* British Equitable, 6 C. B. N. S., 437.

[3] Jeffries *vs.* Union, 1 Fed. Rep., 450, 1 McCrary, 1147.

[4] United Brethren *vs.* White, 100 Pa. St., 12.

[5] Garrett *vs.* Provincial Ins. Co., 20 U. C. Q. B., 200; see also Maynard *vs.* Rhode, 1 Car. and P., 360; Goicoechea *vs.* Louisiana Ins. Co., 17 Am. Dec., 175..

The hard and fast nature of a warranty appears unjust at first sight; but, as Bunyon points out, there is no real hardship necessarily involved in the strict interpretation of the doctrine, because no person is obliged to state as a fact that which is not actually within his knowledge, and if he does so it must be at his peril. When he does not possess positive information, he should preface it with some such words as "to the best of my belief," in which case the warranty is of the belief and not of the fact, and it is sufficient that he believes it to be true.[1]

What constitutes a warranty.—No particular form of words is necessary to constitute a warranty. "There is no magic in the word *warranty*."[2] Any statement upon the literal truth of which in the intention of the parties the validity of the contract is made to depend, whether the exact word *warranty* is used or not, amounts to a warranty.[3] But no particular form of words, not even the use of the expression *warranty* will render a statement or stipulation a warranty if it is apparent from the context or from other parts of the contract that the parties did not intend to make the validity of the contract depend on the literal truth or fulfilment of the statement or stipulation.[4]

In the definition previously given it is stated that a warranty must be inserted on the face of the policy or clearly embodied therein *as a part thereof.*[5] This raises the question,—What constitutes a part of the contract? It is around this point that the controversy has raged in many of the reported cases;[6] for if statements are not a part of

[1] Pawson *vs.* Watson, Cowper, 788, per Lord Mansfield; Clapp *vs.* Mass. Benefit, 146 Mass., 519; Miller *vs.* Confederation Life, 11 Ont. R., 120, 14, S. C. R., 330; Waterbury *vs.* Dakota F. and M. Ins. C., 6 Dak., 468.

[2] Moulor *vs.* American Life, 111 U. S., 342; Clapp *vs.* Mass. Benefit Assn., 146 Mass., 519; Coöperative Life *vs.* Leflore, 53 Miss., 1.

[3] Wright *vs.* Equitable Life, 5 Big. Life and Acc. Ins. cas. 401.

[4] National Bank *vs.* Union Ins. Co., 88 Cal., 496; Clapp *vs.* Mass. Benefit Assn., *supra.*

[5] Burritt *vs.* Saratoga Mut. Ins. Co., 5 Hill (N. Y.), 188; Pawson *vs.* Watson, Cowp., 785; Pawson *vs.* Barnwelt, Doug., 12 N; Gorham *vs.* Sweeting, 2 Saund., 200 b. n.; Jefferson Ins., Co. *vs.* Cotheal, 7 Wend., 72, 22 Am. Dec., 567; Mut. Benefit Life *vs.* Robertson, 59 Ill., 123; Lothian *vs.* Henderson, 3 Bos. and Pull., 499.

[6] Statutory enactments in Canada and in many of the United States have now obviated the difficulty so often experienced formerly in distinguishing between representations and warranties. See page 485 hereof.

the contract, they are not warranties but only representations, which are construed far less strictly than warranties as will presently be shown.

In order to make the statements contained in the application, medical examination, and the like warranties, they must be so referred to as constituting a part of a contract as to clearly evidence an intention that the parties deemed them a part of the policy. The words of reference must be clear and explicit.[1] Statements contained in a paper annexed to the policy but not referred to therein are not warranties.[2] But an entry on the margin of a policy or on a paper referred to in the policy is construed as a warranty.[3] Where the application was specifically referred to in the policy "as forming a part of this policy" and it was stipulated that any misrepresentation or concealment in the application should render the insurance void ;[4] where it was stated in the policy that the same was issued "in consideration of the representations made in the application for the same, which is hereby made a part of the policy it being also understood and agreed that if the representations made in the application for this policy and upon the faith of which this policy is issued should be found in any respect untrue then this policy shall be null and void ;"[5] where the policy stipulated "that the answers contained in the application, which application is hereby referred to and made part of this contract, are warranted by the insured to be true in all respects, and if this policy has been obtained through any fraud, misrepresentation or concealment that this policy shall be absolutely null and void ;"[6] in these cases the answers in the application were deemed warranties. The above examples will serve as illustrations of the explicit language which must be used in order to constitute a warranty.

If there exists any doubt as to whether a statement is an express

[1] Life Assocn. of Scotland *vs.* Foster, 11 C. S. C. (3rd series), 359; Phoenix Life *vs.* Raddin, 120 U. S., 183; Higbee *vs.* Guardian Mut. Life, 66 Barb. (N. Y.), 462; Burritt *vs.* Saratoga Mut. Ins. Co., 5 Hill (N. Y.), 188.

[2] Pawson *vs.* Barnvelt and Bize *vs.* Fletcher, 1 Doug., 12. Both in the Notes. Goddard *vs.* Texas Fire Ins. Co., 67 Tex. 69.

[3] Coöperative Life *vs.* Leflore, 53 Miss., 1; Routledge *vs.* Burrell, 1 H. Bl., 254.

[4] Burritt *vs.* Saratoga, 5 Hill (N. Y.), 188.

[5] Higbee *vs.* Guardian Life, 66 Barb. (N. Y.), 462.

[6] Conn. Mut. Life *vs.* Pyle, 44 Ohio St., 19.

warranty, the Court will lean against that construction which imposes a warranty on the insured.[1] Warranties are not favored at law[2] and statements in an application for a policy of insurance will not be construed as warranties unless the provisions of the application and the policy taken together leave no room for any other construction.[3] Even where the statements in the application were declared to be warranties, it was held they were not to be regarded as such if qualified by other stipulations which afforded a fair inference that the parties themselves did not so intend them.[4] "It was early held, with special reference to contracts of marine insurance that the *scriptum jus* or *apex juris* is not to be laid hold on, but they are to be construed largely for the benefit of trade and for the insured.[5] Having indemnity for its object, the contract is to be construed liberally to that end, and it is presumably the intention of the insurer that the insured shall understand that in case of loss he is to be protected to the full extent which any fair interpretation will give.[6] The spirit of the rule is that where two interpretations equally fair may be given, that which gives the greater indemnity shall prevail, and to the same spirit is due the rule that conditions and provisoes will be strictly construed against the insurers because they have for their object to limit the scope and defeat the purpose of the principal contract; and apparently contradictory clauses will be so construed if possible as to reconcile them with each other and to give to each its due force in furtherance of the main purpose of the contract."[7]

"The insurers may stipulate for any warranty they please and if the insured undertakes that warranty, although it may be something not within her or his knowledge, he or she must abide the consequences. But when the insurers intend that there is a warranty of that sort, they must make it very plain that such is their intention. They must

[1] Wilson *vs.* Conway Ins. Co., 4 R. I., 141; Wall *vs.* Howard Ins. Co., 14 Barb. (N. Y.), 383; National Bank *vs.* Ins. Co., 95 U. S. 673; Clapp *vs.* Mass. Benefit Assocn., 146 Mass., 519; National Bank *vs.* Union Ins. Co., 88 Cal., 497.

[2] Masons Union *vs.* Broademan, 20 Ind. App., 206.

[3] Joyce, Sec. 1960; see also Fowkes *vs.* Manchester, 3 B. and S., 917.

[4] Wheaton *vs.* North British, 76 Cal., 415.

[5] Tierney *vs.* Etherington per Lee Ch. J., cited, 1 Burrows, 349.

[6] Dow *vs.* Hope Ins. Co., 1 Hall (N. Y. Sup.), 166.

[7] May, Sec. 174.

use unequivocal language such as persons of ordinary intelligence may without difficulty understand." [1]

In other words, the view of the Courts is that, as the language of the conditions is framed by the insurers, and as it is therefore in their power to provide for every proper case, the contract is to be construed most favorably to the insured.[2]

The apparently inequitable effect, in numerous instances, of the strict interpretation of warranties is no doubt responsible for the repugnance evinced by many Courts to recognizing warranties when evasion is possible. It is to be feared, however, that, in at least a few instances, the laudable desire of the Judges to succour the oppressed (the insured) and rebuke the oppressor (*i. e.* the insurer) has led them into decisions not absolutely consonant with good law. An examination of the cases on the subject reveals many arbitrary decisions which can hardly be reconciled with recognized legal principles, and these cases should not be relied upon as furnishing precedents of weight.

Latent disease.—Where the applicant is unaware of the presence of hidden disease and in ignorance denies the fact, there seems to be in both British and American Courts a disposition to relax the strict definition of warranty. It has been held that statements, in fact untrue, as to the non-existence of latent disease do not amount to a breach of warranty so as to avoid a policy granted upon such statements.[3]

Thus, in one case, the proposer was in fact afflicted with disease of the liver (resulting in dropsy of which she subsequently died) when she declared herself to be in ordinarily good health. The Court held that her declaration only implied a warranty that she was, according to her knowledge and reasonable belief, free from any disease or symptom of disease material to the risk.[4]

[1] Life Assocn. of Scotland *vs.* Foster, 11 C. S. C. (3rd series), 364, per Lord Deas.

[2] Penn. Ins. Co. *vs.* Mechanics, 37 U. S. App., 706, per Taft, J.; National Bans *vs.* Ins. Co., 95 U. S., 673; Life Assocn. of Scotland *vs.* Foster, 11 C. S. C. (3rd series), 359; Fowkes *vs.* Manchester, 3 Best and Smith, 917; Wall *vs.* Howard Ins. Co., 14 Barb. N. Y., 383.

[3] Bunyon, page 78, Hutchinson *vs.* National Loan, 7 C. S. C. (2nd series), 467; Life Assocn. of Scotland *vs.* Foster, *supra;* see also Swete *vs.* Fairlie, 6 C. and P., I.

[4] Hutchinson *vs.* National Loan, 7 C. S. C. (2nd series), 467.

Similarly where there was a warranty that the insured was in good health, Lord Mansfield charged the Jury that such a warranty could never mean that a man has not the seeds of disorder. "We are," he said, "all born with the seeds of mortality in us." [1]

So also in the case of Supreme Lodge K. H. *vs.* Dickson,[2] where the Court said "It is true that any statement made of a material fact which forms the basis of the contract must be considered as a warranty, and, if false, will vitiate the contract, whether made in good faith, though ignorantly, or wilfully and with knowledge of its falsity. But there is a difference between statements of fact as such, and statements of opinion on matters where only opinion can be expressed. Falsehood may be predicated of a misstatement of fact, but not of a mistaken opinion as to whether a man has a disease, when it is latent and can only be a matter of opinion."

Representations and Misrepresentations.

Definition.—"A representation is an oral [3] or written statement which precedes the contract of insurance and is no part thereof [4] unless it be otherwise stipulated, made by the assured or his authorized agent to the underwriter or his authorized agent, and relates to facts necessary to enable the underwriter to form his judgment whether he will accept the risk and at what premium." [5] If the representation is false and material to the risk the policy is avoided.[6] Such a false statement is termed in insurance a "misrepresentation," which has been defined as the statement of something as a fact which is untrue in fact and which the insured states, knowing it to be untrue, with the intent to deceive the insurers, or which he states positively as true without knowing it to be true, and which has a tendency to mislead; such fact in either case, being material to the risk and adverse to the insurers.[7]

Distinction between representations and warranties.—It does not seem

[1] Reported in 2 Parke, 935; see also Ross *vs.* Bradshaw, 1 Black Rep., 314.

[2] 102 Tenn., 255.

[3] Mut. Benefit Life *vs.* Robertson, 59 Ill., 123.

[4] Burritt *vs.* Saratoga Mut. Ins. Co., 5 Hill (N. Y.), 188.

[5] Joyce, sec. 1883.

[6] Higbee *vs.* Guardian Mutual Life, 66 Barb. (N. Y.), 462.

[7] May, sec. 181; Daniels *vs.* Hudson River, 12 Cush., 417.

possible to lay down any clear positive rule which will invariably determine whether a statement is a representation or a warranty.[1] "The former is a part of the proceedings which propose a contract, while the latter is a part of the completed contract, either expressly inserted therein, or appearing therein by express reference to statements expressly made a part thereof. The falsity of the former may render the contract voidable; but a non-compliance with the latter is an express breach of the contract."[2] "A warranty in insurance," says the Supreme Court of Massachusetts,[3] "enters into and forms a part of the contract itself. It defines by ways of particular stipulation, description, condition or otherwise, the precise limits of the obligation which the insurers undertake to assume. No liability can arise except within those limits. In order to charge the insurers, therefore, every one of the terms which define their obligation must be satisfied by the facts which appear in proof. From the very nature of the case, the party seeking his indemnity or payment under the contract, must bring his claim within the provisions of the instrument he is undertaking to enforce. The burden of proof is upon the plaintiff to present a case in all respects conforming to the terms under which the risk was assumed. It must be not merely a substantial conformity, but exact and literal; not only in material particulars, but in those that are immaterial as well. A representation is, on the other hand, in its nature no part of the contract of insurance.[4] Its relation to the contract is usually described by the term 'collateral'[5] . . . . . Representations to insurers, before or at the time of making a contract, are a presentation of the elements upon which to estimate the risk proposed to be assumed. They are the basis of the contract; its foundation, on the faith of which it is entered into. If wrongly presented, in any way material to the risk, the policy that may be issued thereupon will not take effect. To enforce it would be to apply the insurance to a risk that was never presented."

Statements in application prima facie representations.—We have seen

[1] Daniels *vs.* Hudson River, 12 Cush., 417. [2] Joyce, sec. 1882.

[3] Campbell *vs.* New England Mutual, 98 Mass., 389.

[4] Dewees *vs.* Manhattan Ins. Co., 34 N. J. L., 244; Vandervorst *vs.* Smith, 2 Caines (N. Y.), at 160; Weil *vs.* New York Life, 47 La. Ann., 1405.

[5] Cornfoot *vs.* Fowke, 6 Mees and W., 378; Burritt *vs.* Saratoga Ins. Co., 5 Hill (N. Y.), 188.

that in order to make any statement binding as a warranty, it must be either expressly set out or by reference incorporated in the policy. But statements in the application, even though the latter may be referred to in the policy, are not necessarily warranties.[1] The general rule is that a statement in an application is a representation unless made a warranty by express terms or otherwise clearly referred to so as to become a part of the contract.[2] Either the policy or the application may contain words which qualify the effect of what would otherwise be a warranty, and any ambiguity, contradiction, modification or waiver in either policy or application will be seized upon in order to defeat a warranty.

Thus an application for a life policy contained an agreement that the answers and statements should "be the basis and form part of the contract or policy, and if the same be not in all respects true and correctly stated, the said policy shall be void according to the terms thereof." The policy declared that the insurance was "in consideration of the representations made to the Company" . . . and that *fraud and intentional misrepresentations* vitiated the policy, but did not otherwise refer to the application. It was held that the agreement and statements in the application did not become a part of the policy and were not warranties, and that the policy was avoided only for fraud or intentional misrepresentations.[3]

So, in a Canadian fire insurance case,[4] where the declaration in the application read as follows: "And the said applicant hereby covenants and agrees to and with the said Company that the foregoing is a just, full and true exposition of all the facts and circumstances in regard to the condition, situation, value and risk of the property to be insured, *so far as the same are known to the applicant and are material* to the risk; and agrees and consents that the same be held to form the basis of the liability of the said Company, and shall form a part and be a condition of this insurance contract," the statements in the application were construed as representations.

[1] Benham *vs.* United G. and L. Ass. Co., 21 L. J. Ex., 317.

[2] Daniels *vs.* Hudson River, 12 Cush., (Mass), 416; Phoenix Life *vs.* Raddin, 120 U. S. 183; Cushman *vs.* United States Life, 4 Hun. (N. Y.), 783; Fidelity Mutual *vs.* Jeffries, 107 Fed. Rep., 408.

[3] American Popular Life Ins. Co. *vs.* Day, 39 N. J., 89.

[4] Laidlaw *vs.* Liverpool, etc. Ins. Co., 13 Grant Ch. U. C., 377; see also National Bank *vs.* Ins. Co., 95 U. S. 673.

Similarly in the case of Clapp *vs.* Massachusetts Benefit Association,[1] the policy stipulated that the statements in the application were made a part of the contract and were in all respects true; but in the application the declaration read, "I do hereby warrant each and all the foregoing particulars and statements to be true *to the best of my knowledge and belief*, and that I have not in this application for the above named contract concealed or withheld any *material* circumstance or information concerning the past or present state of my health," etc. It was held that there was not a warranty and that the burden of proof was on the defendant to show that the representations made by Clapp were untrue to the best of his knowledge and belief. The Court remarked that "if the Association had intended to impose a forfeiture of his certificate upon the applicant because of an untrue statement, while it had only required him to warrant that his statements were true to the best of his knowledge and belief, a contract so anomalous should have been clearly expressed. . . . It is reasonable that its words should be construed against itself rather than in such a manner that one dealing with it should by any ambiguity be deceived as to his rights."[2]

Misrepresentations must be material.—Misrepresentations differ from warranties in that they avoid the policy only when material or fraudulent.[3] An untrue representation as to an immaterial point (unless fraudulent) is not fatal to the contract.

In a case where the application contained a question, "Has the party had any sickness within the past ten years?" The assured replied that he had pneumonia and said nothing of a slight attack of chronic pharyngitis several years before. The Court held that, as the answers in the application were in this case representations only, the slight attack in question was an immaterial fact which the assured was not bound to disclose.[4]

[1] 146 Mass., 519.

[2] See also Fowkes *vs.* Manchester, 3 B. and S., 917.

[3] Macdowall *vs.* Fraser, 1 Doug., 260, per Lord Mansfield; Clason *vs.* Smith, 3 Wash. (C. C.), 156; Smith *vs.* Insurance Company, 183 Pa. St., 504, 38 Atl. R., 103; Wall *vs.* Howard Ins. Co., 14 Barb (N. Y.), 383; Dalgleish *vs.* Jarvie, 2 McN. and G., 231, 243, per Rolfe, B.; Mut. Benefit Life *vs.* Robertson, 59 Ill., 123; Williams *vs.* New England Ins. Co., 31 Me., 219.

[4] Wise *vs.* Mutual Benefit, 34 Md., 582, 3 Big. Life and Accident Ins. Cas., 595.

Representations need be only substantially correct.—Warranties, as we have seen, must be literally true. In the case of representations, it is sufficient if they are substantially correct.

Thus, in the Canadian fire insurance case before cited,[1] where a party on applying to effect the insurance stated the cash value of the building to be $4000, and the evidence shewed that the cost was $3837 only; and that he was the owner of the property subject to a mortgage, when he only held a contract of purchase, a portion of the money remaining unpaid, and a mortgage having been agreed for but not executed. Here, the inaccuracies not being intentional or material, and the statements being substantially true, the Court gave a decision against the company.

Unintentional misrepresentations avoid the policy.—As a general rule,[2] an untrue representation, if material, avoids the policy, even though unintentional on the part of the assured.[3] "When the fact is material, it is not the knowledge of the proposer which is of importance, but the actual correctness of his statements; if he has induced the insurers to enter into the contract upon false premises he, of the two innocent parties, must be the sufferer."[4] "It is not necessary in all cases," says the Court of Appeals of New York, "in order to sustain a defence of misrepresentation in applying for the policy, to show that the misrepresentation was intentionally fraudulent. A misrepresentation is defined by Phillips to be where a party to the contract, either purposely or through negligence, mistake or inadvertence

[1] Laidlaw *vs.* Liverpool, etc. Ins. Co., 13 Grant. Ch. (U. C.), 377.

[2] There are some decisions apparently contrary, but they can hardly be relied upon as precedents of weight in view of the preponderance of cases, both British and American, in support of the rule. In regard to concealment, however, as will be shewn below, it is usually held by American Courts that innocent omission of information is not fatal.

[3] Life Assocn. Scotland *vs.* Foster, 11 C. S. C. (3rd series), at page 360; Continental Ins. Co. *vs.* Kasey, 25 Gratt. (Va.), 268; Dennison *vs.* Thomaston Mut. Ins. Co., 20 Me., 125; Duckett *vs.* Williams, 3 C. and M., 348, 3 Big. L. and A. Ins. Cas., 8; Cornfoot *vs.* Fowke, 6 Mees and W., 379, per Lord Abinger; Lindeneau *vs.* Desborough, 8 B. and C., 586; Byers *vs.* Farmers Ins. Co., 9 Ins. L. J., 743; Carpenter *vs.* Amer. Ins. Co., 1 Story C. C., 57; Vose *vs.* Eagle Life, 6 Cush. (Mass), 42; Geach *vs.* Ingall, 14 M. and W. 95; Maynard *vs.* Rhodes, 1 Car. and P., 360, 5 Dowl. and R., 266; Wall *vs.* Howard Ins. Co., 14 Barb. N. Y., 385, per Mitchell, J.

[4] Bunyon, Page 50.

or oversight, misrepresents a fact which he is bound to represent truly, and he lays down the doctrine that it is an implied condition of the contract of insurance that it is free from misrepresentation or concealment, whether fraudulent or through mistake. If the misrepresentation induces the insurer to enter into a contract which he would otherwise have declined, or to take a less premium than he would have demanded had he known the misrepresentation to be untrue, the effect as to him is the same if it was made through mistake or inadvertence as if it had been made with a fraudulent intent, and it avoids the contract. An immaterial misrepresentation, unless in reply to a specific inquiry or made with a fraudulent intent and influencing the other party, will not impair the contract. But if the risk is greater than it would have been if the misrepresentation had been true, the preponderance of authority is to the effect that it avoids the policy, even though the misrepresentation was honestly made." [1]

Thus, where an insurance was effected by a creditor on the life of his debtor, who gave untrue answers to the question, "Who is your medical attendant? Have you ever had a serious illness?" The creditor was ignorant of the misrepresentation, but the policy was held to be void.[2]

Concealment.

Concealment differs from misrepresentation as the negative differs from the positive, or the passive from the active. The number of life insurance cases in which the question of concealment proper has arisen is not large ;[3] probably because the subject is generally so closely interwoven with warranty, misrepresentation and other considerations.

Definition.—The easiest way to approach this subject is probably to discuss first the strict doctrine of concealment which in Great Britain applies equally to all kinds of insurances. A very concise definition runs as follows : "Concealment . . . . . is the failure to disclose any material fact or circumstance which is in fact or law within or which ought to be within the knowledge of one party and of which the other party has not actual or presumptive knowledge." [4]

The British doctrine of concealment.—In a contract of insurance, the

[1] Armour *vs.* Trans-Atlantic Ins. Co., 90 N. Y., 455.
[2] Maynard *vs.* Rhodes, 1 C. and P., 360.
[3] Penn. Ins. Co. *vs.* Mechanics, 37 U. S. App., 738, per Taft, C. J.
[4] Joyce's definition of concealment *re* marine insurance, sec. 1786.

presumption is that the insurer, prior to the contract, has no knowledge whatever of the extent or character of the risk except what is communicated to him by the insured. The statements of the insured are therefore the basis of the contract. Equity requires that the two parties shall contract *pari passu*, and this can only be the case where the insured communicates to the insurer every fact within his knowledge that is material to the risk. Good faith forbids either party, by concealing what he privately knows, to draw the other into a bargain which might never have been made if both parties had possessed equal information of the subject matter of the contract. Not only must the person proposing the insurance abstain from making any deceptive representation, but he must observe the utmost degree of good faith, *uberrima fides*.[1] He ought not only to make a true answer to the questions put to him, but spontaneously to disclose any fact exclusively within his knowledge which it is material for the insurer to know.

Two things should be specially noted about this severe doctrine of concealment, as it will be found that in the United States the law in relation to these two points has of recent years been interpreted with somewhat less strictness.

Concealment need not be intentional.—First: If a material fact has been suppressed, the policy is avoided, even though the omission has been unintentional and innocent.[2] "Concealment," says Arnould, "is the suppression of or neglect to communicate a material fact within the knowledge of one of the parties, which the other has not the means of knowing or is not presumed to know. Whether such suppression of the truth arise from fraud (that is, from a wilful intention to deceive for the party's own benefit), or merely from mistake, negligence or accident, the consequences will be the same. The ground, in short, on which the policy is avoided is that the party has

[1] Moens *vs.* Heyworth, 10 M. and W., 147, 157, per Baron Parke; London Assurance *vs.* Mansel, 11 Ch. D., 363.

[2] Abbott *vs.* Howard, Hayes, 381; Lindenau *vs.* Desborough, 3 C. and P., 353, 8 B. and C., 586; Dalglish *vs.* Jarvie, 2 Mac. and Gor., 243; Macdonald *vs.* Law Union, L, R., 9 Q. B., 328; Cornfoot *vs.* Fowke, 6 Mees and W., 379, per Lord Abinger; Life Assocn. Scotland *vs.* Foster, 11 C. S. C. (3rd series), at page 360; see also Swete *vs.* Fairlie, 6 C. and P., 1, where Lord Lyndhurst is reported as saying, "You use the word *concealment;* I do not choose to use that word, as it may import fraud." And he adopts instead the phrase 'The mere non-communication of facts which, in the opinion of the Jury, were material.'

been in fact deceived, not that the other party has intended to deceive him."

Thus, Baron Rolfe in Dalglish *vs.* Jarvie, 2 Mac. & Gor, 243, says "A party is required not only to state all matters within his knowledge which he believes to be material to the question of insurance but all which in point of fact are so. If he conceals anything that he knows to be material, it is a fraud; but besides that, if he conceals anything that may influence the rate of premium which the underwriter may require, although he does not know that it would have that effect, such concealment entirely vitiates the policy."

And again in Life Association of Scotland *vs.* Foster.[1] "Contracts of insurance are in this, among other particulars, exceptional, that they require on both sides *uberrima fides*. Hence, without any fraudulent intent and even in bona fide, the insured may fail in the duty of disclosure. His duty is carefully and diligently to review all the facts known to himself bearing on the risk proposed to the insurers, and to state every circumstance which any reasonable man might suppose could in any way influence the insurers in considering and deciding whether they will enter into the contract. Any negligence or want of fair consideration for the interests of the insurers on the part of the insured leading to a non-disclosure of material facts, though there be no dishonesty, may therefore constitute a failure in the duty of disclosure which will lead to the voidance of the contract."[2]

Concealment exists even if no inquiry in application.—Second: Even though the party does not omit to answer anything as to which he is interrogated, yet if anything material is known to him[3] and not communicated, the suppression would be fatal.[4] For example, in

[1] 11 C. S. C. (3rd series), at page 360.

[2] See also the celebrated decision of Lord Mansfield in Carter *vs.* Boehm, 3 Burr., 1910, which is too lengthy to be reproduced here, but is well worth reading as being probably the best enunciation of the English doctrine of concealment.

[3] "Here nevertheless, we find a limit to the obligation. For, however important a fact may be, although the assured may be responsible for making an incorrect representation concerning it, he cannot be prejudiced by omitting to communicate information which he does not possess. Were it otherwise, he would no longer contract *pari passu* with the insurers." Bunyon, page 41.

[4] Abbot *vs.* Howard, Hayes, 399, per Joy, C. B.; Morrison *vs.* Muspratt, 4 Bing., 60; British Equitable *vs.* Musgrove, 3 T. L. R., 630.

Hugenin *vs.* Rayley.[1] This was the case of a policy effected on the life of a person who was in prison. The fact of her being in custody was suppressed. The Court found "that there was nothing in the terms of the policy, which required the imprisonment to be stated ; nor was there an omission of the statement of any matter which the office called for ; nevertheless if the imprisonment were a material fact, the keeping it back would be fatal ; and it ought to have been submitted to the Jury, whether the omission of the fact relied on, was or was not a material omission."

Concealment according to United States Courts.—We have seen that concealment, as interpreted by British Law will avoid a policy if the insured has failed to disclose any material fact whether such failure was due to accident, mistake, negligence, or design. That doctrine, in its original strictness, does not now hold in the United States,[2] except in marine insurance where, from the nature of the business, the underwriter must depend to an altogether exceptional extent upon the data furnished by the insured. But in life and fire insurance, the insurer possesses various other sources of information, and, as the business is ordinarily carried on, avails himself of them, through agents, medical examiners, etc. Moreover, in life insurance especially, many cases will arise where the information furnished by the insured cannot be always precise and certain ; many of his replies, whether so expressed or not, must be necessarily statements of opinion rather than of fact. And it is probably owing to these causes *inter alia* that the rigid interpretation of the old law has been gradually[3] relaxed.

[1] 6 Taunt., 186.

[2] Joyce, sec. 1844; Boggs *vs.* American Ins. Co., 30 Mo., 63, per Ewing J.; Hartford Ins. Co. *vs.* Harmer, 2 Ohio St., 452; Horn *vs.* Amer. Mutual, 64 Barb. (N. Y.), 81.

[3] It is evident that this modified doctrine of concealment has grown up in the United States within the last forty years. Thus Phillips (1867) says, "The doctrines of misrepresentation and concealment are common to marine, fire and life policies." And again, "There is not, that I am aware of, either authority or reason for the doctrine that a life policy issued upon a misrepresentation (*sic;* the context shows that he includes concealment in the word) made through negligence or mistake is valid, or that there is in this respect any distinction between a life policy and a marine one." The same views are expressed by the other early writers such as Angell and Bliss. But the later American writers, Joyce, Bacon, May, etc., agree in the distinction they draw between the

American definition of concealment. Not fatal unless intentional.—Concealment, as defined in the latest edition of Judge May's work, "is the designed and intentional withholding of some fact material to the risk which the insured in honesty and good faith ought to communicate to the insurer.[1] It is not mere unintentional silence or inadvertence. It is a positive omission to state what the applicant knows, or must be presumed to know, ought to be stated. It is a suppression of truth whereby the insurer is induced to enter into a contract which he would not have entered into had the truth been known to him." [2]

No concealment if no inquiry in application.—It seems also to be usually held by American Courts that the form of application in which detailed questions are asked is an implied waiver of the right to information upon matters not inquired about. When no inquiries are made, the intention of the insured becomes material, and to avoid the policy it must be found not only that the matter was material, but also that it was intentionally and fraudently concealed.[3]

Thus, in the case of Rawls *vs.* American Mutual Life, 27 N. Y., 282, where the Company defended on the ground that the insured (one Fish) had suppressed the fact that he was of intemperate and dissolute habits which had a tendency to shorten his life. The application did not contain any question that would have elicited this information. The Appeal Judge said in his decision, "The only remaining branch of the charge singled out for exception was the instruction, 'that if Fish answered frankly and truly all the questions put to him, then there was no concealment. The mere omission to state matter not called for by any specific or general question would not be a concealment and would not affect the validity of the policy.' This was not wrong. It may be conceded that if the applicant, when

doctrine of concealment in life insurance and marine. In reading the cases on the subject, the same distinction is also very apparent between early and modern decisions.

[1] Daniels *vs.* Hudson River, 12 Cush. (Mass.), 416, 59 Am. Dec., 192; Clark *vs.* Union Mutual, 40 N. H., 333.

[2] May, sec. 200. See also Joyce, sec. 1844; Bacon, sec. 217; Daniels *vs.* Hudson River, *supra.*

[3] May, sec. 207; Penn. Ins. Co. *vs.* Mechanics Savings Bank, 37 U. S. App., at page 739; Alkan *vs.* N. H. Ins. Co., 53 Wis., at page 142; Washington *vs.* Weymouth, 135 Mass., at page 505; Boggs *vs.* American Ins. Co., 30 Mo., 63, per Ewing, J.

a specific or even general question is put to him, which would elicit a fact material to the risk, untruly stated or concealed the fact, it would vitiate the policy ; but I know of no case, in the law of life or fire insurance, in which the insurers, having framed and put to the insured, and having had fully answered by him, a series of questions calling for such information as they desired touching the subject insured, have been discharged from their contract because the insured did not go farther and state what was not called for in the interrogatories. As was said by the learned judge in the Court below : 'The presumption is that the insurers questioned the party upon all subjects which they deemed material, and all which were in contemplation of the parties at the time, and beyond that, clearly, a party is not bound to disclose.' ''

So, in another Appeal Case, Mallory *vs.* Travelers Insurance Co., 47 N. Y., 52, it was shewn after the death of the assured that he had been insane three or four years previous to making application ; had been sent to an asylum and discharged cured ; and from that time forward had been sane. The application did not contain a question covering the point. The Company defended the policy on the ground that the insanity was a material fact and should have been disclosed. The Court gave decision against the Company, saying : "The judge (*i. e.* of the lower Court) was right in charging that if the deceased did not conceal any fact which, in his mind, was material in making the application, the policy was not void. Cases cited by counsel were cases where false answers were given to inquiries made and have no application to this case." [1]

The subject of concealment, as interpreted by United States Courts, was discussed at length in the case of Penn. Ins. Co. *vs.* Mechanics Savings Bank [2] by Taft, C. J. whose decision is the most clear and thorough that I have been able to find on the subject. The following excerpt from his opinion on that case, though somewhat lengthy, is

[1] This and similar cases suggest the importance to the Company of making the inquiries in the application and medical blank as comprehensive as possible. There has been a tendency in recent years to curtail the number of questions asked in the application, and some companies have made a merit of the simplicity of their proposal forms. The wisdom of such a course is questionable, in view of the peculiar importance attached by American Courts to information specifically called for in the application in comparison with that not so required.

[2] 37 U. S. App., 738, per Taft, C. J.

worth quoting: "The very marked difference between the situation of the parties in marine insurance and that of parties to a fire or life policy has led many Courts in this country to modify the rigor of the doctrine in its application to fire and life insurance, and to lean toward the view that no failure to disclose a fact material to the risk not inquired about will avoid the policy, unless such disclosure was with intent to conceal from the insurer a fact believed to be material; that is, unless the non-disclosure was fraudulent. In marine insurance, the risk was usually tendered and accepted when the vessel was on high seas, where the insurer had no opportunity to examine her, or to know the particular circumstances of danger to which she might be exposed. The risk in such a case is highly speculative, and it is manifestly the duty of the insured to advise the insurer of every circumstance within his knowledge from which the probability of a loss could be inferred, and he cannot be permitted to escape the obligation by a plea of inadvertence or negligence. In cases of fire and life insurance, however, the parties stand much more nearly on an equality. The subject of the fire insurance is usually where the insurer can send its agents to give it a thorough examination and determine the extent to which it is exposed to danger of fire from surrounding buildings or because of the plan or material of its own structure. The subject of life insurance is always present for physical examination by medical experts of the insurer, who often acquire by lung and heart tests and by chemical analysis of bodily excretions a more intimate knowledge of the bodily condition of the applicant than he has himself. Then, too, the practice has grown of requiring the applicant for both fire and life insurance to answer a great many questions carefully adapted to elicit facts which the insurer deems of importance in estimating the risk. In life insurance, not only is the applicant required to answer many general questions concerning himself and his ancestors, but he is also subjected to an extended examination concerning his bodily history. When the applicant has fully and truthfully answered all these questions, he may rightfully assume that the range of the examination has covered all matters within ordinary human experience deemed material by the insurer, and that he is not required to rack his memory for circumstances of possible materiality, not inquired about, and to volunteer them. He can only be said to fail in his duty to the insurer when he withholds from him some fact, which, though not made the subject of inquiry, he nevertheless believes to be material to the risk and actually is so, for fear it would induce a rejection of the risk, or, what is the same thing, with fraudulent intent. . . . . "

"Nor does this rule result in practical hardship to the insurer, for in every case where the undisclosed fact is palpably material to the risk, the mere non-disclosure is itself strong evidence of a fraudulent intent. Thus, if a man about to fight a duel should obtain life insurance without disclosing his intention, it would seem that no argument or additional evidence would be needed to show the fraudulent character of the non-disclosure. On the other hand, where men may reasonably differ as to the materiality of a fact concerning which the insurer might have elicited full information and did not do so, the insurer occupies no such position of disadvantage in judging of the risk as to make it unjust to require that before the policy is avoided it shall appear, not only that the undisclosed fact was material, but also that it was withheld in bad faith. To hold that good faith is immaterial in such a case is to apply the harsh and rigorous rule of marine insurance to a class of insurance contracts differing so materially from marine policies in the circumstances under which the contracting parties agree that the reason for the rule ceases."

Little difference in practice between the two interpretations.—The American definition of concealment, at first blush, seems likely to lead to widely different results from that given on page 466 hereof. Its effect is, undoubtedly towards a more lenient interpretation of the obligations of the insured, but the divergence between the two definitions is perhaps not really so great as would at first appear. Joyce puts the matter very clearly when he says: "It is held that the concealment must . . . . . be not only material but fraudulent, or the fact must have been intentionally withheld. But it would seem that if a material fact is actually known to the assured, its concealment must of itself necessarily be a fraud, and if the fact is one which the assured ought to know, or is presumed to know, the presumption of knowledge ought to place the assured in the same position as in the former case with relation to material facts; and if the Jury in such cases find the fact material, and one tending to increase the risk, it is difficult to see how the inference of a fraudulent intent or intentional concealment can be avoided."

On the whole, the general tenor of American decisions on this subject seems to be fair, reasonable, consistent, and approximates to what May suggests[1] as the true rule: "That there is concealment

[1] May, section 203.

whenever facts are withheld which are known or which must be presumed to be known, because they ought to be known to an ordinarily intelligent person, to be material."

It appears, therefore, that the American interpretation of concealment, although based on a more lenient definition than that which governs in marine insurance and in most British Courts, will work out in practice to nearly the same results.

What Facts are Material.

Definition of material fact.—The question of materiality occupies so important a place in the consideration of both representations and concealments, that it may be worth while to define the phrase "material fact" and consider what facts have been judicially held to be material. We quote two definitions which are practically synonymous, and have been generally accepted as correct. "Every fact and circumstance which can possibly influence the mind of any prudent and intelligent insurer in determining whether he will underwrite the policy at all or at what premium he will underwrite it is material" (Marshall on Insurance, 467). "By a material fact is meant one which, if communicated to the underwriter, would induce him either to refuse the insurance altogether or not to effect it except at a higher premium" (Arnould on Insurance). These definitions afford a general criterion by which the materiality of facts may be tested in the absence of judicial decisions covering the question. We shall proceed to examine some of the facts which have been judicially held to be material.

Age.—A false statement in regard to age is held material and will avoid the policy whether construed as a representation or a warranty.[1] But there are statutory enactments in many States and Provinces which provide that an unintentional misstatement of age shall not vitiate the policy, and that if the age is incorrectly given the amount payable under the policy shall be adjusted in proportion to the true and actual premiums.[2]

[1] Dolan *vs.* Mutual Reserve, 173 Mass., 197; Swett *vs.* Citizens Mutual, 78 Me., 541; Marcoux *vs.* Society of Beneficence, 91 Me., 250.

[2] Examples are Michigan (1907), Act 187; New York (1906), Chap. 326, sec. 101; R. S. C. Chap. 34, sec. 73; R. S. O. Chap. 203, sec. 149.

Residence.—A misstatement in regard to residence is not necessarily material.[1] In some circumstances it would no doubt be regarded as material although I have not met with any cases directly bearing on the point. The term "residence" has been defined to signify "the place of permanent rather than mere temporary abode in the sense of domicile rather than of mere inhabitancy."[2]

Occupation.—Occupation may be a very important element in a life insurance risk, although perhaps not to the same extent as in accident insurance. Some misstatements of occupation are obviously material; for example, where an applicant represented his occupation to be that of printer when in fact he was attending bar, the contract was held to be void;[3] and where the occupation was stated as "farmer" when in fact the applicant was a "slavetaker" the misrepresentation avoided the policy.[4] Other misstatements are just as manifestly unimportant; as, for example, where the occupation was given as "esquire" when the applicant was an ironmonger.[5] The general definition of a material fact is probably a fairly safe guide.

Family history.—The ages of the parents of the applicant at the time of their deaths, the diseases of which they died, and other facts relating to the relatives and family of the applicant may all be material. But the Courts will take a reasonable view, and not avoid a policy for an innocent misrepresentation of too trivial a nature to have influenced the mind of the insurer. Thus, a statement that a sister died of kidney disease, when in fact she died of chronic pneumonia has been held not to be prima facie a material misrepresentation.[6]

Family Doctor.—A question as to the family physician or medical attendant of the applicant is material[7] and must be truthfully answered or the policy will be avoided. Whether the answer is true or not is generally for the Jury to decide.[8] As the object of the question is to

[1] Southern Life *vs.* Booker, 9 Heisk, 606; Perrins *vs.* Marine, etc., Ins. Soc., 2 El. and El., 317, 29 L. J. Q. B., 242.

[2] Mobile Life *vs.* Walker, 58 Ala., 290.

[3] Holland *vs.* Supreme Council, 54 N. J. L., 490.

[4] Hartman *vs.* Keystone Ins. Co., 21 Pa. St., 478.

[5] Perrins *vs.* Marine and General Ins. Co., 2 El. and El., 317.

[6] New Era *vs.* McTavish, 94 N. W. R., 599.

[7] Fidelity Mutual *vs.* McDaniel, 57 N. E. R., 645; Maynard *vs.* Rhodes, 1 Car. and P., 360; Boland *vs.* Industrial Benefit Assocn., 26 N. Y. Supp., 433.

[8] Cushman *vs.* United States Ins. Co., 70 N. Y. 72; Maynard *vs.* Rhodes, *supra.*

obtain the name of a medical attendant who can give information as to the insurability of the life proposed, the failure to give full information may amount to a concealment, as where the applicant gave the name of a casual medical attendant but did not give the name of a physician who had recently attended him for delirium tremens, it was held that the duty of the applicant was to have made a full disclosure.[1] Similarly, where the applicant was asked to state the physician usually employed by him, and if he had none to name any other doctor who could be applied to for information as to the state of his health and he answered "None," and it was shewn that he had occasionally applied to a physician for serious ailments and had been examined for insurance and rejected by another physician, it was held that the failure to state the names of the two physicians was a fraudulent concealment and avoided the policy.[2]

Previous declination.—The existence of other insurance or a previous declination by another company is material.[3] If a false answer is given to a question as to previous insurance the policy is avoided.[4] And if the answer is framed so as to conceal the fact of a previous refusal, it has been held a material concealment and vitiates the policy. Thus, where the applicant was asked if the life had been refused by any office, and if so to name it. There had been numerous refusals, and negotiations with other offices were pending which afterwards resulted in refusals. The reply was that he had been and still was corresponding with other offices as the amount to be insured was large. This was held to be an intentional suppression of facts which vitiated the contract.[5]

Habits, use of liquor, etc.[6]—Intemperance is a material fact, and a representation that the assured is temperate when he is not so will avoid the policy.[7] But it is occasionally a matter of considerable

[1] Hutton *vs.* Waterloo, 1 F. and F., 735.

[2] Horn *vs.* Amicable Ins. Co., 64 Barb., 81.

[3] Anderson *vs.* Fitzgerald, 4 H. L. C., 484. per Lord St. Leonard; March *vs.* Metropolitan Life, 186 Pa. St., 629; London Assurance *vs.* Mansel, L. R., 11 Ch. D., 363.

[4] Eddington *vs.* Ætna Life, 100 N. Y. 536.

[5] *Re* Gen. Provincial Life, 18 W. R., 396 reported in Bliss, 191.

[6] This is a very important branch of the subject and one that has been frequently in dispute, as will be apparent to anyone who scans the numerous cases bearing on the point.

[7] Craig *vs.* Fenn, 1 Car. and M., 43; Lindenau *vs.* Desborough, 8 and C., 586, 3 M. and R., 45; Mutual Benefit *vs.* Holterhoff, 2 Cincin., 379.

difficulty to determine exactly what is meant by the terms 'temperate' and 'intemperate';[1] and it seems hardly possible to give a satisfactory definition. Probably the best way is to quote from a couple of the leading cases.

In Brockway *vs.* Mutual Benefit Life, 9 Fed. Rep., 253, the Court said: "The words 'sober' and 'temperate' are to be taken in their ordinary sense. The language does not imply total abstinence from intoxicating liquors. The moderate, temperate use of intoxicating liquors is consistent with sobriety. But if a man use spirituous liquors to such an extent as to produce frequent intoxication, he is not sober and temperate within the meaning of this contract of insurance."

The Supreme Court of the United States in Insurance Company *vs.* Foley, 105 U. S. 350, where the question in the application was "Is the party of temperate habits?" said: "When we speak of the habits of a person we refer to his customary conduct, to pursue which he has acquired a tendency from frequent repetitions of the same acts. It would be incorrect to say that a man has a habit of anything from a single act. A habit of early rising, for example, could not be affirmed of one because he was once seen on the streets in the morning before the sun had risen; nor could intemperate habits be imputed to him because his appearance and actions on that occasion might indicate a night of excessive indulgence . . . . The representations made are not untrue within the meaning of the policy, although he may have had an attack of delirium tremens from an occasional over-indulgence.[2] It could not have been contemplated from the language used in the policy that it should become void for an occasional excess by the insured, but only when such excess had by frequent repetitions become a habit."

In Union Mutual Life *vs.* Reif, 36 Ohio St., 599, "This habit (of using intoxicants to excess) may manifest itself by daily or periodical intoxication or drunkenness. . . . . Where the general habits of a

[1] E. g., in Thomson *vs.* Weems, 9 App. Cas., 671.

[2] May (sec. 299), dissents from this: "A man who drinks in such a way that he has had the delirium tremens, whether as the result of a single debauch or otherwise, is liable to have another debauch and another attack. He is not a temperate man in the true sense and spirit of the question who is open to such excess, nor does his answer disclose what the Company manifestly desires to know, viz: the danger to his life by reason of his appetite for drink . . . . Technicalities and literalities should not protect the insured any more than the Company."

man are abstemious or temperate, an occasional indulgence to excess does not make him a man of intemperate habits. But if the habit is formed of drinking to excess, and the appetite for liquor is indulged to intoxication, *either constantly or periodically*, no one will claim that his habits are temperate, though he may be daily sober for longer or shorter periods in the intervals between the time of his debauches."

In Thomson *vs.* Weems, 9 App. Cas., 671, a majority of the Judges considered that in estimating the truth of the statement as to temperance, the normal habits of the people in the class and locality where the assured lived must be taken into consideration.

Except in very clear cases, the question as to whether intemperance existed is submitted to the Jury.[1]

Serious illness or accident.—An illness or accident is material when it has an ultimate effect on the party's health, longevity, strength, etc., which might reasonably influence the insurer's rate of premium.[2] The applicant is not expected to mention all his temporary derangements from which he has recovered without impairment of his general health.[3]

Materiality a question of fact.—The question of materiality is a question of fact, and is not dependent on the opinion of the assured.[4] "In all cases of insurance . . . . . the underwriter should be informed of every material circumstance within the knowledge of the assured; and the proper question is, Whether any particular circumstance was in fact material? and not, Whether the party believed it to be so? The contrary doctrine would lead to frequent suppression of information and it would often be extremely difficult to show that the party neglecting to give the information thought it material. But if it be held that all material facts must be disclosed, it will be the interest of the assured to make a full and fair disclosure of all the information within their reach.[5]

[1] North Western Insurance Co. *vs.* Muskegon Bank, 122 U. S., 501.

[2] Holloman *vs.* Life Ins. Co., 1 Woods (U. S. C. C.), 674; Insurance Co. *vs.* Wilkinson, 13 Wall (U. S.), 222; Confederation Life *vs.* Miller, 14 S. C. R., 330; Ill. Masons Society *vs.* Winthrop, 85 Ill., 537.

[3] Fidelity Mutual *vs.* Percy, 63 Blatchford (U. S.), 717; Wilkinson *vs.* Conn. Mutual, 30 Ia., 127.

[4] Lindeneau *vs.* Desborough, 8 B. and C., 586; Dalglish *vs.* Jarvie, 2 Mac. and Gor., 243.

[5] Lindeneau *vs.* Desborough, *supra.*

Question of materiality for the Jury.—Except in cases where the materiality is obvious,[1] it is a question for the Jury whether any particular fact is or is not material when its truth is not warranted or made a condition precedent.[2]

Intentional misstatement always material.—If a concealment or a misrepresentation proceeds from fraud, it avoids the policy, whether material or not, and the fraud precludes all inquiry as to materiality.[3] "Any answer to a question, though concerning a matter not material to the risk, if made with intent to deceive the insurance Company, would avoid the policy."[4]

Materiality not dependent on the outcome.—The materiality of a fact does not depend on whether the outcome shewed that it affected the risk. It makes no difference that death was caused by the fact withheld or not; the sole test being its materiality at the time.[5] Thus in an old English case,[6] the following questions and answers occurred in connection with the application. Q. "Who is your medical attendant?" A. "I have none except Mr. Guy of Chichester." Q. "Have you ever had a serious illness?" A. "Never." It was proved that a few months prior to the application, the applicant had been attended by a Doctor Veitch, a physician, and Mr. Jordan, a surgeon, for an inflammation of the liver, and a fever, and determination of the blood to the head. It was, however, agreed on all hands that the disease of which the assured died had no relation to any of the complaints for which these gentlemen attended him. Nevertheless the Court decided that the facts concealed and misrepresented were material and avoided the policy.

[1] Myers *vs.* Woodmen, 193 Pa. St., 470; Russell *vs.* Canada Life, 32 C. P., 256.

[2] Fidelity and Casualty *vs.* Alport, 67 Fed. Rep., 460; Lindeneau *vs.* Desborough, 8 B. & C., 586; Morrison *vs.* Muskratt, 4 Bing., 60; Boggs *vs.* American Insurance Co., 30 Mo., 63; Fidelity Mutual *vs.* Percy's Executors, 63 Blatch. (U. S.), 717; Clark *vs.* Union Mutual, 40 N. H., 333.

[3] Bliss, sec. 40; Joyce, sec. 1896.

[4] Penn. Ins. Co *vs.* Mechanics Savings Bank, 37 U. S. App., 708, per Taft, C. J.

[5] Maynard *vs.* Rhodes, 1 Car. and P., 360, 5 Dowl. and R., 266, per Abbott, C. J.; Conover *vs.* Massachusetts Life, 4 Big. Life and Acc. Cas., 187; Ross *vs.* Bradshaw, 1 W. Black., 312.

[6] Maynard *vs.* Rhodes, *supra.*

Untruthful answer to direct question is material.—It has already been pointed out that in United States Courts, the question as to whether specific inquiries are made or not has an important bearing on the materiality of a fact.[1] Where a direct question is asked and the applicant makes an untruthful answer, the policy is avoided whether the answers are warranties or representations, because "the parties may by their contract make material a fact that would otherwise be immaterial, or make immaterial a fact that would otherwise be material."[2]

Exceptions to General Rules.

Exceptions.—There are several exceptions to the general rules regarding misrepresentations and concealment. The chief of these are set forth in a celebrated decision of Lord Mansfield[3] which has been frequently quoted. He says: "There are many matters as to which the insured may be innocently silent. (1) He need not mention what the underwriter knows—*scientia utrinque par pares contrahentes facit.* An underwriter cannot insist that the policy is void because the insured did not tell him what he actually knew; what way so ever he came to the knowledge. (2) The insured need not mention what the underwriter ought to know; what he takes upon himself the knowledge of. (3) Or what he waives being informed of. (4) The underwriter needs not be told what lessens the risk agreed and understood to be run by the express terms of the policy."

What the underwriter knows.—"The underwriter cannot insist that the policy is void because the insured did not tell him what he actually knew." Thus, in Pimm *vs.* Lewis, 2 F. & F. 778, where it was alleged that the insured omitted to communicate to the insurers a matter material to be made known to them, in order to enable them to judge of the risk, viz.: that he was in the habit of grinding rice-chaff in his mill. The policy provided that the contract should be void if the insured should "omit to communicate any matter material

[1] May, sec. 185; Bliss, page 70; Bacon, sec. 212.

[2] Phoenix Life *vs.* Raddin, 120 U. S., 189, per Gray, J.; See also London Assurance *vs.* Mansel, 11 Ch. D., at page 369, per Jessel, M. R.; Wilson *vs.* Conway Assurance Co., 4 R. I., 141; Anderson *vs.* Fitzgerald, 4 H. L. C., 484; Miller *vs.* Mutual Benefit, 31 Ia., 232.

[3] Carter *vs.* Boehm, 3 Burr., 1905.

to be made known to the insurer." Held, that this meant some matter, not only material, but also unknown to the insurers ; and as the Company had knowledge of the fact through their officer, they were liable.[1]

The question of the effect of agent's knowledge arises here. As a general rule,[2] "knowledge by or notice to the agent of the inaccuracy of a statement in the proposal, on which a policy is issued after such notice or knowledge, binds the Company and prevents them from availing themselves of the inaccuracy as a defence to a claim on the policy." [3]

This principle is illustrated in an English accident insurance case where the applicant had subscribed to the statement "I am in good health . . . . . and have no physical infirmity." The agent knew that he had lost one eye, but the Company was not directly informed of the fact. After the policy had been issued an accident occurred which rendered the assured totally blind, and the Company resisted payment on the ground of the misstatement as to the physical infirmity. The Court of Appeal held that the knowledge of the agent was the knowledge of the Company.[4]

What the underwriter ought to know.—"The insured need not mention what the underwriter ought to know." Thus, in the case of Swift *vs.* Massachusetts Mutual Life,[5] a question put was, "Have the parents, brothers or sisters of the party been afflicted with insanity or with pulmonary, scrofulous, or any constitutional disease?" to which the answer was in the negative. The Court said, "In considering the question of the truth of this particular answer, plaintiff's answers to other questions upon the same subject must be borne in mind. For if the answers to other questions informed defendants truly of the matters inquired into, the answer in question cannot be said to be untrue so as

[1] See also, Royal Canadian *vs.* Smith, 5 Russ. and Geld. (Nova Scotia), 322.

[2] The subject of agency is, of course, too large and complicated to be discussed with any degree of fullness in this paper.

[3] Bunyon, page 303; See also Cotton *vs.* Fidelity and Casualty Co., 41 Fed. Rep., 506; Bowden *vs.* London, etc. Ass. Co., 1892, 2 Q. B., 534; Cruikshank *vs.* Northern Accident Co., 23 Sc. L. R., 134.

[4] Bowden *vs.* London, etc. Ass. Co., *supra.*

[5] 2 N. Y. Supreme R., 303, reported by Bliss; see also Bates *vs.* Hewitt, L. R., 2 Q. B., 595.

to avoid the policy. The question was put to the plaintiff to enable the Company to know whether insanity, pulmonary, scrofulous, or other constitutional disease existed in the family of her husband. If the answers conveyed to them the information that scrofula was a disease existing in the family, they had the information they needed to enable them to determine the propriety of taking the risk. Plaintiff, in answer to a subsequent question, told them that her husband's mother died of scrofula. And in answer to another question she told them that her husband's sister died of disease of the blood. It seems to me that when the fact was disclosed that the mother died of scrofula, and one of the daughters of disease of the blood, the existence of scrofula in the family was as clearly stated as if they had been told it was hereditary."

What the insurer waives being informed of.—"The policy cannot be avoided for non-disclosure of a fact which the insurer waives being informed of." Examples of this principle may be seen in the cases cited below[1] in which American Courts have held that if an answer to a question in an application is obviously incomplete and the Company issued the policy without further inquiry, it is to be regarded as having waived its right to the information.

It has been held that the acceptance of a premium by the insurer with knowledge that a fact has been misrepresented amounts to a waiver of that ground of defence. Thus, in the case of Armstrong *vs.* Turquand,[2] the policy contained a proviso that if the assured should be guilty of fraud in procuring the said policy, it should be void. The Company pleaded that the assured had, at the time of effecting the insurance, in conjunction with the agent of the Company, fraudulently concealed the fact of his having met with an accident, from the effects of which he was then suffering paralysis. The Plaintiff replied that the Company, after they had knowledge of the facts pleaded, received a second premium from the assured, and thereby elected to affirm the policy. Held, that the meaning of the proviso was that the policy should be void in the particular event in case the Company should elect to treat it so; and inasmuch as they had elected to treat it as subsisting by the receipt of the subsequent premium, they were liable under the policy.

[1] See pages 484, 485, etc. hereof.

[2] 23 Big. Life and Acc. Ins. Cas., 350.

What lessens the risk agreed to be assumed.—"The underwriter needs not be told what lessens the risk agreed and understood to be run by the express terms of the policy." This is an almost self-evident exception. If a fact does not increase the risk, it is not material and immaterial facts need not be disclosed.[1]

Qualified Answers.

If the answers given by a person in his application are limited or qualified in such a way as to render them statements of belief or opinion instead of facts, the policy cannot be avoided even for material inaccuracies unless the misstatements are fraudulent.[2]

Thus, where the answers in an application were qualified by the following words at its foot. "The above is correct, as near as I can remember it," it was held that the policy should not be avoided unless the Jury were satisfied that the answers were untrue in some respect materially affecting the risk and that the assured knew of their incorrectness.[3]

In another case[4] where the applicant stated that the facts recited in his application were "true to the best of his knowledge and belief," it was held that the contract could not be avoided except it could be shewn that an untrue or fraudulent statement had been made by him with the knowledge or belief that it was untrue or fraudulent.

Similarly in another case[5] where the question was whether any relatives had been afflicted with certain hereditary diseases and the applicant replied, "Not to my knowledge," it was held by the Supreme Court of the United States: "The affirmation was restricted and narrowed down to what the applicant himself personally knew touching the subject. . . . . To make out the defence sought to be established by the insurers, three things were, therefore, necessary to be shewn; that the alleged insanity of the uncle had existed; that it was hereditary; and that both these things were known to the applicant when he answered the question."

[1] Lexington Ins. Co. *vs.* Paver, 16 Ohio, 324, 334.

[2] Miller *vs.* Confederation Life, 14 S. C. R., 330; Fowkes *vs.* Manchester, 3 B. and S., 917; Jones *vs.* Provincial, 3 C. B. N. S., 65; Stackpole *vs.* Simon, quoted by Parke, 932.

[3] Ætna Life *vs.* France, 94 U. S., 561.

[4] Clapp *vs.* Mass. Benefit Assocn., 143 Mass., 519.

[5] Insurance Co. *vs.* Gridley, 100 U. S., 614.

Equivocal Interrogatories—Opinions.

If an inquiry in an application is equivocal or calls for an answer which involves, not a question of fact, but of opinion, the Courts will allow the applicant every reasonable latitude in his answer. Thus, in fire insurance, inquiries as to what buildings endanger the property insured,[1] or as to whether there is a livery stable 'in the vicinity,'[2] are questions as to which intelligent men may differ, and it is enough to answer them as men of ordinary intelligence should. Similarly, in life insurance applications, if the inquiry be as to whether the applicant has suffered from any derangement of certain functions,[3] or had any 'serious illness,'[4] or disease 'tending to shorten life,'[5] or any other inquiry which may be variously understood or involves a matter of opinion, an honest though erroneous answer has been regarded as not a misrepresentation or concealment.[6]

Equivocal Answers.

The applicant should give full and complete answers to inquiries made in the application. If answers are incomplete, or equivocal, or framed so as to convey an impression which is untrue, (even though they may be true as far as they go), the policy may be avoided for misrepresentation or concealment.

Thus, where in response to an inquiry whether the life had been proposed to or declined by any other office, and if so its name, the applicant replied that he had been and still was corresponding with other offices, as the amount to be insured was large. In fact, the life had, prior to that time, been refused by eight offices, and before the policy was issued, it was refused by eight more. It had been accepted by one. The Judge said[7]: "If he knew he had been already refused by eight other offices, what justification had he for the answer he gave?

[1] Dennison *vs.* Thomaston Mutual, 20 Me., 125.
[2] Haley *vs.* Dorchester Mutual, 12 Gray (Mass.), 545.
[3] Higbee *vs.* Guardian Life, 53 N. Y., 603.
[4] Jones *vs.* Provincial Assurance Co., 3 C. B. N. S., 65.
[5] Hogle *vs.* Guardian Life, 6 Robt. (N. Y. Superior Ct.), 567.
[6] May, sec. 210.
In *re* Gen. Prov. Life Ins. Co., 18 Week. Rep., 396, reported by Bliss, section 72.

Did it not imply that he had not been refused? In order to tell a falsehood, it was not always necessary to use express words, it might be done by implication. . . . . He was bound to conclude that there was here suppression, and intentional suppression of facts, which, if known to the office, would have prevented them granting the policy. The suppression vitiated the contract." And similarly in other cases.[1]

But a distinction is drawn by American Courts between an answer which is apparently on the face of the application incomplete and imperfectly answered, and one which is apparently complete but in fact incomplete, untrue and calculated to mislead.[2] In numerous cases, where questions in an application have been unanswered or incompletely answered, and the insurers without further inquiry have issued the policy, it has been held that he had waived his right to the information, and the policy cannot, in absence of clear proof of the fraudulent suppression of the facts, be avoided on the ground of concealment.[3]

Thus in a leading case in the Supreme Court of the United States, the following question was asked in the application: "Has any application been made to this or any other Company for assurance on the life of the party? If so, with what result? What amounts are now assured on the life of the party and in what Companies? If already assured in this Company state the number of the policy." The only answer written opposite this question was, "$10,000. Equitable Life Assurance Society," which was true as far as it went. It was shewn however, that less than three weeks before the application for the policy in suit, applications for additional insurance had been made to the Equitable Life and the New York Life, both of which had been declined. The Court held that the answer to the question was manifestly incomplete, and that the Company by issuing the policy without further inquiry "waive the want or imperfection in the answer, and render the omission to answer more fully immaterial."[4]

[1] E. g., Wainwright *vs.* Bland, 1 M. and R., 481; London Assurance *vs.* Mansel, 11 Ch. D., 367; etc.

[2] Joyce, sec. 1872.

[3] Daly *vs.* John Hancock Mutual Life, 65 Ind., 6; American Ins. Co. *vs.* Mahone, 56 Miss., 180; Higgins *vs.* Phoenix Mutual, 74 N. Y., 9; Hall *vs.* Peoples' Ins. Co., 6 Gray, 185; Lorillard Ins. Co. *vs.* McCulloch, 21 Ohio St., 176.

[4] Phoenix Life *vs.* Raddin, 120 U. S., 183.

It is doubtful whether this decision would have been rendered in Canada or in Great Britain,[1] but the principle involved seems to have been firmly established by a long series of decisions in American Courts. The whole subject emphasizes the importance of careful scrutiny of the answers to questions in applications, and strict insistence on their being fully answered. If inquiries are worth printing in the application, it would appear that it is equally worth while to demand full and unequivocal responses.

Statutory Enactments.

The effect of the Common Law principles, with which this paper has mainly dealt, is considerably modified in various respects by Statutory Enactments which override[2] the Common Law.

Misstatement of age.[3]—Reference has already been made to the Statutes protecting policies from forfeiture in case of unintentional misstatement of age.

Representations, not warranties.—In certain States and Provinces there are other Statutes which enact that statements in the application, in the absence of fraud, shall be regarded as representations and not warranties ; and in most cases, it is further stipulated that a transcript of the entire application shall appear in the policy.[4]

The provision in the proposed new Canadian Insurance Act[5] will serve as a fair type of such enactments. "The policy and the endorsement thereon shall constitute the entire contract between the parties, and all statements made by the insured shall, in the absence of fraud, be deemed representations and not warranties, and no such statement shall be used in defence to a claim under the policy unless it is contained in a written application, and a copy of such application shall be endorsed upon or attached to the policy when issued."

The virtual effect of a clause like the above is to remove the subject of warranty out of the realm of practical insurance law.

[1] Vide Forbes *vs.* Edin. Life Ass. Co., 10 C. S. C., 451; London Assurance *vs.* Mansel, 11 Ch. D., 367.

[2] Penn. Ins. Co., *vs.* Mechanics Savings Bank, 37 U. S. App., 692; Hermany *vs.* Fidelity Mutual, 151 Penn. St., 17; Fidelity Mutual *vs* Ficklin, 74 Md., 172.

[3] See page 473, *supra.*

[4] Examples are N. Y. (1906), Chap. 326, sec. 58; Minn. (1905), Chap. 19, sec. 1623; Michigan, 1907, Act 187; R. S. O., 1897, Chap. 203.

[5] Section 97 (c).

Misstatements must be fraudulent or material.—Substantially the same result is attained by a clause to the effect that no oral or written misrepresentation or warranty shall be deemed material or defeat or avoid the policy, unless made with actual intent to deceive, or unless the matter misrepresented or made a warranty increased the risk.

The Ontario Statute [1] may be cited as an example. "No contract of insurance . . . . . shall contain or have endorsed upon it or be made subject to any term, condition, stipulation, warranty or proviso providing that such contract shall be avoided by reason of any statement in the application therefor, or inducing the entering into of the contract by the corporation, unless such term, condition, stipulation, warranty or proviso is limited to cases in which such statement is material to the contract, and no contract . . . . . shall be avoided by reason of any such statement, unless it be material to the contract."

The Ontario Statute contains this further provision : "The question of materiality in any contract of insurance whatsoever shall be a question of fact for the Jury, or for the Court if there be no Jury." [2]

Indisputable Policies.

We have also to consider those enactments which stipulate that the policy shall be incontestable after a stated period except in case of fraud, for non-payment of premiums, or (in some cases) for military or naval service without a permit. These laws have become quite popular during recent years and are now on the statute books of a good many states.[3]

Effect of indisputable clause.—The effect of an indisputable clause in a policy is to make it absolutely binding upon the insurer, in the absence of fraud, at the expiration of the time specified.[4] Even if a material fact has been misrepresented or concealed, the policy is indefeasible after the stated period, unless the Company is able to

[1] R. S. O., 1897, Chap. 203, sec. 144; see also R. S. C. (1906), Chap. 34, sec. 72; and others.

[2] "This enactment is very far reaching and really puts the insurer in a most difficult position."—Hodgins, Life Ins. Contracts in Canada.

[3] Examples are N. Y. (1906), Chap. 326, sec. 101; Mich. (1907), Act 187; proposed Dominion Insurance Act sec.

[4] North American Life *vs.* Elson, 33 S. C. R., 383; Mareck *vs.* Mutual Reserve, 62 Minn., 39; Simpson *vs.* Virginia Life, 115 N. C., 393.

prove fraudulent intent. When it is considered that these questions do not usually arise until after the death of the subject of the insurance, and that the question of fraudulent intent is for the Jury to decide,[1] it will be conceded that the chances of a company successfully contesting a claim under one of these policies are not very bright. The moral, of course, is that the time for detecting errors and misrepresentations is at the inception of the contract.

Examples of incontestable clause.—If a policy is stated to be incontestable after a specified period it cannot be avoided after that period has elapsed, even though the insured has violated one of the other conditions of the policy. In a Canadian case,[2] the incontestable provision in the policy read as follows: "After being in force three years, the only conditions which shall be binding upon the holder of this policy are that he shall make the payments hereon as herein provided, and that the provisions as to military and naval services, proofs of age and death and limitation of time for action or suit shall be observed. In all other respects, after the expiration of the said three years the liability of the Company under this policy shall not be disputed." There was also another clause which stipulated that "this policy is issued and also accepted by the insured upon the following additional provisoes and agreements therein made a part thereof, and (*inter alia*) if without a permit the insured engages . . . . in the employment of a railroad . . . . this policy shall thereupon become null and void." etc. The assured, after taking out the policy changed his occupation to that of brakesman on a railroad, and, a few days after the three year period had elapsed, was killed by an explosion on the railroad. The Supreme Court of Canada upheld the Supreme Court of British Columbia in the decision that the effect of the incontestable clause "was to provide an automatic cutting off at the end of the triennium of all defences arising after the coming into force of the policy except such as are reserved in the clause itself" and held that the Company was liable.

So also, in an American case,[3] the policy in suit contained the provision that: "After five years from the date of this certificate, it is incontestable for any cause, except non-payment of dues or mortuary

[1] Mallory *vs.* Travellers' Life, 47 N. Y., 52; Houghton *vs.* Manufacturers, 8 Met., 121, 41 Am. Rep., 489; Burritt *vs.* Saratoga Ins. Co., 5 Hill, 188.

[2] North American Life *vs.* Elson, 33 S. C. R., 383.

[3] Mareck *vs.* Mutual Reserve, 62 Minn., 39.

assessments, the age of the applicant being correctly stated in the application for the certificate." There was another clause to the effect that "death of the member by his own hand, whether voluntary or involuntary, sane or insane, is not a risk assumed by the Association under this contract." The age of the assured was correctly stated in his application and all dues and mortuary assessments were duly paid up to the time of his death. More than five years after the date of the certificate, the insured committed suicide. Held, that the "incontestable clause" applied and that the Company was liable for the full amount of the policy.

Fraud a necessary exception.—Fraud is necessarily excepted from the scope of these indisputable clauses. "An agreement that the insurer will not raise any objection, even in the case of direct personal fraud, is a void condition. It has even been questioned whether it would not be sufficient to render the policy itself wholly void *ab initio* as an illegal contract. In these cases,[1] then, fraud, if not mentioned, must be assumed to be excluded, since that construction is always to be preferred which will support a contract, and it is never to be supposed that the parties to it intend an illegal stipulation where a lawful meaning can be given to their words." [2]

Insurable interest, another exception.—There appears to be also another exception, if we may be guided by the case of Anctil *vs.* Manufacturers' Life, 1899, A. C. 604. The policy in this case contained the following clause: "After this policy has been in force for one full year, it will be incontestable on any ground whatever, provided that the premiums stipulated have been promptly paid and that the age of the assured has been admitted." The assured died after the policy had been current for more than a year. The Company then refused payment on the ground that the beneficiary had no insurable interest. The Supreme Court of Canada, and subsequently the Privy Council, held that this was a good defence, notwithstanding the condition in the policy. The decision of the Privy Council was stated to be based "upon general principles of public policy and expediency, and which cannot be defeated by the private convention of the parties. Any other rules would lead to the sanction of wager policies."

[1] I. e., where fraud is not specifically stated to be an exception to the operation of the indisputable clause.

[2] Bunyon, Page 129.

Bibliography.

The chief books of reference used in preparing the paper have been Joyce (Ed. 1897) ; Bliss (2nd Ed.) ; Angell (2nd Ed.) ; Bunyon (4th Ed.) ; Porter (4th Ed.) ; Bacon (3rd Ed.) ; May (4th Ed.) ; Phillips (5th Ed.) ; Holt, *Insurance Law of Canada;* Hodgins, *Life Insurance Contracts in Canada;* the *Insurance Law Journal;* Bigelow's *Life and Accident Insurance Cases;* and the various legal Encyclopedias and Digests.

(Presented by Mr. M. M. Dawson.)

Permanent Disability Benefits.

BY C. W. JACKSON.

Having recently had occasion to consider the question of permanent disability among insured lives, I now present some of the results of the investigation to the Society, hoping they may be of service.

One very great difficulty which arose early in the investigation was that of finding sufficient reliable data to work upon.

In this country permanent disability benefits have for some time been granted by fraternal societies ; and of late by several insurance companies. The latter, however, have been working too short a time and the experience is too scanty to afford reliable data. I was therefore driven abroad for statistics to work upon.

As benefits for permanent disability have been granted for years in Germany by state societies and insurance companies, several tables are to be found there. Rates of invalidity based upon the experience of German railway employés, engaged in train service and otherwise have been calculated by Behm, Zimmerman, Bentzien and Karup.

The following are the probabilities of becoming permanently disabled as obtained by those investigators and used by several companies :

Age.	1868-1873 Behm dem Deutsch Anker.	1868-1884 Zimmerman. Germania.	1868-1889 Bentzien.	1868-1884 Zimmerman (Non-train service) Karlsruhe.	1877-1889 Karup Gotha
25	.00053	.00072	.00072	.00054	.00065
35	.00212	.00284	.00287	.00220	.00237
45	.00662	.00811	.00910	.00698	.00831
55	.02320	.02940	.03190	.02690	.03050
65	.06760	.10000	.11740	.09750	.12350
75	.13310	.20700	.23010	.20620	.25470

The first three sets of rates were based on the experience of employés engaged in train-service as engineers, conductors, etc., as well as of non-train-service employés ; the two latter on the experience of non-train-service men only. It will be observed that, the later the period of the investigation, the heavier the rate of disability becomes ; and the question is suggested : Is this due to increasing sickness or to the effect of laws giving to railroad employés, who are civil servants of the States, pensions if permanently disabled ?

In 1854 the Prussian Government passed a law compelling membership in Sickness Relief Societies and was followed by most of the other northern states. In 1871 owners of railways, mines and factories were rendered liable for injuries to their employés caused by accident, when the employés were not to blame.

In 1883 a Sickness Insurance Law was passed compelling workmen to become members of sickness or accident societies, their employers being made responsible for the premiums, as a rule paying one-third themselves and deducting two-thirds from the workmen's wages. In the event of sickness the workman received one-half the wages of an ordinary day labourer in his locality. The effect of the latter law is, I think, shown by the following figures taken from the non-train-service railway employés. From 1877–1883 there were 869,672 exposed to risk and of these 8,741 became permanently disabled ; from 1884–1889 of 847,424 exposed to risk 12,028 were disabled.

In 1889 the German Government passed a very comprehensive law for insurance against sickness and old age to embrace all the labouring classes. The rate of invalidity used was that of Behm for all workmen and was much lower than his rates for the non-train-service staff up to age 60, but higher afterwards, very much lower up to age 70 than that used by Dr. Karup but slightly higher after that age. The probabilities of death among invalids used were those calculated by Zimmerman from the experience of pensioned railway employés. As full rates have been published both by Zimmerman and Karup I have used them for calculating the extra premium above the net American Experience premium for the benefit of cessation of premium payments in event of permanent disability, using the American Experience rates of mortality and Karup's and Zimmerman's rates of invalidity.

In Table I, F_x = Karup's rates of disability or the probability of becoming permanently disabled at age x.

$^{a}l_x$ = table of active members, *i. e.*, premium paying members.
$^{i}l_x$ = table of disabled = l_x (Amer. Ex.) — $^{a}l_x$.
u_x = Zimmerman's rate of disability.

Taking as radix the number of living by the American Experience table at age 20

$$^{a}l_{20+n} = l_{20}(p_{20} - F_{20})(p_{21} - F_{21}) \ldots (p_{20+n-1} - F_{20+n-1}).$$

Tables II and III give the extra premiums above the net American Experience premium for the privilege of ceasing premium payments

upon becoming permanently disabled. These extra premiums for Life and Limited Payment Life are found from the formulæ

$$^{i}P_x - P_x \; ; \; {}_{n}{}^{i}P_x - {}_{n}P_x.$$

where P_x and ${}_{n}P_x$ are net American Experience premiums.

$$^{i}P_x = \frac{M_x \text{ (Amer. Ex.)}}{{}^{a}N_x} \cdot {}_{n}{}^{i}P_x = \frac{M_x \text{ (Amer. Ex.)}}{{}^{a}N_x - {}^{a}N_{x+n}}$$

Table IV is a comparison of the table formed by using American Experience rates of mortality and Karup's rate of invalidity, with that used by Karup for the Gotha Life. Karup's table however, begins at age 15.

Both these tables, however, are to my mind defective, in that the same rate of mortality is assumed both for invalid and active members which is far from being the case; it has also the effect of unduly diminishing the active list by attributing to it too many deaths, and thereby increasing the invalidity premium. Not being satisfied with these results nor with the data from which they were obtained, as conditions in Germany, especially under the laws above mentioned, are very different from those prevailing here, I tried to find another experience more nearly corresponding to conditions prevailing here.

A great deal of sickness experience has been given in the investigations of the Manchester Unity in Great Britain, but unfortunately it is not in such a form as to be available for my purpose. The British Government's reports—Sutton's Tables—on Friendly Societies also contain a mass of information on sickness, but the greater part is not suitable. On examining these reports, however, I found both the mortality and sickness experience, males, from 1861–1870 given in such a form as to be of service.

We are given the number at each age, exposed to risk of sickness and death, the number of weeks of sickness experienced and also the number at each age who had been sick continuously for two years of sickness, the number entering upon this state and the deaths among these invalids.

The experience of the first two years of membership was omitted, as the societies had varying rules as to granting sickness benefits, some not granting them until two years had elapsed. In order therefore to obtain correct data and to embrace all cases of sickness experienced, Sutton excluded the first two years' experience. The experience comprised more than 770,000 years of exposure and 24,445 deaths.

From the total experience comprising both sick and healthy members Sutton calculated unadjusted rates of mortality. These rates I have graduated by G. F. Hardy's Friendly Societies Formula as modified by Mr. Spencer, viz :

$$\frac{1}{12}\Big[u_0\Big\{[3]+[7]\Big\}-(u_{-5}+u_{+5})\Big][3][5]\div 10.$$

The terms at the beginning and end of the table are obtained by extrapolation. Table V shows the actual and expected deaths, the deviation and accumulated deviation. With these mortality rates I have formed a general mortality table l_x and d_x.

Considering that all those who have been continuously sick two years or more, would be considered permanently disabled and unable to pursue a gainful occupation, I have taken these and from their experience have calculated the probability of mortality among invalids, ${}^{i}q_x$, graduating it by the graphic method. Table VI shows the actual and expected deaths, deviation and accumulated deviation.

These rates of mortality are very curious, being very high at age 20 and diminishing rapidly until they attain a minimum between ages 55 and 60 then gradually increasing and not attaining the rate prevailing at age 20 until age 88 is reached. They are much heavier than those formed by Zimmerman and used by the German Government in 1889, until age 70, after which age they are lighter.

From the experience of those continuously sick two years or more I also deduced rates of being disabled, *i. e.*, of having become totally and permanently disabled at age x or earlier and remaining disabled at age x. As a step towards ascertaining the probability of becoming disabled at age x, I have taken the numbers entering upon their third year of invalidity at age $x+2$ and assuming that the same rate of mortality prevailed among those who have been disabled less than two years as among those disabled two years or more have thus arrived at the number who became disabled at age x by the following formula

$$\frac{Y}{{}^{i}p_{x+1}\cdot{}^{i}p_x}$$

Y = Number entering upon third year of sickness at age $x+2$.

This is not strictly correct as the mortality rate in the first two years is probably heavier than that prevailing among those who have already experienced two years of such disability. The effect of this is to bring out a slightly smaller number than the true one. It has the effect, also, however, of increasing the present value of the income during

total and permanent disability. And since the value of insurance against disability is measured by the product of the probability of becoming disabled and the present value of this income, this goes to offset the error in practice.

From the number of those who are disabled I have calculated the chance of being disabled i_x, graduating it graphically. Table VII gives a test of the graduation. From these rates aggregate tables showing the number who are disabled at any given age, have been formed. Table IX.

l_x and d_x Living and dying among the total membership, including sick and disabled.

i_x Probability of being disabled at age x.

${}^a l_x$ Number of active members at age x.

${}^i l_x$ Number who are disabled at age x.

$l_x = {}^a l_x + {}^i l_x$.

${}^i l_x = i_{x-1} l_{x-1}(1 - {}^i q_{x-1})$.

This latter formula is not strictly accurate as it assumes that the disabled are exposed to the risk of death a whole year whereas those who become disabled that year are on the average exposed only half a year ; as these numbers, however, are comparatively small, the error is not very serious.

From these tables I calculated extra premiums by the same method used to calculate those in Table II, but these would still be too high because those entering at the older ages are made to assume the burden of those already disabled, having become so at younger ages.

To avoid this error a different method is necessary. The aggregate table has been used for the active members as this, except for the assumption made as to the members disabled two years or less, represents the real state of affairs. The probability for age x of an active member becoming disabled $i_{(x)}$ has been found from the same data and graduated by graphic method, (for test of graduation see Table VIII) and commutation columns.

$$C'_x = \tfrac{1}{2} v^x {}^a l_x i_{(x)} \left\{ v^{\frac{1}{2}} {}^i a_x + v^{-\frac{1}{2}} {}^i a_x \right\}$$

$$M'_x = \Sigma C'_x.$$

To obtain ${}^i a_x$ a mortality table was formed using the probability of death ${}^i q_x$ which represents the rate of mortality among those who have been sick two years and more. ${}^i a_x$ thus represents the value of a life annuity from age x on the life of a person who is disabled. (For this mortality table see Table X.)

Assuming that at age 80 all will cease paying premiums—beyond that age very few indeed would be considered capable of pursuing a gainful occupation—single and annual premiums for an annuity due of 1 per annum to be entered upon when becoming permanently disabled or attaining age 80, have been calculated from the formulæ

$$\frac{M'_x - M'_{80} + {}^aN_{80}}{{}^aD_x} \text{ and } \frac{M'_x - M'_{80} + {}^aN_{80}}{{}^aN_x - {}^aN_{80}}$$

and the extra to be added to the whole life annual premium for the cessation of premium payment on becoming permanently disabled

from the formula* $P_x \frac{M'_x - M'_{80} + {}^aN_{80}}{{}^aN_x - {}^aN_{80}}$ See Table XI.

It will be observed by comparing Tables II, III and XI that the rates obtained from the British experience, differ very widely from those obtained by using German rates, especially at the higher ages; in consequence of the high German rates, the Gotha Life does not issue policies with the permanent disability feature, except when the premium-paying period ceases on or before age 60.

The difference in the rates may in part be due to the fact that the German rates were deduced from the experience of railway employés, the British from insured lives.

I am greatly indebted to my chief, Mr. M. M. Dawson for valuable suggestions and assistance and to Mr. Führer, Actuary of the Germania Life for a translation of the parts of Dr. Karup's "Reform des Rechnungswesens der Gothaer Lebensversicherungsbank A. G." relating to permanent disability, and other tables.

The authorities consulted are several Journals of the Institute, Sutton's British Government Friendly Societies Reports, Karup's "Reform des Rechnungswesens der Gothaer Lebensversicherungsbank, A. G." Meyer's "Beitrage sur Pensionsversicherung," Schaertlin's "L'Assurance en Cas d'Invalidité."

* This provides for the return of the net life annual premium only.

TABLE I.

	Am. Ex. and Karup.			Zimmerman.
Age.	F_x	${}^{a}l_x$	${}^{i}l_x$	u_x
20	.00039	92637	0	.00021
21	.00043	91878	36	.00026
22	.00047	91117	75	.00033
23	.00053	90355	116	.00040
24	.00059	89588	163	.00047
25	.00065	88818	214	.00054
26	.00071	88044	270	.00062
27	.00078	87265	331	.00071
28	.00086	86481	397	.00080
29	.00095	85692	468	.00085
30	.00107	84897	544	.00096
31	.00123	84089	632	.00113
32	.00143	83270	730	.00131
33	.00170	82435	842	.00156
34	.00201	81576	975	.00187
35	.00238	80692	1130	.00220
36	.00280	79777	1313	.00248
37	.00325	78830	1523	.00282
38	.00373	77847	1764	.00310
39	.00423	76825	2037	.00341
40	.00477	75763	2343	.00382
41	.00535	74660	2681	.00437
42	.00597	73514	3053	.00488
43	.00667	72322	3460	.00554
44	.00745	71079	3906	.00626
45	.00835	69780	4393	.00698
46	.00940	68418	4927	.00771
47	.01063	66984	5513	.00887
48	.01208	65468	6159	.01026
49	.01377	63859	6872	.01178
50	.01574	62143	7661	.01375
51	.01799	60309	8533	.01609
52	.02055	58347	9494	.01838
53	.02346	56250	10547	.02075
54	.02681	54012	11694	.02373
55	.03074	51624	12939	.02687
56	.03540	49078	14286	.03059
57	.04098	46365	15739	.03507
58	.04766	43476	17303	.04061
59	.05557	40406	18979	.04695
60	.06474	37162	20755	.05445
61	.07513	33764	22607	.06174
62	.08656	30252	24491	.07039
63	.09887	26687	26343	.07914
64	.11190	23142	28088	.08814
65	.12550	19700	29641	.09752
66	.13970	16437	30924	.10851
67	.15460	13422	31869	.12009
68	.16980	10708	32425	.13166
69	.18510	8333	32557	.14479
70	.20000	6317	32252	.15781
71	.21410	4662	31516	.17085
72	.22710	3349	30381	.18374
73	.23920	2341	28902	.19246

TABLE I—*Continued.*

Age.	Am. Ex. and Karup. F_x	$^{a}l_x$	$^{i}l_x$	Zimmerman. u_x
74	.25070	1594	27144	.19975
75	.26140	1055	25182	.20617
76	.27090	680	23081	.21197
77	.27850	426	20904	.21730
78	.28420	260	18701	.22226
79	.28940	155	16515	.22692
80	.29720	90	14384	.23134
81	.31250	50	12333	.23537
82	.34100	26	10393	.23922
83	.38810	13	8590	.24409
84	.45850	5	6950	.25046
85	.55310	2	5483	.25914
86	.66610		4193	.27164
87	.78380			.29125
88	.88790			.32641
89	.96350			.40773
90	1.00000			.80000

TABLE II.

Am. Ex. and Karup $3\frac{1}{2}\%$.

EXTRA PREMIUMS FOR PERMANENT DISABILITY.

Age.	Whole Life.	20-P. Life.	15-P. Life.	10-P. Life.
20	1.30	.14	.10	.07
25	1.92	.33	.26	.23
30	2.89	.77	.62	.55
35	4.52	1.68	1.41	1.30
40	7.41	3.69	3.11	2.95
45	12.79	8.22	6.91	6.51
50	23.70	18·85	16.07	14.78
55	48.56	44.30	39.82	36.35
60	115.29		106.98	100.71
65	341.58			327.58

TABLE III.

Am. Ex. and Zimmerman $3\frac{1}{2}\%$

EXTRA PREMIUMS FOR PERMANENT DISABILITY.

Age.	Whole Life.	20-P. Life.	15-P. Life.	10-P. Life.
20	1.15	.12	.08	.05
25	1.70	.28	.22	.19
30	2.55	.65	.53	.48
35	3.98	1.43	1.20	1.12
40	6.47	3.14	2.63	2.48
45	11.04	6.97	5.86	5.47
50	20.19	15.79	13.54	12.52
55	40.31	36.34	32.68	30.31
60	90.67		83.25	79.06
65	240.15			229.34

TABLE IV.

Comparison of Table Formed from American Experience and Karup's Rate of Disability with Tables as used by Karup.

	Karup Gotha Life Table.			Am. Ex. and Karup.		
Age.	l_x	$^{aa}l_x$	$^{i}l_x$	Am. Ex. l_x	$^{a}l_x$	$^{i}l_x$
20	102826	102668	158	92637	92637	0
25	100000	99606	394	89032	88818	214
30	97369	96603	766	85441	84897	544
35	94511	93072	1439	81822	80692	1130
40	91152	88302	2850	78106	75763	2343
45	86879	81650	5229	74173	69780	4393
50	81660	72672	8988	69804	62143	7661
55	74779	59825	14954	64563	51624	12939
60	65490	42287	23203	57917	37162	20755
65	53965	22034	31931	49341	19700	29641
70	40467	7109	33358	38569	6317	32252
75	26140	1272	24868	26237	1055	25182
80	13233	127	13106	14474	90	14384
85	4670	5	4665	5485	2	5483
90	941		941	847		847
95	67		67	3		3
100	1		1			

Age.	q_x Karup	q_x Am. Ex.
20	.00583	.00780
25	.00521	.00806
30	.00567	.00843
35	.00646	.00895
40	.00860	.00979
45	.01096	.01116
50	.01500	.01378
55	.02207	.01857
60	.03265	.02669
65	.04719	.04013
70	.07096	.06199
75	.10614	.09437
80	.16188	.14447
85	.23179	.23555
90	.34674	.45455
95	.51440	1.00000
100	.77055	

TABLE V.

B. F. S. Males 1861–1870.

Test of Graduation of Mortality Table.

Age.	Expected Deaths.	Actual Deaths.	Deviation.	Accumulated Deviation.
20-39	8137	8116	+21	+21
40-59	9966	9971	— 5	+16
60-79	5885	5898	—13	+ 3
80-98	467	460	+ 7	—10
All Ages	24455	24445	+10	+10

TABLE VI.

TEST OF GRADUATION OF ${}^{i}q_x$ THE RATE OF MORTALITY AMONG THE PERMANENTLY DISABLED.

Age.	Expected Deaths.	Actual Deaths.	Deviation.	Accumulated Deviation.
20-39	223	221	+ 2	+ 2
40-59	518	530	—12	—10
60-79	1231	1215	+16	+ 6
80-90	238	243	— 5	+ 1
All Ages	2210	2209	+ 1	+ 1

TABLE VII.

TEST OF GRADUATION OF i_x = PROBABILITY OF BEING DISABLED.

Age.	Expected.	Actual.	Deviation.	Accumulated Deviation.
20-39	3501	3523	—22	—22
40-59	9913	9790	+123	+101
60-79	17650	17753	—103	— 2
80-95	369	363	+ 6	+ 4

TABLE VIII.

TEST OF GRADUATION OF $i_{[x]}$ = PROBABILITY OF BECOMING DISABLED.

Age.	Expected.	Actual.	Deviation.	Accumulated Deviation.
20-39	821	803	+18	+18
40-59	1626	1634	— 8	+10
60-79	2247	2273	—26	—16
80-91	63	39	+24	+ 8

TABLE IX.

B. F. S. Males 1861–1870.

Age.	l_x	d_x	i_x	${}^{a}l_x$	${}^{i}l_x$
20	100000	650	.00210	100000	
21	99350	660	.00212	99197	153
22	98690	667	.00215	98529	161
23	98023	674	.00218	97855	168
24	97349	681	.00221	97176	173
25	96668	687	.00225	96491	177
26	95981	693	.00233	95800	181
27	95288	700	.00245	95100	188
28	94588	708	.00261	94390	198
29	93880	717	.00281	93670	210
30	93163	727	.00305	92937	226
31	92436	737	.00331	92191	245
32	91699	751	.00359	91433	266
33	90948	764	.00388	90660	288
34	90184	781	.00418	89874	310
35	89403	798	.00450	89069	334
36	88605	814	.00483	88247	358
37	87791	832	.00517	87408	382
38	86959	850	.00552	86551	408

TABLE IX—*Continued.*

B. F. S. Males 1861–1870.

Age.	l_x	d_x	i_x	${}^a l_x$	${}^i l_x$
39	86109	867	.00589	85676	433
40	85242	887	.00630	84783	459
41	84355	911	.00676	83868	487
42	83444	934	.00728	82926	518
43	82510	961	.00787	81957	553
44	81549	991	.00853	80956	593
45	80558	1024	.00925	79923	635
46	79534	1061	.01007	78853	681
47	78473	1100	.01104	77742	731
48	77373	1139	.01221	76580	793
49	76234	1180	.01360	75369	865
50	75054	1220	.01520	74105	949
51	73834	1265	.01705	72789	1045
52	72569	1313	.01915	71416	1153
53	71256	1367	.02155	69983	1273
54	69889	1426	.02435	68482	1407
55	68463	1485	.02760	66904	1559
56	66978	1544	.03167	65247	1731
57	65434	1601	.03640	63490	1944
58	63833	1663	.04199	61651	2182
59	62170	1731	.04848	59713	2457
60	60439	1812	.05592	57677	2762
61	58627	1901	.06431	55530	3097
62	56726	2000	.07371	53271	3455
63	54726	2098	.08503	50896	3830
64	52628	2182	.10147	48367	4261
65	50446	2254	.12000	45581	4865
66	48192	2313	.14019	42654	5538
67	45879	2356	.16286	39705	6174
68	43523	2384	.18760	36709	6814
69	41139	2420	.21395	33713	7426
70	38719	2468	.24150	30740	7979
71	36251	2514	.27150	27807	8444
72	33737	2544	.30070	24889	8848
73	31193	2558	.32895	22114	9079
74	28635	2536	.35610	19498	9137
75	26099	2481	.38200	17070	9029
76	23618	2411	.40760	14849	8769
77	21207	2333	.43104	12798	8409
78	18874	2244	.45404	10949	7925
79	16630	2149	.47600	9260	7370
80	14481	2039	.49600	7736	6745
81	12442	1905	.51582	6387	6055
82	10537	1756	.53175	5191	5346
83	8781	1586	.54048	4175	4606
84	7195	1384	.54175	3355	3840
85	5811	1181	.53730	2720	3091
86	4630	993	.52536	2207	2423
87	3637	824	.51230	1793	1844
88	2813	664	.49784	1434	1379
89	2149	525	.48194	1136	1013
90	1624	412	.46500	991	733
91	1212	320	.44400	687	525
92	892	245	.42300	526	366
93	647	183	.40200	397	250
94	464	136	.38100	296	168
95	328	112	.36000	216	112
96	216	96			216
97	120	80			120
98	40	40			40

TABLE X.

B. F. S., Males 1861–1870.

MORTALITY TABLE OF DISABLED FORMED FROM $^{i}q_{x}$.

Age.	$^{i}l_{x}$	$^{i}d_{x}$	$^{i}q_{x}$
20	10000	2695	.2695
21	7305	1734	.2375
22	5571	1147	.2059
23	4424	843	.1905
24	3581	634	.1770
25	2947	492	.1670
26	2455	391	.1595
27	2064	316	.1530
28	1748	259	.1480
29	1489	212	.1425
30	1277	174	.1370
31	1103	146	.1320
32	957	122	.1265
33	835	101	.1210
34	734	84	.1145
35	650	71	.1090
36	579	60	.1040
37	519	52	.1005
38	467	45	.0970
39	422	40	.0945
40	382	36	.0925
41	346	31	.0910
42	315	28	.0890
43	287	25	.0875
44	262	23	.0865
45	239	20	.0860
46	219	19	.0850
47	200	17	.0845
48	183	15	.0845
49	168	15	.0844
50	153	13	.0843
51	140	11	.0841
52	129	11	.0839
53	118	10	.0838
54	108	9	.0837
55	99	8	.0836
56	91	8	.0835
57	83	7	.0835
58	76	6	.0835
59	70	6	.0835
60	64	5	.0836
61	59	5	.0837
62	54	5	.0839
63	49	4	.0842
64	45	4	.0845
65	41	3	.0852
66	38	4	.0861
67	34	3	.0880
68	31	2	.0905
69	29	3	.0935
70	26	3	.0970

TABLE X—*Continued.*

B. F. S. Males 1861–1870.

MORTALITY TABLE OF DISABLED FORMED FROM $^{i}q_{x}$

Age.	$^{i}l_{x}$	$^{i}d_{x}$	$^{i}q_{x}$
71	23	2	.1010
72	21	2	.1050
73	19	2	.1095
74	17	2	.1145
75	15	2	.1205
76	13	2	.1265
77	11	1	.1330
78	10	2	.1400
79	8	1	.1480
80	7	1	.1570
81	6	1	.1670
82	5	1	.1780
83	4	1	.1910
84	3	0	.2070
85	3	1	.2240
86	2	0	.2420
87	2	1	.2600
88	1	0	.2770
89	1	0	.2925
90	1	1	.3055
91	0	0	

TABLE XI.

B. F. S. Males, 1861–1870, $3\frac{1}{2}\%$.

EXTRA ANNUAL PREMIUM FOR CESSATION OF ANNUAL PREMIUM PAYMENTS ON BECOMING PERMANENTLY DISABLED, OR ATTAINING AGE 80.

Age.	
20	.28
25	.39
30	.57
35	.84
40	1.30
45	2.08
50	3.46
55	5.99
60	10.78
65	19.89
70	35.95

(Presented by Mr. M. M. Dawson.)

Note on the Calculation of Insurance Values Based on any Mortality that Follows Makeham's Law and an Arbitrary Rate of Interest by Means of Available Tables.

BY CHRISTIAN JENSEN.

A paper on Makeham's formula by Dr. J. P. Gram appears in the Scandinavian *Actuarial Journal*, 1904, in which he extends the use of Makeham's formula by using the force of mortality. As the paper is in Danish I desire to call attention to the method and to develop the theory so as to make it of more practical use in this country.

The formula for the force of mortality is

$$\mu_x = \alpha + \beta e^{\gamma x} = \alpha + \beta c^x$$

where α, β, γ are constants, e is the basis of the natural logarithms, $c = e^{\gamma}$.

$$\mu'_x = \alpha' + \beta' e^{\gamma' x} = \alpha' - \alpha + \alpha + \beta e^{\gamma \frac{\gamma'}{\gamma} x + \operatorname{Log}_e \beta' - \operatorname{Log}_e \beta}$$
$$= \rho + \alpha + \beta e^{\gamma(\lambda x + \epsilon)} = \rho + \mu_{\lambda x + \epsilon}$$

where $\rho = \alpha' - \alpha$, $\epsilon = \dfrac{\operatorname{Log}_e \beta' - \operatorname{Log}_e \beta}{\gamma}$, $\lambda = \dfrac{\gamma'}{\gamma}$.

We may, therefore, use a given force of mortality, $\mu_x = \alpha + \beta e^{\gamma x}$, instead of any other force of mortality, $\mu'_x = \alpha' + \beta' e^{\gamma' x}$, if we make at most three modifications, namely, (a) multiplication of the age by a constant, (b) addition of a constant to the thus transformed age and (c) addition of a constant to the force of mortality.

Dr. Gram has

$$\bar{a}_{x.\overline{n}|} = \int_0^n \frac{D_{x+t}}{D_x} dt = \int_0^n e^{-\int_0^t (\delta + \mu_{x+t}) dt} dt$$

where δ is the continuous rate of interest.

We have also

$$a_{x.\overline{n}|} = \sum_{t=1}^{t=n} \frac{D_{x+t}}{D_x} = \sum_{t=1}^{t=n} c^{-\int_0^t (\delta + \mu_{x+t}) dt}$$

From the last two formulas we see that a constant addition ρ to δ is equivalent to a constant addition ρ to μ_x in the calculation of annuities. This holds for any law the force of mortality may follow.

A change in β in Makeham's formula causes an algebraic addition to the age in the corresponding annuities, while a change in γ leads to a more complicated transformation which applies only to continuous annuities.

$\bar{a}'_{x.\overline{n}|}$ based on $\delta' + \mu'_x = \delta' + \alpha' + \beta' e^{\gamma' x}$ is expressed by continuous annuities based on $\delta + \mu_x = \delta + \alpha + \beta e^{\gamma x}$, by the equation

$$\bar{a}'_{x.\overline{n}|} = \frac{1}{\lambda} \bar{a}_{z\overline{\nu}|}$$

where $z = \lambda x + \zeta$, $\zeta = \dfrac{\log_{10} \beta' - \log_{10} \beta - \log_{10} \lambda}{\log_{10} c}$, $\nu = \lambda n$

and $\delta + \alpha = \dfrac{\delta' + \alpha'}{\lambda}$ determines the value of δ, the value of α being given. We see thus that if we have annuity tables based on a certain mortality following Makeham's law and so many different values of the continuous rate of interest δ that $\bar{a}_x$ for an arbitrary value of δ can be found by interpolation we can calculate the annuity values corresponding to any other mortality following Makeham's law and an arbitrary rate of interest by means of interpolation for age and rate of interest and multiplication by a constant.

1. By means of this theory I will, in the first place, investigate how a constant addition ρ to the force of mortality will influence premiums and reserves for Life and Endowment policies.

$P_x((\delta + \rho) + \mu)$ is based on force of mortality μ_x and continuous rate of interest $\delta + \rho$,

$P_x(\delta + (\rho + \mu))$ is based on force of mortality $\rho + \mu_x$ and continuous rate of interest δ.

Ordinary Life

$$(P_x(\delta + (\rho + \mu)) + d)(1 + a'_x) = 1,$$

$$P_x(\delta + (\rho + \mu)) = \frac{1}{1 + a'_x} - d = \frac{1}{1 + a'_x} - d' + d' - d,$$

hence

$$P_x(\delta + (\rho + \mu)) = P_x((\delta + \rho) + \mu) + d' - d,$$

where $a'_x = a_x(\delta + (\rho + \mu)) = a_x((\delta + \rho) + \mu)$ is based on force of mortality μ_x and continuous rate of interest $\delta + \rho$, $d = 1 - e^{-\delta}$, $d' = 1 - e^{-(\delta + \rho)}$.

Endowments with premiums payable during the whole term may be dealt with in the same way.

$$P_{x\overline{n}|}(\delta + (\rho + \mu)) = P_{x\overline{n}|}((\delta + \rho) + \mu) + d' - d.$$

Limited Payment Life.

$$\begin{aligned} {}_nP_x(\delta + (\rho + \mu)) &= P_x(\delta + (\rho + \mu)) \frac{N'_x}{N'_x - N'_{x+n}} \\ &= (P_x((\delta + \rho) + \mu) + d' - d) \frac{N'_x}{N'_x - N'_{x+n}} \\ &= {}_nP_x((\delta + \rho) + \mu) + (d' - d) \frac{N'_x}{N'_x - N'_{x+n}} \end{aligned}$$

N' is based on force of mortality μ_x and continuous rate of interest $\delta + \rho$.

Reserves,

$${}_tV_{x\overline{n}|} = 1 - \frac{1 + a_{x+t.\,\overline{n-t-1}|}}{1 + a_{x.\,\overline{n-1}|}}$$

As a constant addition ρ to the continuous rate of interest is equivalent to a constant addition ρ to the force of mortality in the calculation of annuities, reserves for Ordinary Life policies and Endowment policies with premiums payable during the whole term may be found by interpolation for rate of interest.

Limited (n) Payment Life. Reserve end of t' year.

$$\begin{aligned} {}_tV_x^{(n)}(\delta + (\rho + \mu)) &= {}_tV_x(\delta + (\rho + \mu)) \\ &\quad + P_x(\delta + (\rho + \mu)) \frac{N'_{x+n}}{N'_x - N'_{x+n}} \cdot \frac{N'_x - N'_{x+t}}{D'_{x+t}} \\ &= {}_tV_x((\delta + \rho) + \mu) \\ &\quad + (P_x((\delta + \rho) + \mu) + d' - d) \frac{N'_{x+n}}{N'_x - N'_{x+n}} \cdot \frac{N'_x - N'_{x+t}}{D'_{x+t}} \\ &= {}_tV_x^{(n)}((\delta + \rho) + \mu) \\ &\quad + (d' - d) \frac{N'_{x+n}}{N'_x - N'_{x+n}} \cdot \frac{N'_x - N'_{x+t}}{D'_{x+t}} \end{aligned}$$

N' and D' are based on force of mortality μ_x and continuous rate of interest $\delta + \rho$.

2. A change in β in Makeham's formula causes an algebraic addition to the age in the corresponding premiums and reserves, and may, therefore, be dealt with by interpolation in available tables.

3. The formulas become more complicated when γ changes. In this case we must interpolate on continuous insurance values, and it may be expedient first to find the annuities and from these to calculate the desired insurance values.

The formulas may especially be of use in dealing with lives with a super-standard mortality.[1] It appears, according to Dr. Gram, that the mortality among substandard lives insured by life insurance companies can very often be represented by Makeham's formula with sufficient accuracy for ages 25 and upward to be used in satisfactorily determining premiums and reserves. The mortality among cigar-makers, the mortality among spinners and weavers, etc., each one represented by Makeham's formula, are given in Dr. Gram's paper. The tropical experience of New York Life Insurance Company is represented by Makeham's formula in the *Transactions* for 1894. It is not very laborious to calculate premiums and reserves based on a super-standard mortality represented by Makeham's formula with the same c (or γ) as a standard mortality according to which sufficiently extensive tables have been calculated. We may, therefore, try to undertake the graduation of a mortality experience on substandard lives in such a way that this will be the case and only in case this does not give satisfactory results, determine all three constants in Makeham's formula from the mortality experience in question.

In *Transactions*, No. 29, Mr. Hunter dealt with reserves for lives with super-standard mortality. He assumed the standard mortality multiplied by a constant factor or increased by a constant addition. If, instead of assuming as Mr. Hunter, that the constant factor or the constant addition is determined with regard to q_x we assume that the constant factor or the constant addition is determined with regard to μ_x the two cases mentioned may be dealt with according to the presented formulas.

In case of a constant factor to μ_x we shall have

$$\begin{aligned}\mu'_x &= k\mu_x = k\alpha + k\beta e^{\gamma x} = (k-1)\alpha + \alpha + \beta e^{\gamma x + \operatorname{Log}_e k} \\ &= (k-1)\alpha + \alpha + \beta e^{\gamma(x+\epsilon)} = (k-1)\alpha + \mu_{x+\epsilon}\end{aligned}$$

where $\epsilon = \dfrac{\operatorname{Log}_e k}{\gamma} = \dfrac{\log_{10} k}{\log_{10} c}$.

In using American Experience tables instead of Makehamized American Experience tables the results will be approximately correct.

In case of a constant addition to μ_x the reserves will be found to agree very closely with Mr. Hunter's reserves in case of a constant addition to q_x. For instance

[1] Lives with a higher mortality than American Experience I call substandard lives.

$\mu'_x = \text{Am. Exp. } \mu_x + .005$. Interest 3%.

$\delta = \text{Log}_e 1.03 = .0295588$, $\rho = .005$, $\delta + \rho = .0345588$,

$i' = e^{\delta+\rho} - 1 = .035163$, $d' - d = e^{-\delta} - e^{-(\delta+\rho)}$

$= .004842$.

Amount of policy $1,000.

	Ord. Life. ${}_{10}V_{40}$	20 Paym. Life. ${}_{10}V^{(20)}_{40}$	20 Year End. ${}_{10}V_{40\,\overline{20}|}$
$i = .035$	166.89	261.10	396.66
$i = .04$	157.19	241.12	385.51
$\Delta =$	— 9.70	— 19.98	— 11.15
$2 \times .0163\,\Delta =$	— .32	— .65	— .36
$i = .035163$	166.57	260.45	396.30

$$1000\,(d' - d)\,\frac{N'_{60}}{N'_{40} - N'_{60}}\,\frac{N'_{40} - N'_{50}}{D'_{50}} = 19.36$$

$${}_{10}V^{(20)}_{40}(\delta + (\rho + \mu)) = 279.81$$

Mr. Hunter has in the *Transactions* as corresponding reserves based on mortality Am. Exp. $q_x + .005$ and interest 3% ${}_{10}V'_{40} = 166.78$, ${}_{10}V^{(20)\prime}_{40} = 279.99$, ${}_{10}V'_{40\,\overline{20}|} = 396.16$.

A universal table which will enable us to calculate annuities based on any mortality that follows Makeham's law and an arbitrary rate of interest, readily, is published in the Scandinavian *Actuarial Journal* following Dr. Gram's paper.

Annuities and insurances on joint lives based on the mortality among substandard lives may be dealt with in the same way as annuities and insurances on single life based on the same mortality. It will also hold for different classes of lives if there is the same constant c in Makeham's formula for the mortality among the different classes of lives.

I will calculate an annuity on joint lives based on Makehamized American Experience and interest 3% by means of the universal table, in order to illustrate the use of this and the transformation from joint lives to single life.

$$\mu_{xy} \ldots (n \text{ lives}) = \mu_x + \mu_y + \ldots = \alpha + \beta e^{\gamma x} + \alpha + \beta c^{\gamma y} + \ldots$$
$$= n\alpha + \beta e^{\gamma x}\,(1 + e^{\gamma(y-x)} + \ldots) = \alpha' + \beta' e^{\gamma x}$$
$$= \mu'_x$$

where $\alpha' = n\alpha$, $\beta' = \beta\,(1 + e^{\gamma(y-x)} + \ldots) = \beta(1 + c^{y-x} + \ldots)$.

Here log means the logarithm with 10 as basis.

In Makehamized American Experience $a = .0075913$, $\log \beta = \bar{5}.5059982$, $\log e^{\gamma} = \log c = .04579609$. A continuous annuity based on $\delta' + \mu'_x = \delta' + a' + \beta' e^{\gamma x} = \delta' + a' + 10^{\log \beta' + x \log c}$ is to be calculated by interpolation on logarithms of continuous annuities based on $\delta + \mu_x = .1s + 10^{-4} e^{.1x} = .1s + 10^{-4 + .043429 \ldots x}$.

The numbers of transformation are

$$\lambda = \frac{\gamma}{.1} = \frac{\log c}{.043429\ldots} = \frac{.04579609}{.04342945} = 1.054494.$$

$$\log \lambda = .0230441.$$

$$z = \lambda x + \zeta = \lambda x + \frac{\log \beta' + 4 - \log \lambda}{.043429\ldots}$$

$$= \lambda x + \frac{\log \beta + 4 - \log \lambda}{.043429\ldots} + \frac{\log (1 + c^{y-x} + \ldots)}{.043429\ldots}$$

$$z = 1.054494x - 11.9054 + \frac{\log (1 + c^{y-x} + \ldots)}{.043429\ldots}.$$

For two lives,

$$.1s = \frac{\delta' + a'}{\lambda}, \quad s = \frac{\delta' + 2a}{.1\lambda} = \frac{.0447414}{.1054494} = .424293$$

$x = 30$, $y = 40$

$z = 33.263$

		$s = .42$	$s = .42429$	$s = .44$
$x = 33$	$\log \bar{a}_x =$	1.22218		1.21118
$x = 34$	$\log \bar{a}_x =$	1.21542		1.20462
	$\Delta =$	— .00676		— .00656
	$.263\ \Delta =$	— .00178		— .00173
	$\log \bar{a}_z =$	1.22040		1.20945
	$\Delta' =$			— .01095
	$\frac{.429}{2} \Delta' =$	— .00235		
	$\log \bar{a}_z =$	1.21805	for $s =$	.42429
	$\log \lambda =$	.02304		
	$\log \bar{a}_{xy} =$	1.19501	as $\bar{a}_{xy} = \frac{1}{\lambda} \bar{a}_z (s = .42429)$	
	$\bar{a}_{xy} =$	15.668		
		.504		
	$1 + a_{xy} =$	16.172		

The same result may be obtained from Mr. Hunter's tables in the *Transactions* if allowance is made for 2′ difference in the interpolation there.

The Genesis of the American Experience Table.

DAVID PARKS FACKLER.

Last Autumn the Secretary of the Society wrote me that, at the meeting of the Council held in Toronto on October 10th, a resolution was passed asking me, "to prepare an account of the construction of the American Experience Table." The Secretary's letter adds, "you are the only man now living that has a knowledge of the materials from which the table was constructed and the method of graduation, and the Council accordingly felt that a statement by you on this subject would be of great value for insertion in our *Transactions.*"

Recognizing the reasonableness of this request, as well as the honor done me, I have endeavored to refresh my recollection on all points and have consulted many old books and papers, but fear that I shall not be able to do much in the way of giving a really satisfactory account.

In 1858 Mr. Homans compiled the experience of the Mutual Life Insurance Company for its first fifteen fiscal years, ending February 1st, 1858, and his report was published by the Company in a quarto volume, very few copies of which are extant. The Company's experience was elaborately stated in five different classifications based on residence, and also in three divisions, for life, endowment and term policies. On pages 20 and 21 he presented an adjusted table of mortality based on the experience of the whole Company for the fifteen years. This table starts with 100,000 persons living at the age of 10, and ends with one person living at 99 and dying at that age. It will be noted immediately that this differs from the American Table, which makes 95 the oldest age attained. The rates of mortality in this 1858 table are stated only up to age 74 inclusive, though the number of persons surviving to ages 79, 84, 89, 94 and 99 are shown, as well as the number of deaths occurring at those ages. On making up the rates from the numbers of living and dying, they are found to be the same as by Professor Gill's Table. I should state that Professor Gill was

the first Actuary of the Company. He based the Company's premiums upon a graduation of the Carlisle and other English tables prepared by himself. He stated that this table should be modified by future actual American experience.

The oldest person in the Company was only 78 years old and Mr. Homans felt that the Company had no experience at these advanced ages upon which any rates of mortality could properly be based, and therefore assumed the same rates as in Professor Gill's Table. Mr. Homans refers to the twelve-year experience of the Mutual Benefit as agreeing with his own Company's experience "in a remarkable manner," and states that he believes that "if such observations are continued we will soon have the requisite data for framing a reliable *American* Table of Mortality which will be an accurate representation of the relative value of life amongst the classes of assured lives at the different ages." The adjusted rates of mortality by this (1858) table of Mr. Homans are shown further along in this article where they are placed side by side with those of the American Table.

Before proceeding, however, it must be remarked that no one can read this volume, showing the mortality experience of the Mutual Life Insurance Company for its first fifteen years, without profound admiration for the industry, ability and genius shown by Mr. Homans, who had no knowledge of life insurance prior to his appointment as Actuary. He was chosen Actuary of the Mutual Life without his knowledge, while serving as an astronomer in a United States Government expedition on the Western plains, and was then in his 25th year of age. Shortly after I joined him in the service of the Company, he once remarked to me that he did not know what the word "premium" meant until after he became the Actuary of the Company. I think he did not actually enter this Company's office until January, 1856. When it is considered that Mr. Homans had nothing to guide his studies except the very few and imperfect English publications then in existence, and the scanty records left by his predecessor, Professor Gill, it is amazing that he was able to study up the subject and compile the Company's experience, as he did so well, within less than three years. Had Mr. Homans been framed in a strictly mathematical or scientific mould it might not have seemed so remarkable, but as we know, he was intensely human, being always sociable, companionable and fond of the enjoyments of life.

In the years 1859 and 1860 Mr. Homans continued the compilation of the Company's experience, and in 1860 framed what was afterwards known as the American Experience Table. While based in the main upon the experience of the Mutual Life Insurance Company, it, in a considerable degree, represented Mr. Homans' personal opinion of the probable rate of mortality among insured lives after the immediate effects of medical selection had expired. The adjusted table of mortality prepared in 1858 showed the mortality experience of the entire Company, and was much affected by the heavy mortality in the extreme Southern and Western States. Mr. Homans' new table was intended to represent the death rate among insured lives residing in salubrious districts after the effects of medical selection were eliminated. When these two points are considered it will be understood how it is that, for age thirty-five and under, Mr. Homans' new table generally shows a somewhat lower rate of mortality than does the table deduced from the Company's general experience, while above age thirty-five the rate is higher than by the Company's general experience.

In the Spring of 1861, just as the Civil War was breaking out, Mr. Homans went to England to consult British actuaries as to the best method of dividing surplus and also as to some proposed new tables of premiums for survivorship annuities.

After his return these tables were prepared and were first published in the Company's rate book for 1862 with a preface in which it is stated; "These rates of premium have been calculated at great labor and upon an entirely new and original basis, which has been submitted to, and received the unanimous approval of, some of the most experienced and able actuaries in Great Britain as well as some of the leading insurance officials in this country." These rates are based on the American Table with a loading of thirty per cent. They occupy eight pages of the Company's rate book for 1862 and were published by that Company and the Equitable Life for many years afterwards. From the quotation it would appear that the mortality table and the mode of computing the rates had been shown to the English Actuaries in the Spring of 1861, while the calculation of the premiums in detail was made after Mr. Homans' return from England later in 1861.

From the foregoing it will be seen that the American Table was in existence early in 1861, and so early that I felt justified in stating that it was constructed in 1860 after the experience for the

years ending February 1st, 1859 and February 1st, 1860 had been compiled. Mr. Homans went to England so early in 1861 that he could not have had time to complete the computation of the Company's experience for the year ending February 1st, 1861, and also prepare the American Table from this additional experience before his departure.

Mr. Homans did not give his table its present name, and it was not called the American Experience Table until Superintendent Barnes proposed it as the legal basis for the State of New York, in 1868 and 1869. In his annual report in 1869 Mr. Barnes said, "The American Experience Table of Mortality was constructed mainly on the actual experience of the Mutual Life Insurance Company of New York. The Actuary of that Company, Mr. Sheppard Homans, who compiled the experience and constructed the table, availed himself of all the other statistics at hand for ascertaining the laws of mortality applicable to healthy insured lives in this country . . . . . and all the standard European Tables were also used for the purpose of comparison in graduating and modulating the American Table."

I entered the Mutual Life office in July, 1859, as one of the assistants in the Actuarial Department, and was employed in extending the Company's experience but had no part in graduating it, or in making what is now known as the American Table. It was at first called simply and familiarly the "Homans' Table," and I have many old manuscript tables bearing that designation. As previously stated, I believe the American Table was framed in 1860 while I was too young and inexperienced to be consulted thereabout, having been in the Company's office only about a year.

It has often been thought that the American Table was based on the statistics contributed by the Companies represented at the American Underwriters Convention which first met in May, 1859. That, however, is entirely a mistake, although in the semi-centennial history of the New York Life there is a statement to that effect on page 55. I asked Mr. Homans regarding this point and he wrote me, March 3d, 1896, saying "the statement in regard to the basis upon which the so-called American Experience Table was constructed is incorrect. The statistics from the thirteen companies referred to were not used in the preparation of that table."

My first work in the Mutual Life Insurance Company was in the preparation of statistics for the joint experience of those

thirteen companies. The work, however, was abandoned, owing to various causes, and nothing ever resulted therefrom. The later statistics of American companies, from which Mr. Meech's tables were prepared, were made up afresh and entirely independently of the earlier attempt.

The graduation of the Mutual Life's 15 year experience to 1858 was made graphically, as I was positively informed by one who saw the work done; but my informant knew nothing as to the method employed for the American Table. I remember that Mr. Homans once spoke of having employed a graduation formula in connection with that table, but did not mention which. His son, Smith Homans, recently showed me a pamphlet on mortality tables that was presented to his father in 1860 by Dr. Farr, and we both thought it might have aided his father in his work, but we could not discover any marks to indicate that he had employed any of its formulæ.

It is very remarkable that this table, based largely upon judgment rather than actual experience, should be so nearly the same as if it had been based on the Makeham formula of several years later. A comparison with the rates of mortality by the Makehamized American Table of Mr. Arthur Hunter will be interesting in this connection, and can be made by examining the exhibit at the close of this article.

Before closing this it may be proper to speak further of Professor Charles Gill, the first Actuary of the Mutual Life Insurance Company. He appears to have been the first American to frame a mortality table and compute independent rates. He was a fine mathematician and published a duodecimo volume of one hundred pages entitled "Angular Analysis"—of which I have a copy. This system of analysis afforded solutions of the problems to which the Diophantine analysis is usually applied. Mr. Homans' report speaks of him as the author of a book entitled "Assurance Tables," and states that in 1851 he made a report upon the Company's Experience, for its first eight years, among its members residing in the Northern States; and President Winston in his introductory remarks prefacing Mr. Homans' report speaks of reports having been presented by Professor Gill both in 1851 and 1853 and says they were "*marked* by the general ability which distinguished his acts." Without derogating from Mr. Homans' credit it must be conceded that Professor Gill's work must have aided Mr. Homans

considerably. If I have some leisure this summer I will endeavor to learn more of Professor Gill's life, if any biography was published, for as he appears to have been the first person on this Continent that served as an actuary in a company's office, the record may interest all of us.

At the beginning of this paper I stated my fears that I could not give you much information regarding the Genesis of the American Table, and now, in closing, I am afraid you will all be disappointed, though I have done my best to furnish the information desired. It is greatly to be regretted that Mr. Homans never published any account of the method employed by him in preparing the table.

COMPARISON OF THE RATES OF MORTALITY OF THE AMERICAN EXPERIENCE TABLE WITH THOSE OF VARIOUS TABLES.

(Percentages.)

Age.	Prof. Gill's Table.	Homans' 1858 Table.	Homans' American Table 1860.	Makehamized American Hunter, 1902.
10	.526	.741	.749	.765
15	.615	.763	.763	.773
20	.710	.790	.781	.784
25	.818	.826	.807	.803
30	.943	.866	.843	.836
35	1.089	.907	.895	.891
40	1.264	.932	.979	.984
45	1.488	1.010	1.116	1.142
50	1.852	1.341	1.378	1.408
55	2.381	1.666	1.857	1.858
60	3.195	2.361	2.669	2.615
65	4.525	3.659	4.013	3.888
70	6.494	5.935	6.199	6.003
*	*	*	*	*
74†	8.847	8.847	8.703	8.644
*	*	*	*	*
79†	13.006	13.006	13.173	13.746
*	*	*	*	*
84†	18.968	18.960	21.136	21.739
*	*	*	*	*
89†	29.238	29.221	39.586	33.651
*	*	*	*	*
94†	51.630	51.643	85.714	49.565
95	——	——	100.000	53.448
*	*	*		*
99	100.000	100.000		100.000

† As before stated, these are the only advanced ages shown in Mr. Homans' 1858 Report.

Abstract of the Discussion of Papers Read at the Previous Meeting.

Valuation of Policies on the Select and Ultimate Basis—H. N. Sheppard.

Vol. X, Page 141.

Written Discussion.

Mr. Peiler: Mr. Sheppard's adaptation of the accumulation method by u and k columns will be found very useful in determining reserve values on the Select and Ultimate basis under special forms of insurance. The accumulation method is extremely simple and capable of general application wherever the underlying net premiums have been determined. Thus, in the case of varying premiums it is only necessary, in the process of accumulation, to add the premiums corresponding with the years of insurance to the reserve fund and, in the case of varying insurance, to deduct the costs covering the yearly benefits.

In the office with which I am connected we constructed D and N and u and k columns on the Select and Ultimate basis, soon after its adoption, and proceeded with some nicety as the results obtained from accurate tables may be verified by differences.

In dealing with special forms of contract the average net premium and the terminal value of the fifth year can always be determined, but it frequently occurs that only the u and k or the D and N columns are available for valuation purposes. In such cases Mr. Sheppard's adaptation of the accumulation method may be applied inversely, from the fifth terminal value to the fourth and so on, resulting with the initial value for the first year of insurance. The process is clumsy but may be applied under almost any condition.

Several other methods are herewith stated which have been found of practical value in special cases.

Given, the level premium and fifth value, to obtain π_x, the initial value for the first year:

$$\pi_x = v - \frac{(1 - {}_5V_x)D_{x+5} + (P_x + d)(N_{[x]+1} - N_{x+5})}{D_{[x]}}$$

Given the initial value, π_x, terminal value, ${}_nV_{[x]}$, and mid-year value, ${}_{n+\frac{1}{2}}V_{[x]}$, under one form of level premium contract, to determine similar values under another form, π'_x, ${}_nV'_{[x]}$, ${}_{n+\frac{1}{2}}V'_{[x]}$, from the difference of the net premiums, $P'_x - P_x$:

$$\pi'_x = \pi_x + (P'_x - P_x)\frac{N_x - N_{[x]+1}}{D_{[x]}}$$

$${}_nV'_{[x]} = {}_nV_{[x]} + (P'_x - P_x)\frac{N_x - N_{[x]+n}}{D_{[x]+n}}$$

$${}_{n+\frac{1}{2}}V'_{[x]} = {}_{n+\frac{1}{2}}V_{[x]} + (P'_x - P_x)\frac{N_x - N_{[x]+n+1}}{D_{[x]+n+\frac{1}{2}}}$$

where $$\frac{1}{D_{[x]+n+\frac{1}{2}}} = \tfrac{1}{2}\left(\frac{1}{D_{[x]+n}} + \frac{1}{D_{[x]+n+1}}\right) = \tfrac{1}{2}\frac{1+u_{[x]+n}}{D_{[x]+n}}$$

Given the fifth terminal value and level net premium, to determine consecutively the terminal values for the fourth, third, second and first years and the initial value for the first year:

$$1 - {}_nV_{[x]} = vp_{[x]+n}(1 - {}_{n+1}V_{[x]}) + P_x + d$$

$1 - {}_nV_{[x]}$ is applied in the next preceding operation after ${}_nV_{[x]}$ has been read by complement and recorded.

$$1 - \pi_x = vp_{[x]}(1 - {}_1V_{[x]}) + d.$$

The last two formulae are applicable to joint life contracts. A table of $vp_{[x]+n}$ furnishes the necessary data for the factor $vp_{[x]+n,\,[y]+n}$ for two lives.

Mr. J. H. Woodward: Mr. Sheppard's brief paper on the computation of select and ultimate reserves is useful and timely. In cases where the present value of the assumed mortality gains have already been computed for another purpose, the method suggested provides a means by which the reserves may be readily obtained. Mr. Sheppard recommends a continuous retrospective valuation, which has the advantage of being applicable to all plans of insurance, and he furnishes the necessary select accumulation columns on the 3½% basis.

When the amount of insurance is constant and the net premium from the ultimate table is level, select and ultimate reserves can be conveniently obtained without previously computing the present value of the assumed mortality gains by using the method brought forward by Mr. M. H. Peiler in a pamphlet recently copyrighted by him. Mr. Peiler's method is to multiply the difference between the net ultimate premium on the policy to be valued and the net ordinary life ultimate premium at the same age by an auxiliary factor, of which he has tabulated the values, and to the product so obtained add the select and ultimate ordinary life reserve.

A proof of the equivalence of these two methods may be of interest to students and is as follows:

With the usual notation, n being less than the period of selection, we have

By Mr. Sheppard's formula

$${}_nV'_{(x)} = ({}_{n-1}V'_{(x)} + \pi_x)\,U_{(x)+n-1} - K_{(x)+n-1} \qquad (\alpha)$$

By Mr. Peiler's formula

$$ {}_{n}V'_{(x)} = {}_{n}V_{(x)} + \left(\frac{N_x - N_{(x)+n}}{D_{(x)+n}}\right)(\pi_x - P_x) $$

which may be written

$$ {}_{n}V'_{(x)} = ({}_{n-1}V_{(x)} + P_x)\,U_{(x)+n-1} - K_{(x)+n-1} + \left(\frac{N_x - N_{(x)+n}}{D_{(x)+n}}\right)(\pi_x - P_x) \qquad (\beta) $$

Equating the right-hand sides of equations (α) and (β) and observing that $K_{(x)+n-1}$ disappears, we have

$$ ({}_{n-1}V'_{(x)} + \pi_x)\,U_{(x)+n-1} = ({}_{n-1}V_{(x)} + P_x)\,U_{(x)+n-1} + \left(\frac{N_x - N_{(x)+n}}{D_{(x)+n}}\right)(\pi_x - P_x) $$

whence

$$ ({}_{n-1}V'_{(x)} - {}_{n-1}V_x + \pi_x - P_x)\,U_{(x)+n-1} = \left(\frac{N_x - N_{(x)+n}}{D_{(x)+n}}\right)(\pi_x - P_x) $$

Or, in commutation symbols,

$$ \left\{ \left(\frac{N_x - N_{(x)+n-1}}{D_{(x)+n-1}}\right)(\pi_x - P_x) - \left(\frac{M_x - M_{(x)+n-1}}{D_{(x)+n-1}}\right)(1-1) + (\pi_x - P_x) \right\} \frac{D_{(x)+n-1}}{D_{(x)+n}} = \left(\frac{N_x - N_{(x)+n}}{D_{(x)+n}}\right)(\pi_x - P_x $$

Dividing both sides by $(\pi_x - P_x)$,

$$ \left(\frac{N^x - N_{(x)+n-1}}{D_{(x)+n-1}} + 1\right)\frac{D_{(x)+n-1}}{D_{(x)+n}} = \frac{N_x - N_{(x)+n}}{D_{(x)+n}} $$

whence

$$ N_x - N_{(x)+n-1} + D_{(x)+n-1} = N_x - N_{(x)+n} $$

or

$$ N_{(x)+n-1} - D_{(x)+n-1} = N_{(x)+n} $$

From the elementary properties of commutation columns this equation is known to be true; whence follows the equivalence of equations (α) and (β).

And I may say that in the current number of the *American Underwriter* it is pointed out that Mr. Peiler's method is preferable where the select and ultimate reserves have to be computed on a large number of plans at the same time, for the reason that his multiplying factor can be set up on the arithmometer once for all, and then the difference between the net ordinary life premium and the net premium on the plan to be followed can be multiplied through in each case on all at the same time.

MR. J. M. CRAIG: I regret to say that I have not prepared a written discussion, Mr. President, and the only thought that was in my mind on the paper was on the cause that prompted its preparation.

The paper itself, as you all are aware, is on a method of calculating reserves on the select and ultimate basis, and while it is probable that all the companies had prepared their valuation sheets in anticipation of what they were required to do, yet it was very kind on the part of Mr. Sheppard to prepare his u and k columns, because they become valuable for special forms of contract. But, in his preliminary statement, he says:

"As it is necessary by Section 84 of Chapter 326, Laws of the State of New York, for every life insurance company doing business in that State to report the excess of the valuation of its policies over the legal minimum standard, it is necessary to consider the best method of calculating reserves on the Select and Ultimate Basis."

Now, it is a curious fact, that, so far as I am aware, no life insurance company transacting business in the State of New York values for its own purposes on the Select and Ultimate Table, and the only purpose for which such a valuation is made is to comply with the law, so that the difference between the valuation which any company uses for its own purposes and a like valuation on the Select and Ultimate method may be stated somewhere in its report to the Superintendent of Insurance, and just why the increased labor and expense was imposed by law upon the companies to make this valuation which is of no use to them or to the Insurance Department, it is hard to determine.

I do not mean to be understood as criticising the Select and Ultimate Method of Valuation, because I believe that it is preferable to the preliminary term, in that it requires the full reserve to be maintained at the end of five years, and it is evident to every thoughtful man that new companies do need some help in that line. But, the obection in my mind is to a law requiring a company to do certain work that is of no benefit to anybody.

RECENT INSURANCE LEGISLATION—E. E. RHODES.
VOL. X, PAGE 145.

WRITTEN DISCUSSION.

MR. IRELAND: In his paper on "Recent Insurance Legislation," Mr. Rhodes has given us a valuable compilation of certain insurance laws passed since 1906 by some twenty-six of the States of the Union; he has given us a double index to the paper, and has throughout given specific references to laws and sections, the whole involving a good deal of work that was very much like drudgery, and also a good deal of brain work, for only a person who intelligently understood the meaning and bearings of the various laws could satisfactorily prepare a paper of this kind. The whole paper constitutes an index and a summary, and is not only of use to the person who wishes to find what stand a given State has taken on some specified point, but also to the student who desires a bird's-eye view of the legislation referred to, wherein he can see grouped about each point of the discussion sketches of the positions that have been taken by the several States.

Mr. Rhodes has divided the subject of legislation into 39 parts, and it is perhaps safe to assume that the subjects of his 39 articles are more familiar to some of this audience than are the 39 articles generally referred to, in serious assemblies, under that title. In the lists given in the paper of the States and Territories that have and have not enacted laws of the class under consideration, no mention is found of the District of Columbia, Iowa, Kentucky, Maryland, Mississippi, Nevada, Ohio, Oklahoma, Vermont, Virginia, or Washington, but it is believed that these might properly have been included in the list of States that have not enacted any new laws relating to life insurance.

If now we want to get any adequate conception of the amount of insurance lawmaking that there has been in the United States during the last two years and a half, we need to add to the contents of this paper the various matters that are specifically excluded therefrom. In the first place the paper relates to laws "enacted subsequently to the Armstrong legislation in New York," and the last named great mass of most important lawmaking is therefore to be found and studied elsewhere. Secondly, the paper does not include the various State laws that "relate solely to companies of local origin;" doubtless Mr. Rhodes thought that each company would be familiar with the laws of its own State, and that it would only be a source of confusion and of inconvenient increase in bulk to include these laws in his summary. Thirdly, in section number 40 he says, "It has not been deemed necessary to refer to the legislation in Texas which resulted in the enforced withdrawal of practically all the established companies doing business in that State. Neither have I referred to the legislation in Wisconsin." He proceeds to give some particulars of the new Wisconsin laws, and to state difficulties in the way of giving a complete summary of them. Finally he notifies us that the article

is intended "to cover only regular forms of insurance." If all these excluded matters should be brought together and added to the paper, one would get some idea of the effect of what may be termed the French Revolution of Life Insurance.

In the paper itself we find an interesting and startling exhibition of the effect of legislation by nearly 30 different bodies on the same subject matter, going, in many cases, very much into detail. The whole attitude of the people of the country and of their mouthpieces in the lawmaking bodies had been completely changed in a very brief period. What the men who had been engaged in carrying on life insurance for many years had learned about the subject, what they believed to be for the best interest of the company as a whole, and of the individual policyholders, was largely set aside, and the legislators voted according to their several lights as to the precise form of policy to be used by a company,—as to the number of days of grace that should be allowed, as to the safeguards that a company might, or might not, make use of in its dealings with individuals, and as to many other matters of great importance or of minor importance as the case might be. Almost every privilege that had been granted to their policyholders by some of the reputable companies was by law forced into the contracts of all companies. The differing opinions of experienced and honest insurance men, as to what privileges could wisely and safely be put into the contract and what things it would be more prudent to omit, were practically thrown overboard, and the discretionary power of the managers of life companies was very greatly abridged. Naturally, the various States did not agree as to details and methods, and a mutual company desiring to do business in a number of States, and to give fair treatment and equal benefits to all its policyholders, found it a difficult thing so to do; the practical effect has been in a general way, to require such a company to give in all the States the benefits and privileges that might be required in any one of them. Even then the trouble did not end. For example, when one State prescribed *a month* for the time of giving any specified notice or grace, and another State prescribed *thirty days*, or when any other discordant provision appeared, there was at once a discrepancy that had to be got around in some way; and the submission of forms of application and policies, first to one State and then to another, and the harmonizing of the requirements of the different commonwealths, made an amount of trouble that many of you fully and painfully appreciate. By way of elucidation one may refer to the four and a half pages given in Mr. Rhodes' paper to section 8, on the distribution of surplus; to the three pages given to section 9, on non-forfeiture; and the more than three pages given to section 24, on valuations.

Another feature in the business that has grown and become more prominent in these later years is the freer exercise of the wide discretionary power given to the insurance officials of the various States: such interpretations of law as they saw fit to make had to be accepted, at least for the time being; such additional requirements not to be found in any law, as they thought proper to put upon the companies were of binding force, and added to the burden.

It is an evidence of the inherent strength and majesty of the business, and of the faith and common sense of the real insurance men who are in the management of it, that with all these laws in operation policies have been made to conform to what was demanded, the business goes on, and the hope of all concerned is that the results of present laws may prove to be somewhere near as good as their friends hope, and that what there is of evil in them may be modified when the light of experience indicates clearly that such action should be taken and the cooler times and calmer views of the public on the subject permit unprejudiced consideration of the questions involved.

Oral Discussion.

Mr. Welch: Mr. President, I have little to say in regard to this paper, which is not one that would require criticism.

I wish to second the remarks of Mr. Ireland in saying that the whole society is under a debt of gratitude to Mr. Rhodes for the care with which he has analyzed these various laws which have given us so much trouble during the past few months.

All that has been said in criticism of insurance legislators for their throwing aside of conservatism and enacting extreme laws, I agree to, and yet none of you gentlemen who have had to meet these insurance commissioners personally, who have discussed with them the nature of these requierments upon which you may have had a difference of opinion, could have failed to learn from them some very pertinent reasons for the enactment of these laws, and, having learned these reasons, could not fail to have had a greater sympathy for the legislators in enacting, and the insurance commissioners in enforcing these laws. I confess that since I first faced the situation and saw how unjustly the policies of honest companies in the field were criticised and how such policies had to be twisted and pulled to be brought under all the requirements of this mass of new legislation, I have undergone a change of heart, in that I now have a greater sympathy for the men in the West who have not had to deal with companies such as most of us here represent, but have been forced to deal with other companies issuing contracts entirely different from those with which we are most familiar and against which almost entirely this legislation was directed.

I think as Mr. Ireland does, that there will be a modification of this; but I believe that these laws will be modified sooner and on more conservative lines if we who are the hardest hit will make up our minds that this legislation was not directed against us but enacted for the good of the business which we are in. I believe that we, by showing reasonableness towards these men who are put under obligation to carry out these laws, can bring them into sympathy with us and our ideals and aims—in the same way that some of us have been forced to come into sympathy with them in their endeavor to meet conditions of which we were totally ignorant—and that in this way the desired modifications will the sooner be made.

MR. HANN: Mr. President, I think this is a question we ought to be very decided on from an economic standpoint as well as from a legal one.

If there is anything of a deterrent character in the working of the act now in force, so as to stult the business in the future, or to interfere with the great beneficent business of life insurance, to limit its usefulness, it certainly will have to go. It matters not who is at present at the back of it; when the people see that it is working in the opposite direction to that which they intended, they will immediately rise in arms against it. I believe that that form of legislation is the best that leaves the force of contract as far as possible to the people themselves, because it leaves it to healthy competition. It is to be deplored that there were causes which led to severe legislation, but there was a great deal injected into the Act that was inconsiderate. I take it that we have stood before the bar. Those who were interested in the business did not have as clear an opportunity of making known the real state of things as we would have had if we could have appeared and given our full explanations of what things were and what we were doing.

The hope of the future is in the honorable discharge of the duties that fall upon the managers of insurance companies. You cannot legislate people into honesty. You cannot accomplish much in that way. But you can accomplish a great deal by educating the people up to what is honorable and noble in business methods. Having done that you will soon find that when the company resorts to methods that are questionable it will be brought up before public opinion that will demand the use of better methods.

I will repeat that if the effects of the operation of the present law should turn out to be baneful, it will be repealed because the public is ever ready to reject any such enactments that are found wanting.

MR. PAPPS: Mr. Rhodes is not able to be here this morning, and in his absence I would just like to say one word in regard to what Mr. Ireland said as to some States not being mentioned in Mr. Rhodes' paper. I believe Mr. Rhodes' reason for not mentioning them was that they had not had a meeting of the legislatures during the period covered by the paper.

There is one thing Mr. Rhodes asked me to say, that is, that in the paper there were one or two omissions, as follows:

The states of Tennessee, Nebraska, and Arkansas passed laws preventing the removal of suits to the Federal Courts.

In Minnesota the law requiring policies, renewals, etc., to be stamped "Old Line Plan" was repealed.

VALUATION AND DISTRIBUTION—H. MOIR.
VOL. X, PAGE 179.

WRITTEN DISCUSSION.

MR. YOUNG: Mr. Moir in his paper on Valuation has given us one of those important papers which bring together the best that has been written on a particular subject for the convenience of busy men and students. He brings the subject up to date, his treatment is original, and he has succeeded in making a well-worn and much discussed subject fresh and interesting.

It is somewhat startling to read that "Authoritative definitions as to what constitutes a net premium method of valuation are very hard to find." Yet if we were required to produce a definition that would be acceptable to the Courts, we should probably agree with Mr. Moir. The net premium reserve, however, is a quantity that admits of perfect definition. If we were asked to calculate the net premium reserve for a particular contract of a certain duration on the American 3% basis, we should each of us arrive at the same result. The determination of the table of mortality and rate of interest to be used are matters of judgment, and have no part in the definition of the abstraction we call net premium reserve. Mr. Moir considers as objections to the method that the actual premiums charged and the incidence of the expenses are ignored, but I should rather call these intrinsic features of the method. The actual premiums charged vary with different companies and different conditions; the incidence of expenses is not the same throughout the world. These have no place in the fundamental definition of the net premium reserve. "Serious blunder and misunderstanding," have resulted from the use of the net premium method of valuation, as Mr. Moir declares, but this is not due to defects in the method but to misjudgment in applying it. The net premium reserve is our basic conception of valuation, and as such I do not think it can be superseded. Mr. Moir has proposed a definition in the latter part of his paper, which involves conceptions of a "true table of mortality," and of a "true rate of interest approximating reasonably to the probable experience of the Company." The objection to his definition is that probably no two of us would agree in identifying the concrete representatives of these conceptions. The rule seems lacking in the vital characteristic of a definition—it does not define. Further, I think it would be objectionable to incorporate in a rule of valuation for general application that "the net premiums should be those on which the office premiums were based." This might be desirable for one purpose and not for another, and if we incorporate it in a general definition we should further state under what circumstances this condition would be applicable.

Mr. Moir brings out clearly the two main purposes of valuation: 1. to determine the solvency of a company; 2. to ascertain what surplus should

be distributed. I do not think that any actuary at the present time would contend that the net premium method of valuation should be used as a final test of solvency or that any fixed basis of valuation could be devised applicable to the conditions of all companies. Our valuations, fortunately, are usually undertaken in order to determine the amount of surplus that can safely be distributed to the policyholders. The amount of surplus to be distributed at any time cannot be determined automatically by a mathematical process, but is a matter of judgment, and the valuation we make is our principal guide. A natural process to adopt for this purpose would be to value the sums assured on a conservative basis of mortality and interest, and deduct the value of the future premiums with a certain deduction using annuities calculated on the same basis. The portion of the premium to be valued as an asset is a matter of judgment, but it would be natural to deduct a percentage not only for expenses, but to equalize the profit from loading in future years. In the majority of cases this process would lead simply to a net premium valuation. This method of valuation would indeed be a poor guide to our judgment of the amount of surplus to be distributed, if we did not ascertain what proportion of the future premiums was reserved for future expenses and profits. I do not think we have any method of valuation which is likely under present day conditions to supersede the net premium valuation as a guide in the distribution of surplus, in established companies.

The defects of the net premium method of valuation as a Governmental Standard are similar to the objections to its use as a test of solvency. Mr. Moir's discussion of the recent "brood" of methods for the purpose of Governmental valuation is an interesting and valuable portion of his paper. In this discussion he says, in reference to a net premium valuation, that "the existing surplus is taken from the present policyholders, and carried to reserve;" elsewhere he speaks of a certain valuation disturbing the equities of policyholders. I prefer the old-fashioned view that in valuing a company's liabilities as a guide in determining the amount of surplus to be distributed the actuary is not considering the equities of the individual policyholders. The object of the valuation is to ascertain what aggregate amount the company should hold in order to best conserve the interests of the whole body of policyholders. We are only concerned with the aggregate amount of reserve and do not take cognizance of the shares of individuals or of classes of policyholders in that aggregate. The equities of the policyholders are determined in the actual distribution of the surplus.

Mr. Moir's remarks on British Valuation are interesting and instructive. He explains that the gross premiums include an addition equivalent to the value of a certain reversionary bonus, and that the companies, in order to provide for the future bonuses, make their valuation at a low rate of interest. Mr. Moir goes on to say that such a valuation is not a net premium valuation, because the low rate of interest is only adopted to approximate to a valuation at a higher rate of interest with an allow-

ance for the future reversionary additions provided for in the premiums and promised. It seems to me that the British Valuation is correctly described as a net premium valuation and I think exception might properly be taken to the statement that the bonuses are "*promised.*" The companies so far as I am aware, nowhere state the basis of calculation of their premiums or the amount of bonus they have assumed; their contracts do not state that any specified amount of bonus will be declared, and when publishing the amount of bonus actually declared in the past, they are, I think, always careful to point out that such statements of past results are not to be considered as promises for the future.

Mr. Sheppard: The first part of Mr. Moir's paper discusses what he calls (and what is generally called) the "Net Premium Method of Valuation." This is hardly a correct description, as the method is a special, not a general one, and it should be rather called the "Full Level Net Premium Method of Valuation." Suppose we have settled on the office premiums for a particular benefit, whether those office premiums be level or increasing, or decreasing or irregular, each of them may be considered as divided into two parts, net premium and loading, the division into these two parts being subject only to the two conditions: (1) That at the inception of the insurance the present value of the variable net premiums is equal to the net single premium for the benefit and (2) that the reserve in hand at the end of any policy year resulting from the net premiums previously paid, together with the net premium then due, is sufficient to cover the following year's cost of insurance. If surrender values are guaranteed, they are naturally limited by the reserve resulting from these variable net premiums. Subject to these conditions, we can vary our net premiums in any way we choose and obtain what is strictly a *net* premium method of valuation. This is the fundamental idea of the various valuation methods described in the Report of the Wisconsin Insurance Investigation Committee. Our difficulties arise from the fact that the laws of the various States recognize only the full level net premium method, except for the modifications described in Mr. Moir's paper.

Mr. Moir says that an objection to the net premium system (which applies even in its general form as stated above) is that no provision is made for expenses after the policy is paid up, but this (at any rate in the case of non-participating policies) could be adjusted by reserving a portion of the loading as suggested in the chapter on Policy-Values in Part II of the Text-Book of the Institute of Actuaries, and as it is a necessary result of the step from continuous to limited premiums, I do not see that it can be considered as a valid objection to the net premium method as a whole. Perhaps one of these days a reserve for such expenses on a limited payment policy will be recognized by the Legislature as a necessary addition to the net level premium reserve on the policy itself. This question will be again referred to when the two factor method of distribution of profits is discussed.

The main question now left is, how far should the Legislature recognize

the current office practice in the payment of commissions and other initial expenses as affecting the reserve? I think the time has come for actuaries to take a firm stand in declaring that the payment of a heavy initial expense in obtaining new business affects the company as a whole, and that this initial expense should *not* be debited against the policies on whose account it is incurred, in other words, this expense may be considered as merely the present value of an annuity, the payments of which are spread over the total future premium income of the company, advanced out of surplus and repaid out of the future profits of the company as a whole.

The chief objection to the full level net premium system then resolves itself into the hardship to which a new company is subjected, if it is compelled to put up the reserve by that method. In order to avoid a monopoly of the business by companies now firmly established, it is only just that relief be granted *to those that need it.* There seems to me to be an objection to all the laws that grant such relief (with the exception of the New Jersey law), viz., that the relief is not withdrawn when no longer needed, and I think that when a certain amount of surplus has been accumulated by a new company on the insurance side of the business (as shown by successive Gain and Loss Exhibits) this relief should be withdrawn.

In criticising the New Jersey Valuation Law, Mr. Moir omits what seems to me an important point, viz.: that the reserves so held are not the result of accumulating net premiums that satisfy the criterion that their present value at the commencement of the policy is equal to the net single premium for the benefit.

Mr. Moir says that "it is objectionable to take surplus from policyholders to whom it now belongs." In my judgment a much stronger word should be used in this connection, if you can prove that it belongs to them. But there are three elements that have to be considered, first, the insurance law, which is *not* based on equity; second, equity as it is concerned with what happened in the past; third, equity as it is concerned with the provision to be made for the future. We actuaries are the only judges of what is equitable in a highly technical business like that of life insurance, but the New York Insurance Law, by limiting the amount of the surplus, has taken out of our hands the right to decide how much of the undistributed profits are to be allotted to present policyholders. We must therefore act according to what we consider the rules of actuarial equity within the limits imposed by law. I should like Mr. Moir to state what he considers to be the surplus that *belongs* to a policyholder. I fail to see how gains from lapse and surrender can under any rule of equity be considered to belong to persistent policyholders, as the rate of discontinuance otherwise than by death is not involved in the calculation of premiums and reserves. I think the directors of a company, if the law did not prohibit it, would be justified in devoting the whole of the fund thus formed to the extension of the business.

Mr. Moir discusses the question of what a true net premium valuation

would be if the company decided to reduce the valuation rate of interest. I am afraid I cannot agree with his formula. It can be proved actuarially that the net premium after the interest rate is changed, which satisfies the condition that the present value of the future net premiums when the policy was issued was to be equal to the present value of the future benefits at that time (which I hold to be a necessary condition in the case of a company which distributes profits by the contribution method) must be equal to the difference of the reserves at the two interest rates when the interest rate is reduced, divided by the value of the annuity at that time at the reduced interest rate for the balance of the period of the benefit together with the net premium at the reduced rate of interest. It may be objected that this can become greater than the gross premium in the case of a limited payment policy towards the end of the premium paying period, but as it is impossible to put up a higher reserve on a paid up policy except by either taking the difference in toto from past profits, or by making a deduction from future profits as they arise or by a combination of both, the objection resolves itself into dividing the portion of the difference of the reserves not taken out of surplus into two parts, one to be taken out of loadings yet to be received, the other to be taken out of profits after the policy is paid up, as they arise. In any case, if the profits are not sufficient to make good the amount so found, the company must either make up the deficiency out of the surplus in hand, or become technically insolvent.

Mr. Moir does well in recalling to our notice that the British method of using what seems to us an unnecessarily low rate of interest is merely a method of artificially keeping up the bonus rate, but it has its justification if the premium actually charged is found to be equivalent to a non-participating rate for the sum assured, and the bonuses that result from the method used. This should be of interest to actuaries trained in offices using the contribution method of distribution of profits, as showing how two entirely different methods can claim to be equitable between different plans and ages.

Mr. Moir claims that the value of the general agency contracts should be taken into consideration in a true valuation of the company's liabilities. I fail to see how this can be correct in the case of a net premium valuation, as these commissions are to be paid out of future loadings for which credit is not taken. Of course it would be possible for a company to be solvent under a net premium method of valuation and insolvent under a gross premium method with these contracts as liabilities. Who then would be the preferred creditors, the agents or the policyholders? This is a legal question which I would rather leave to others to discuss. It seems to me that a great injustice would be done to policyholders if their contracts were reduced in order to pay excessive commissions to the agents of the company.

In the second part of his paper Mr. Moir discusses what is perhaps the most debatable question that we have to consider. As I have already taken more than my fair share of your time, I will confine myself to one

or two points that have occurred to me in reading it. I don't think Mr. Moir appreciates sufficiently the importance of the idea that the legal reserve on the policy is something that we hold to the credit of the policyholder. I know the arguments that have been brought against this idea, but nevertheless it is strongly entrenched in the law courts and is intimately connected with the idea of loan and surrender values. It is also the fundamental idea of the contribution system, which seems to me to have become officially recognized, at any rate in the case of States which under their statutes require a Gain and Loss Exhibit.

I should like hear to say a word in favor of the distribution of the profits by the two factor method, using select tables of mortality in accordance with the average experience of the company, the balance of the dividend after interest profit has been allowed, being a percentage of the loading based on such select tables and independent of the duration of the policy, in accordance with what was said above on the subject of first year's expenses. The introduction of the Cost of Insurance is merely a method (and not a very satisfactory one) of correcting the inequity arising from the use of an aggregate table or of a table showing a higher rate of mortality than that experienced by the company.

In order to meet, if possible, the objection referred to by Mr. Moir in the last sentence of the section under the heading of "Loadings" on page 203, I should like to suggest a modification of the method stated above, and that is to fix, even in the case of a company issuing nothing but participating policies, a schedule of non-participating loadings for each class of policy and to distribute the loading profit proportionately to the excess of the participating over the non-participating loading. If the total loading profit distributed is greater than the total excess of the participating over the non-participating loading required, then the balance should be distributed proportionately to the non-participating loading assumed in each case.

Reference was made above to the case of limited payment policies after they have become paid up. By the two factor method the dividend will consist entirely of interest profit. As, however, there is necessarily some expense still attached to these policies, the interest allowed will be reduced unless some correction is made. Assuming one-eighth of one per cent. to cover this expense, I should suggest that the loading on these policies during the premium paying period be reduced by an amount, which accumulated at the valuation rate of interest with benefit of survivorship, will amount at the end of the period to the deficit of the reserve at the valuation rate of interest as compared with that at one-eighth of one per cent. lower. The amount of the immediate forborne annuity thus created represents the addition to the tabular reserve on the policy. When the policy is paid up, the reserve will be calculated at the lower rate and the interest profit will remain unaltered.

In conclusion I should like to express the pleasure I have received in studying a paper which is of so practical a nature and which at the same time discusses a variety of questions from an independent point of view.

Mr. Hutcheson: The question of "Surplus Distribution" is one regarding which, as Mr. Moir states, we can never hope for uniformity, as we all have different questions to deal with. We may all decide that the contribution method is the best, but we will each have our ideas as to the number of factors to be used and as to the best method of assessing expenses against policies of different classes and durations.

Sources of Surplus: Apart from profits from investments, the principal items of profit and loss which go to make up the surplus may be set out as follows:—

Credits: (1) Loading, (2) Excess interest, (3) Favorable mortality, and (4) Surrender charges, and
Debits: (5) Insurance expenses, and (6) Investment expenses.

The "two factor" system: I shall confine my remarks to the "two factor" system of dividing surplus, the two factors being (1) excess interest, and (2) all other surplus items.

Excess Interest: We are pretty well agreed that "Excess interest" less "Investment expenses" should be divided in accordance with the reserves held, but even here there is much diversity of opinion as to the details. Some suggest using the rate of interest derived from the company's operations of the year, whereas others use a rate indicated by the trend of the rate of interest as shown by successive years' operations. In addition, the "reserve" on which the interest is calculated may be either the initial reserve or the mean reserve. In either case, excess interest from non-participating policies, annuities, etc., is left intact.

Loading, Mortality, and Surrender Charges: Regarding the division of the other items of profit and loss, there is a much greater divergence of opinion. There are the "two factor" and the "three factor" schools, as Mr. Moir points out. In the "two factor" school the "gains from mortality" and the "surrender charges" are deducted from the "insurance expenses," and the balance of these expenses is assessed against the "loadings." The excess of the loading over the assessment is added to the excess interest on the reserve, and the total is the annual dividend.

Assessment of Expenses: The question of the assessment of expenses as between policies of different classes, durations and amounts is, however, one of great difficulty. There are various systems in use: under one the assessment is a uniform percentage of either the premium or the loading, irrespective of the class or duration of the policy, whereas under another the assessment is a percentage graded according to the duration of the policy—the percentage being larger for recently issued policies, and decreasing yearly with the duration of the policy. Under another system the assessment depends partly upon the premium and partly upon the sum insured, but not upon the duration of the policy, and so on.

New Business: Regarding the assessment of expenses incurred in connection with the procuration of new business, there are two diametrically opposite views. The one is that these expenses should be assessed against the new business, whereas the other is that the cost of procuring this

new business should, to some extent at least, be paid for by the existing policyholders. In the case of tontine business, where all items of profit and loss are assumed to be kept separate for each class, it is doubtless correct that initial expenses should be assessed against new business if each year's new business is considered as a separate class, but, in the case of annual dividend business where there are no such classes, I do not think that the whole of the initial expenses should be assessed against the new business. If anyone outside of Wisconsin held the opinion that the entire initial expenses should be assessed against the new business, he would do so on the theory that any new business obtained ought to pay its own way, and this would amount practically to saying that new business was not necessary, but that the company would write it provided it could get it without cost to the old policyholders. Nearly all of us, however, hold that new business is vital and necessary, and we therefore hold that the existing policyholders should pay at least part of the cost of getting it.

Owing to the large initial cost of getting new business, the surplus of the company is drawn upon temporarily, and the dividends of the existing policyholders suffer for a few years in consequence. Ultimately, those of the existing policyholders who survive gain by the new business in lessened mortality and expenses. This suggests that the expense of getting the new business should be assessed most heavily against the recently issued existing policies, and most lightly against the existing business of some years standing.

Aim of Management of Company: Before proceeding further, however, I wish to state what I consider the object of the management of a company should be, as the assessment of expenses to some extent depends upon this. The object should be to keep the company in a healthy condition by writing as much new business as possible, at as small cost as can be. Other things being equal, a large new business should ultimately mean small cost to individual policyholders, but if growth be too rapid, a diminution in dividends will take place meantime, unless the company has a large surplus to draw upon. I can hardly agree with the statement that dividends ought not to rise and fall according to the volume of new business, because if the existing policyholders are assessed to pay for new business, they must be assessed more if the business increases to any appreciable extent. My opinion is that in ordinary circumstances new business should not be increased so rapidly in any year as to cause a decline in dividends, provided of course that the company is in a healthy condition and is writing a normal amount of new business. In such a company, there will be a point beyond which new business cannot be written without increasing the assessment against the existing members. Of course there are often circumstances where new business should be written at the expense of dividends. For example, in the case of a moribund concern, it may be necessary to write a comparatively large amount of new business in order to keep the company out of the receivers' hands, and in such case, the cost of this new business will naturally use up surplus and

cause a decline in dividends. As to the rights which a company has to use surplus in this way, I think there can be no doubt. No one would hold the opinion, I think, that a railroad company would not be justified in passing its dividends, and putting its whole net income of any one year into necessary improvements if it could not raise the additional capital necessary for this purpose in other ways. In like manner, if a life insurance company has run down, it would be justified, apart from legal restrictions, in spending every dollar of surplus in getting new business, and the worse the condition of the company, the more justified it would be in such expenditure.

Dividends and Commissions: In the "two factor" dividend system, one of the factors consists of "loading" plus "mortality savings" plus "surrender charges" minus "insurance expenses." One of the principal insurance expenses is commissions, and the commissions increase more rapidly than the other items if a large and increasing new business is done. The result is that the expense assessment increases with any increase in new business. Within limits, therefore, we can tell in any company what the assessment will be if a certain amount of new business be done at certain rates of commission and we can, therefore, tell in a rough way beforehand what the dividends will be. If we could foretell the amount of new business which we would do provided the rates of commission were slightly increased or slightly reduced, we could in like manner estimate the expense assessment which would be necessary, and consequently the dividends which could be paid. If we could do this, there would be nothing to prevent us from deciding upon a scale of dividends for one year in advance, and then we could estimate the amount of business we would have to write, in order to make that dividend possible. We could then decide what commissions we would have to pay to get that amount of new business. At present we have to give the policyholder in dividends what is left after paying the agent. If the plan I have outlined could be worked out, it would result in our paying the agent what is left over after deciding what dividends the policyholder was to get for his premium. If we could do this, we could each year in advance fix upon the scale of dividends desired, and then the scale of commissions could be derived by a simple calculation. Of course I know that this is not practical owing to the fact that competition fixes the commission rates to some extent, and also because we cannot predict the profits quite as nearly as this system would require.

Necessity of New Business: To return, however, to the case of a healthy company, if new business is not necessary or desirable, it should not be obtained; but if it is either necessary or desirable for the good of the company then the existing members ought to pay for getting it. In the case of a life company with a fair volume of existing business of a good quality, the question of how much the old policyholders should pay for new business should depend upon the ideas of the management as to the amount of new business which should be written and as to what the policyholders should pay for their insurance. On the one hand, we should

not forget that the company was given its charter to write insurance and that new business is vital to its existence, and on the other hand, we should remember that the success of the company may depend upon the maintenance of its present position as a dividend payer. Every company has a different problem to solve, and probably a different one every few years. My attitude is that in getting new business, a company should gain (1) first-class risks which will tend to diminish the average mortality in future, and (2) permanent contributors to the expenses of the company, thereby lightening the future burden upon all. Surely the cost of bringing such risks into the company should not be assessed solely against the new members, but, rather against the old members who are to reap the advantage of the low mortality and the smaller future expenses. Then again, if the new business of one year is to be considered as old business in the next year, that is, before the company has been recouped for the initial expenses, should such one year old business not be called upon to pay for part of the initial expense of the next year's new business?

Again, the rates for dividends must be the same for each $1,000 insured, and it is evident, therefore, that the assessment of expenses on a $100,000 policy must be one hundred times that on a $1,000 case, although the actual expenses, apart from commissions and taxes, cannot show anything like such a ratio.

Enough has been said, I think, to show that new business is necessary, and that the old policyholders must pay some part of the acquisition expenses, and further, that we can hardly hope for any system of assessing expenses which will be mathematically correct. In the circumstances, any refinements in the assessment of expenses, and therefore in the calculation of dividends, is undesirable. Or, as Mr. Moir puts it, it is better in practice to have a simple method, so long as it is roughly equitable, than it is to have a very accurate and fair, but complex method of distribution. All that can be hoped for is a system which will give rough justice.

Graded Percentage System: The system for assessing expenses to which I wish to confine my remarks is the one under which the assessment is a varying percentage of the loading, constant for policies of the same duration, and decreasing with the age of the policy. Broadly speaking, if there is to be any distinction at all in the assessment, I think it should be in favor of those who have stood by the company and paid the greatest number of premiums. Various reasons can be advanced for grading the expense assessments in this way, amongst which are the following:

First: The new members being younger on the average will live to get greater benefits from the influx of new lives, and should therefore be taxed more for the cost of getting such new business.

Second: The surrender charges on the business issued some years ago are generally greater than those on policies of recent issue, so that the old policies contribute more through surrender charges towards initial expenses, and should not therefore be assessed so heavily for expenses as policies of shorter duration.

Third: A considerable part of the aggregate excess interest earned comes from paid-up non-participating policies issued in continuation of other policies, and this excess interest may be considered as a contribution towards expenses. A greater part of this excess interest comes from old policies than from new, so that the old policies should be assessed less for expenses.

Practical Working of Graded System: Each company should, as far as possible, decide upon a relative scale of assessments and having ascertained the actual surplus at the end of each year, it should decide whether larger or smaller assessments will be necessary, which means of course whether smaller or larger dividends can be given. If the surplus earnings of the year are exceptionally large, and not likely to be repeated, the additional surplus should be laid aside so as to equalize the dividends in the future. Or, if it is found that more new business is desired, this amount which has been laid aside can be so applied.

If a uniform amount of new business is written in succeeding years, and if profit and loss items are more or less constant, the dividends allotted to similar policies of like durations in successive years should be uniform. If a certain rate of interest be fixed upon as the dividend rate, and if a scale of percentages of loading be adopted for each duration, then the Gain and Loss Exhibit each year will show whether this rate of interest and this scale of percentages can be used.

An examination of the rate of interest derived from the Gain and Loss Exhibit each year will show whether or not the dividend rate of interest can be used, and if not, what changes should be made in it. My opinion is that the trend of the rate earned should be followed in fixing the dividend rate, year to year fluctuations being ignored, and any exceptional profit or loss being thrown into or taken out of a dividend equalization fund.

If the loadings on the participating business in force each year be grouped by calendar year's issues, and if the amount of loading in each group be multiplied by the graded dividend scale of percentage assessments, and if the amounts of these products be summed, we will get the total assessment for expenses on the scale proposed. A comparison of this total with the insurance expenses less the surrender charges and less profit from favorable mortality, will show what changes, if any, are necessary from year to year.

If either the surrender charges or the mortality profits are exceptionally large any year, they might well be placed to an equalization dividend fund, to be drawn upon if profits from these sources fall below the average any subsequent year.

If less new business than the average be written in any year, there will be a saving in insurance expenses, which saving may well be placed in the dividend fluctuation fund, unless it is intended to write the average amount during the next year. If more than the average amount be written, the dividend equalization fund should be drawn upon or the assessment should be increased, resulting in smaller dividends.

By some such process as this, wild fluctuations in the dividends can be avoided for policies of the same duration, and increasing dividends will be given on individual policies.

Mr. St. John: In his scholarly paper presented at the last meeting of the Actuarial Society of America Mr. Moir has treated the subject of "Surplus Distribution" in so liberal a manner, free from the usual method of doctrinaires, that there appears little occasion for rigorous criticism.

In respect to the treatment of surplus under policies lapsing within the first three years, the opinion is expressed that there is injustice practiced in the forfeiture of the dividends *nominally* pertaining to this class of insurance. This seems, however, to ignore any distinction between conditional and unconditional assignment of surplus.

The method which credits these contracts with dividends at so early a period, assumes in effect that through their continuance the heavy initial expenses of procuring these risks—as distinguished from that pertaining to their maintenance—would be refunded from their future contributions to the premium income, or that it will be borne by the persisting insurance.

The opinion respecting a contingent reserve for current fluctuations in mortality and interest is certainly well founded, especially with reference to the business of immature companies or under especial plans of insurance, *e. g.*, that commonly known as renewable term, which may also be considered as whole life insurance under an ascending scale of premiums, requiring for its successful administration especial methods of valuation and distribution.

Proceeding to a review of the well known contribution plan of dividends an analysis is presented of the surplus into its constituent parts, loading, mortality, and interest. and the discussion follows of the principles involved in the equitable distribution, and the conclusion is reached that first year's expenses be separately charged, and renewal expenses be put in the form of a percentage of the premium and a constant for each thousand dollars of insurance.

Here the question arises as to the equities involved in the division of expenses between the first and subsequent years—concerning this, opinions may differ widely.

The advantage gained by existing policyholders from the acquisition of new business will not be nearly of equal value in different companies, nor in the same company at different periods of its development.

Having attained a fair basis for the application of the laws of average, in respect of number of risks and amount of assets (invested funds), rapid growth is not essential, moderate increments of new business and consequent economy will then best serve the general interests of the insured: a selection of homogeneous risks of good quality is the prime condition of financial stability.

The opinion that renewal expenses be charged as a percentage of the

premiums and a constant depending upon the sum assured suggests a very suitable method of loading, as well as an equitable scheme of apportionment.

The method proposed for the distribution of surplus interest earnings as a percentage of the fund held in behalf of each policyholder—reserve and accrued surplus—is usually adopted in practice.

In respect to the distribution of the so-called mortality profit, it is proposed that in the event of its separate apportionment a mortality fluctuation fund be formed during the earlier years of insurance, to provide against the heavier losses inevitable from the increasing duration of the risks.

This is a convenient substitute for the more precise method of valuation by "select tables" of mortality as proposed by a number of distinguished authorities, notably by Dr. T. B. Sprague in the XXII volume of the *Journal of the Institute*, and Mr. J. A. Higham in the first, as well as by scholarly American writers at an earlier date than the first mentioned.

Proceeding to the discussion of the "one factor" method of distribution, the difficulties respecting its application to limited payment life policies is noted and the suggestion is made that the leading factor employed be the same as that under ordinary life policies, and provision be made, through a special reserve, for surplus distribution after the expiration of the premium paying period; a similar method was proposed many years ago by Mr. Emory McClintock, then actuary of the Northwestern Mutual Life Insurance Company, though it has seldom been adopted in practice by American Life Insurance Companies.

In again treating of the "mortality profits" two causes of variation are noted, first the incidental fluctuations (analogous to the "errors of observation" in physical research) and the systematic variations caused by the progressive rates of mortality during the earlier years of insurance, and special reserves for these are advocated.

In treating of surrenders and lapses the suggestion is made that gains from these sources be applied to reduce the initial expenses— or cost of obtaining new business—an opinion which ought to meet with general approval.

A special investment fluctuation fund (consisting of exceptional profit) is advocated, but under skillful and conservative management this fund is essential chiefly to provide against any prejudice to a company arising from arbitrary valuations by supervising authorities; the investments of any well conducted life insurance company are notably of a permanent and not speculative character.

In treating of "Class mortality" reference is made to an excessive mortality under Renewable Term classes effecting renewal at the expiration of the first term. If rigorous scrutiny be made in the selection of applicants for insurance, conservatism be practiced in expenditures (including cash surrender values and other forms of non-forfeiture) then the accumulation of funds may prove sufficient to admit of renewal for

several successive terms without increase of the initial premium—this has been the experience during a period of forty years under a considerable body of this form of insurance.

It may be further stated that though, through considerations of policy, this kind of insurance be classed as "Renewable Term" it may in effect be whole life insurance (with requisite premiums) based upon assumptions of mortality and interest derived from experience, and more liberal than those prescribed by arbitrary legislation, and I find support for this view from the author of the valuable paper here reviewed, who has advanced the opinion that a company may employ—for specific purposes—a mortality table following closely its own experience, carefully investigated.

In all questions relating to the distribution of surplus none is of greater importance than the equitable assessment of expenses,—this alone is wholly within the control of a company's managers. The surplus arising from the incidence of mortality will be practically determined by a skillful selection of risks under natural law, and the gains from surplus interests by economic conditions, and successful investment of funds, and their distribution, will be equitably determined through recognition of these facts.

In the instructive discussion on the assessment of expenses it is stated that "Loadings are introduced for two purposes,—firstly, to meet expenses, and secondly, to furnish surplus." While this is true a more extended explanation would be, that besides the expense provision loadings supply a fund available in the event of adverse contingencies from whatever source they arise, and apparently there is no essential reason why money should be paid into the treasury of a life insurance company solely to be refunded, less expenses of administration—at the option of its managers—in the form of dividends even though it be fructified by interest earnings. The investment function of life insurance has always furnished the tenable grounds for adverse criticism.

The inequality in uniform percentage assessment of administrative expenses upon the premiums of both old and young policyholders is succinctly illustrated, and will meet with general acceptance, and is followed by numerical illustrations of the difference between charging expenses as a percentage simply, or as a percentage and a constant.

An analysis is given of expenses—according to their natural incidence—into their elements, initial, renewal and administrative, and a practical formula is presented for the determination of the "residual constant" (to be apportioned on the outstanding sum assured)—after deducting from the total expenses the two first stated elements.

It is apparent that a general algebraic formula for the distribution of surplus may be applied, with equity or partiality according to the weight given to the symbols for the separation of the total expenses—*e. g.*, what proportion shall be assigned to the acquisition of new business? This will depend largely on the relative advantages of moderate or rapid development.

The latter portion of the paper consists so largely of illustrative and pertinent *facts* that it introduces little material for discussion, but the emphatic statement that equity can only be attained by harmony in Premiums, Valuations, and Distribution is abundantly demonstrated in Mr. Moir's valuable contribution to the technical literature of our Society.

The final opinion that valuation as a test of solvency should be something entirely different from valuation for the purpose of distributing surplus is a logical deduction from the principles enunciated in the essay, and is amply proven by the unfortunate experience in the field of American life insurance during the decade following the years '69 and '70; an experience resulting in a great degree from the rigorous application of the principles of net valuation.

Mr. Henderson: This paper is divided into two parts, the first relating to valuation and the second to surplus distribution, and my reason for entering into this discussion is that I do not consider it would be proper to permit a paper, in which the attitude is taken regarding net premium valuation, which is adopted by the author in this case, to pass without remark. My objection is to what appears to me to be an attempt to unwarrantably extend the meaning of net premium valuation so as to include a method which appears to the author of this paper to be the best possible method of valuation. This attempted extension arises naturally and probably unconsciously from the fact that it has been assumed for many years that the net premium method of valuation is the best possible method.

The natural result of this is that any one who considers a certain special method of valuation the best possible is liable to assume as a corollary that it is entitled to be called a net premium method. I assume that it will be admitted without question that in any net premium method of valuation the present value, according to a specified mortality table and rate of interest, of the future net premiums is deducted from the present value on the same basis of the future benefit guaranteed. The only question, therefore, is, on what basis the net premium should be computed, with possibly a subsidiary question as to whether or not the mortality table used in valuation must have been actually derived from experience. I do not think, however, that this latter question is a very important one, nor that the distinction between hypothetical and true mortality tables is one which it is necessary to make. Mortality tables which start by being true tables have a marked tendency to become obsolete as such, and to become largely hypothetical in their nature.

With regard to the former question, however, of the basis of the net premium, it appears to me to have been the consensus of actuarial opinion for more than a generation that the net premium did not depend in any way on the actual amount of gross premium charged nor on the method by which it was computed. In chapter 16, article 71 of Part II of the Institute of Actuaries' Text-Book, the net premium is defined as "That

premium which, according to the mortality table and rate of interest adopted, will exactly provide the benefit contracted for." From this definition it follows that the present value, at the time the policy is issued, of the future net premiums is exactly equal to the present value of the future benefit, so that the reserve is zero. It has further been the consensus of opinion that, in any net premium method of valuation as distinguished from a gross premium method or a modified gross premium method, the net premiums to be valued in the case, for example, of an Ordinary Life policy are level throughout life including the first policy year. An attempt has been made in recent years to get away from this restriction by speaking of modified net premiums or adjusted net premiums, but it appears to me that premiums modified or adjusted in any of the ways which have been proposed are no longer net premiums. Net premiums have no relation whatever to expenses; they are entirely provided for in the loading. This free and easy way of looking at net premium valuation culminated in the visionary valuation legislation recently proposed in Wisconsin.

I do not desire to be misunderstood as contending that the select and ultimate method of valuation or some modified gross premium method is not a better test of solvency than the net premium method of valuation, but what I do contend is that the fact of its being a satisfactory method of valuation as a test of solvency does not make it a net premium method of valuation. If the select and ultimate method of valuation can be called a modified net premium valuation, so also can a valuation based on an entirely arbitrary series of policy values for the successive years.

The second part of this paper constitutes a convenient summary of different possible methods of distribution of surplus. The tables given under the heading of practical results, however, are vitiated for practical purposes by the fact that the relative proportions of profit from the different sources are likely to be widely different in an Annual Dividend Company from those in a Deferred Dividend Company, and the results appearing in the tables given are largely derived from deferred dividend business.

Oral Discussion.

Mr. Dawson: Mr. President and Members of the Society, I feel that it would be unwise to let the discussion pass over without calling attention to a few things that have arisen in the discussion, as well as in the paper.

One of those is in relation to the difficulty of definition even concerning these things which have been common to actuaries for a generation.

The definition of a net premium which has been given in the discussion to-day, and which differs fundamentally from the one given by Mr. Moir is that it is necessarily a premium which follows the gross premium; that is, if the gross premium is level, it is level; if the gross premium is increasing, it is increasing; if the gross premium is decreasing, it is decreasing. I, perhaps, have extended the definition a little in making this statement, but I think it is a correct statement.

Now, the difficulty in the definition of that term has been, after the term has been in use by the profession for a great many years, that we have not come to some common understanding as to the meaning of the term, and most frequently this confusion is found to have resulted from the term having been made a shibboleth by a part of the profession. The fact is that the term net premium originally had a definite meaning. It was the premium calculated without reference to addition by loading for expenses or otherwise. Later on a system of valuation was introduced which led to these elements of confusion.

Now, the introduction of the net premium valuation in Great Britain, which was the first country to adopt it, was not accomplished, as perhaps some actuaries may think, without effort. All the great men of our profession there were opposed to it, absolutely without exception. The Institute of Actuaries frowned upon it from the beginning; those names which were at that time the great names were all quoted against it, and quoted against it because they regarded it as artificial and all that sort of thing. It finally came in after a comparatively authoritative definition was given for it in a prize essay by a man who was then a young man and who is now an older man, comparatively, Mr. Henry William Manly. There was a prize given for his essay on this very subject, and he laid it down as a fundamental conception on the subject that the premium was obtained by adding to the net premium an amount for expenses, and that the method of valuation was justified because at the end of a certain term, if they had used precisely that amount for expenses, and had a mortality exactly as expected, and realized interest exactly as expected, that was exactly the sum it should have been.

Now, it so happens, that in our own country, when the net premium system of valuation was introduced, it was introduced after a struggle. Elizur Wright brought it to the attention of the public by enforcing it through his arbitrary will. It was not fixed upon by law. Not on the ground which you have taken here to-day at all. But, on the ground that it did provide for expenses, and that expenses were taken into account.

Now, as to these difficulties of definition; I have a little incident to relate which I think will be of interest, that shows how valuable it is to get a comparatively authoritative definition clearly. You all remember some tests taking place during the last year as to the select and ultimate method. Now, if we were to have similar tests in the State Department or in Court such as we did have in the States of Vermont and Massachusetts relative to the meaning of net premium valuation, you all can see the difficulty we would encounter as to what that term actually means. Fortunately, the select and ultimate valuation given us at the time this test arose, or at least at the time the Armstrong laws were passed, was defined in three formal documents, one written by Mr. Moir, two by myself, one of which appeared in the Actuarial Society's *Transactions* and the other one in the German Society's *Transactions*, and, in the discussion that took place on those various papers in the Actuarial Society, the International Congress of Actuaries, and in Germany. And fortunately,

for the particular issue that arose, one of those very matters about which there was a contest had been examined by one of the men who was responsible for the formula. So that it became an easy matter to define the meaning of select and ultimate and to cover certain particular things by its application.

I do not feel that I ought to pass one thing that has been brought out in this discussion, which, it seems to me, is a thing that ought always to be mentioned guardedly and with especial care to avoid misconception or the adoption of any definition of net premium valuation which could be construed to admit of the so-called distribution of expenses absolutely at the arbitrary option of the company or its managers. Now, the fact is that premiums are not made with that view, and valuations should not be made that way; and any definition of a term for which we all have so much respect as that of net valuation, which appears to give any support whatever to the theory that a distribution of expenses is permitted,—a distribution of the so-called expense provision is permitted,—which widely and dangerously departs from the absolute requisites in regard to such expenditures, is a thing that we should carefully guard against.

I have been very much interested also in the discussion concerning the question of surplus, and wish to call attention again to the matters two years ago relative to this matter; and that is, the importance, if possible, under the existing conditions, such as for instance the actual provision of law, that the surplus earned in a given year is what must be used for distribution, and of getting away from the old idea that there needs to be first an indirect, and very difficult, and, to my mind actually impossible analysis of the so-called sources of that surplus. It seems to me that the remarks of Mr. Sheppard, as made, on that matter, deserve to be carefully considered. Certainly the norm is that the man should receive, at the very best, if there were no expenses in life insurance, his insurance at the net level premiums fixed by a table of mortality corresponding fairly to the actual facts, and by a rate of interest corresponding fairly to the actual conditions, and as far as the rate of interest is concerned, to distribute back to him upon his investment, the extra earnings of that investment. It is perfectly evident we will approach closer to the norm which would exist if we took this business without expenses, if we give back the surplus as a reduction of the loading, over the net premium computed in that manner. And, when the cost of new business is permitted to absorb, as it does absorb in all the net level premium companies of the country, practically, either all or the larger part of the gains by mortality because of efficient selection, it is manifest also that you may properly lose in that distribution omitting from both sides of the account those expenses and provision therefor, applying the net ultimate premium instead of the net select.

In this connection I would like to say just a word in closing concerning the total to be distributed. It has always seemed to me,—while there

is certainly latitude for all sorts of opinions upon the subject as to the treatment of the new policyholders and distribution of surplus, varying from the idea that all the cost of new business should be charged against them and not against a reserve in some manner by reducing the sum of gains through surplus, to the other extreme perhaps that the distribution should be made strictly on the net level premium method and absolutely disregarding all expenses,—that the following appears to be reasonable both from the standpoint of convenience and equity: while in the ordinary management of the life insurance companies the policyholders may be dealt with on the basis of the actual costs under their particular policies, there is every reason in the world why, when you reach the distribution of surplus no such distinction should be made. We all know that there is a reason.

We all of us appreciate that it is convenient in order to avoid expense, trouble and unnecessary explanation, and it is a desirable thing to do.

I put forward this idea with some reluctance and hesitation. But it seems to me that it may be possible to meet on that common ground.

Mr. Macaulay: Mr. President, I will not attempt to give any lengthy criticism of Mr. Moir's paper. I do not know that we fully realize its magnitude and scope. It is really a treatise. There are nearly fifty pages of matter, and to deal at all exhaustively with a book of this description would take a great deal more time than you could give now, and a great deal more time than I have been able to give in the past.

I would say, however, that I have read Mr. Moir's paper with much satisfaction. It has pleased me greatly, not merely because of the facts and information which it gives, but still more, because of its suggestiveness. It will certainly stimulate thought, and will do much good in that way. I do not say that I agree with everything Mr. Moir has said, but certainly I do agree with a great deal of it, a very great deal.

There are a few points I would like to refer to.

Mr. Moir has done yeoman service in emphasizing the distinction between the two bases of valuation. There are naturally two such bases, one for solvency purposes and another for the distribution of profits. These are absolutely distinct and separate things, and to my mind nothing but evil can come from uniting the two. The fact that of recent years we have come to realize that a distinction should be drawn between them, is a very encouraging sign.

I remember how our honored friend, Mr. McClintock, a few years ago frequently drew attention to the fact that for solvency purposes a gross premium valuation was desirable, and I remember also my own feeling that Mr. McClintock was indulging in visions; that talk along those lines did not get down to solid earth; it was perhaps all right for Great Britain, but the attitude of the legislative mind and of the actuarial mind on this continent was such that these ideals were not here within the range of practical politics. The very fact that we are now facing this problem in a different frame of mind ourselves, and realizing that it

is not settled to the extent that we formerly thought, is itself encouraging.

Mr. Moir has drawn attention to the peculiarity of the New York law by which a company is not considered to be insolvent unless it has fallen below the American four and a half per cent. table. I have been under an impression that that was a slip of the law-makers. If it was their intention to not repeal the old standard and thus to authorize two standards, then the law reflects credit upon those who framed it. That is the first enactment, so far as I know, anywhere, which recognizes a distinction between a basis for licensing a company and a basis for putting a company into insolvency. I do not say it is ideal. It is not the basis I would adopt. It is probably not the basis most of us would adopt. But the fact that there is now in New York a difference between the standard for solvency and the standard for merely licensing a company, is far more important than the exact bases adopted.

The idea of limiting a weak company so that it cannot appeal to the public for new assurances, particularly on the participating plan, but nevertheless can go on and work out its old business, is, I think, a decided step forward in legislation. If there had been such a law in the time of the Atlantic Mutual, that company would certainly have been saved.

The change in opinion which has taken place in recent years in regard to standards of solvency, is greater than even the New York law which I have quoted would indicate. There have been other alterations. The select and ultimate method of valuation has been adopted for general use in that State, while elsewhere, some form of the preliminary term system is fairly generally recognized as admissible.

In the early seventies many companies were sacrificed to the net premium standard of solvency. Another testing time is coming, and before many years will have passed, I think we will have a chance of judging for ourselves whether these changes which have been made in the laws will be sufficient to avert catastrophes such as those which happened thirty-five years ago.

When we speak of the preliminary term system, there is one point we ought to remember, and that is, that so long as that method of valuation is made to depend upon the terms of the policies, it is apt to defeat its own purposes, because if a company must write its policies in such a way that all competing agents can point out that at the end of the first year the assurance has no value, it is difficult in practice to hold the business. This is the weak point. I think for solvency purposes any basis of valuation which may be adopted, should depend not upon the phraseology of the policies, but upon the law. Of course we have when determining reserves to take into account the surrender values which the companies give, but apart from that, the legal basis of solvency should not, I think, depend on the insertion or non-insertion in the policies of some special clause dealing with this point.

Mr. Moir's remark that the basis of valuation is legally fixed in Great Britain at four per cent., is possibly subject to misinterpretation. The British act provides no basis whatever for the determination of the sol-

vency or insolvency of a company, leaving that entirely to the discretion of the court, which of course acts on expert advice. The Act of 1872 merely states that in the case of a company which has been actually declared insolvent and is in process of being wound up, if a valuation of its policies should become necessary, presumably for the purpose of distributing the assets in cash, such valuation shall be by the Seventeen offices table with four per cent. interest and net premiums. It will no doubt be remembered that when Mr. Ryan was present at one of our meetings he stated, in reply to a question, that in his personal opinion, a valuation for solvency purposes should be by the H^M four per cent. table, with 12½ per cent. off the gross premiums, and all negative values excluded.

I would like to emphasize in closing, my obligation to Mr. Moir, and to repeat that his paper is one of the most valuable and suggestive that we have had in a long time.

Mr. Hann: Mr. President, this question of distribution of expenses has been going on for the past five years, and the more you discuss it the longer it will go on.

A net premium valuation is well understood here. We know precisely what is meant by that; in Great Britain it is quite another matter. They were never bound down there to any net premium valuation such as we are by law. The notion in regard to net was, if you take a cask of sugar and put it on the scale and weigh it, and then take the sugar out and weigh the cask, the difference between them was the net. They believe the proper thing to do is first to analyze what the expenses of the company would be, and then, having that before them, they would throw off from their premium a certain percentage or a certain proportion whatever method they adopted, and the result could be called a net premium valuation.

You could go on forever discussing this matter as to what is net and what is not net. Then we have this "pure premium." One actuary asked what is a "Pure premium method?" "Is any other an impure method?"

Our net premium method here, I think will be granted by all present, to be a very good method. No company ever suffered from it. That is, a good going concern that could put up its reserves on the basis existing in most of our States will be all right; but some of these attempts to reduce the reserve in the early years may have the support naturally of young companies, because they have to struggle against the initial expenditure which presses heavily on them. They have to struggle with, of course, small funds, but they must expect to put up capital, and out of that capital they have to stand the racket of the early expenses until they are firmly established. After that there is far greater danger of having a plan or method by which in the early years the reserve can be reduced than there is in compelling the net premium reserves to be maintained from the beginning.

Mr. Moir: Mr. President, I have but little to say, except to thank the speakers who have so carefully examined the paper and read it and commented upon it.

The origin of the paper was rather interesting to me. I was in Canada in February of last year, and got the report of the Canadian Commission. On reading it I saw the language which I quoted, that British companies value at 2½% to 3½%, and I knew that this statement must cause a great deal of misapprehension and misunderstanding. I prepared the outlines of this paper in the five or six hours on the train coming from Canada. That was the origin, and the paper gradually developed and became much larger than I intended at first.

If American actuaries are prepared to say that the British companies use net premium valuation at two and a half per cent., I have no reason to object. My own personal view is they do not make a net valuation at this low rate at all; that they are simply approximating a net valuation at some higher rate with a provision for the future, and that it is a great mistake to call it net premium valuation; it is so entirely different from the general conception of what net premium valuation is.

Then, again, I find that many people here are influenced by the terms used in Great Britain. They call it net premium valuation, and with them it is perhaps natural and proper, but it is net premium valuation from a different view-point altogether from what it is on this side of the Atlantic; therefore, there ought to be a distinction and I want to call attention to that distinction.

Many of the remarks that have been made regarding this paper are such that I would like to answer, but it is practically impossible to recall them. Mr. Young was very kind in furnishing me with a copy of what he proposed to say, so that I have had a little more opportunity of studying his remarks than anyone else's. He said: "If we were asked to calculate the net premium reserve for a particular contract of a certain duration on the American three per cent. basis, we would each of us arrive at the same result." Now I beg to differ; that is not so. You may be told that a certain contract is a whole life policy, and asked to value that whole life policy, ten years in force taken at age thirty. In one State the net value is one thing; in another State it is another. If the policy contract provides for term insurance the first year, the net value in Vermont is entirely different from the net value in Massachusetts, although the wording of the Vermont statute was copied from that of Massachusetts. Then again in New York State if the office premium is less than the net premium, the deficiency has to be taken into consideration as part of the calculation. Accordingly, the so-called net value of such a policy is entirely different in different States. Some States do not make any provision for charging the difference between the actual premium and the net premium when the former is less than the latter. They simply say it shall be a net valuation.

Of course, one may say that the New York law does not change the net value of the policy; that it ceases to be a net valuation when you make

the additional charge. If you say so, it is equivalent to saying that the New York standard of solvency is not net premium valuation. You get lost as to what things are and what they are not.

Mr. Young says: "The objection to his definition is that probably no two of us would agree in identifying the concrete representatives of these conceptions. The rule seems lacking in the vital characteristics of a definition, it does not define."

Well, I would only ask Mr. Young and others to give a better.

One criticism of Mr. Young is perfectly correct: In one part of my paper I was careful to say that dividends in Britain are "virtually promised," and in another part I fell into the slip of saying that they were promises. Of course, that is not so. They are not promised by the companies. They are virtual promises, because the expectation is held out; not only so, but the statement is emphatically made that the surplus will be distributed in such and such form. And they also make the statement that for the past n years the additions have never been less than a certain figure. All actuaries look upon that as a promise of reversionary additions. They also regard it as a virtual promise of the amount; so that, while it is not a legal promise, and while, if they had large losses, they would be perfectly justified in passing the bonus period (as once in a while they have to do though very rarely), still they hold themselves as morally bound to maintain this bonus. At the same time, it was a slip on my part to say they are promises. They are promises by the agent, just as they are in this country, and become virtual promises by the company itself.

Mr. Hutcheson referred to the assessment of expense as a varying percentage of the loading. It seems a little doubtful as to how this would work if you did not take into account the premiums. The point is that the loading might be exactly the same on a ten year endowment and on a whole life policy; yet the assessment of the expenses might be different if the Company were paying renewal commissions of five per cent. Other refinements become necessary if you start with an initial refinement.

I was much interested in the remarks made by Mr. Sheppard, who dealt so particularly with many points, some of which I have noted; but I think it better to let the whole matter pass, allowing readers to form their own opinions.

I am very much pleased with the manner in which the paper has been received.

ON SURPLUS DISTRIBUTION—D. E. KILGOUR.

VOL. X, PAGE 224.

WRITTEN DISCUSSION.

MR. HANN: Any paper of merit like Mr. Kilgour's will always receive a welcome at these meetings even though we may not fully agree with its conclusions.

Whenever a company's balance is struck, it is the rule in the United States to consider it as the difference between the present value of the Assets and the Liabilities. Standard tables are alone used chiefly because of State requirements. No company, however, considers the whole of the surplus thus derived as available for distribution. There are many other elements that enter into liability which need careful analysis as intimated by the author. The present system of valuation of liabilities is due largely to the influence of State supervision.

Mr. Kilgour's plan is to first correct the usual surplus, by using a hypothetical table of mortality deduced from the experience of a company with such variations as may be deemed necessary. Having found the amount to be added to the actual surplus for all ages, kinds and durations, the sum is added to the actual surplus to arrive at the "true distributable surplus" for the year. This amount is then apportioned for all ages, kinds and durations. Having found in this manner the "true apportionable surplus" for each individual policy, the amounts added to the actual surplus in gross, are to be deducted from the individual policies. The amounts to be deducted are positive for early durations and negative for later ones. The net result would therefore be to reduce dividends during the early years of a policy and increase them in later years.

The policyholder would reasonably conclude that when the "true distributable or apportionable surplus" is ascertained he should receive that amount without any reduction.

The author does good service in pointing to the desideratum of true tables. The Select table referred to would certainly be unsuitable to the varying conditions of companies in the United States. Since we do not possess the best thing, let us content ourselves with the best attainable, namely the Standard table, or any other better suited to our needs.

The barriers in the way of obtaining suitable select tables are twofold; firstly, as the duration extends, the number of lives under observation is reduced below the point that affords safe averages; secondly, the amount of labor in their construction is enormous. If we had true tables, the so-called saving from mortality in the early years would greatly diminish. Well-managed companies are not misled by the light mortality in the early years, they are guided by an analysis of their experience extending through many years.

The point is emphasized that we use a more severe table than accords with the facts. This is no drawback to a firmly established office.

If we keep in mind the great changes which have been introduced of late years into the practice of the day, tables now in use may be materially modified hereafter.

We cater for the man who retires, we grant valuable options without compensating extras, we guarantee loans that unquestionably precipitate surrenders. Formerly, many men took surrender values whether in good or in impaired health, now a man as a rule, retires if he is a good risk and borrows if he is a bad one. These work in one direction and will probably produce less favorable mortality results than are involved in our present tables which were framed under different conditions. I think therefore, the aggregate table is an appropriate one to use, until we possess select tables constructed on sufficiently broad lines. If we possessed a true table to-day, it does not follow, with the ever-changing conditions that it would be reflected in the future. For these reasons it is a dubious plan to live from hand to mouth in respect to mortality.

All things considered it is fair to claim that the contribution method metes out equity between the members sufficiently close for all practical purposes. It is a well-defined plan, is easily understood both by agents and the insured and the method is direct, which cannot be said of the plan suggested. We had better hold fast to the familiar track.

It might be well to point out that policy values depend less on rates of mortality that they do on the progression of the fall in the annuity.

The corrective function finally deduced is sometimes negative, at other times positive and is consequently not the most desirable to work with. At the head of the numerical illustration there is a misprint in the second term. For $-P(V_n + P)$ read $-\rho(V_n + P)$. This is given correctly at foot of page.

The method is somewhat laborious, not easily understood, necessitates a double valuation, and after aiming at a refinement, appears to upset the whole equity in the treatment of expense.

We must not overlook the fact that the insured expect an increasing dividend, while the office urged on by competition, seeks to pay large profits in the early years.

Whatever plan is followed it must be based on a judicious compromise between theory and experience.

Mr. Papps: One method of ascertaining the divisible surplus of a company is to deduct from the total funds the liabilities, which for the most part are based upon a certain standard mortality table. If we calculate the policy liabilities by a table of mortality and interest, which differs from the standard table, and which is based upon rates of mortality and interest which correspond more closely than the standard table with the rates experienced by the company, we will have what Mr. Kilgour calls the "true surplus."

Mr. Kilgour does not propose that this 'true surplus' should be distributed to the policyholders, but suggests that it might be taken into account in making a distribution. This is to be done by allotting

the true surplus to the policies and then reserving the difference between the true surplus and the surplus shown by the standard table, as part of the reserve for each policy. Mr. Kilgour does not attempt to show how the allotment should be made, but from the suggestion he makes it would appear to be his own opinion that the functions upon which the allotment depends should be taken from what may be called the true mortality table.

By a somewhat lengthy process Mr. Kilgour arrives at an expression following equation number (12) which, he says, is "the quantity which must be deducted from the true surplus. . . . in order to arrive at the divisible surplus disclosed by the standard reserves adopted." In other words, this expression is supposed to represent the difference between the true and ordinary surplus. Now equation number (9) gives another expression for this difference, and these two expressions differ by $\sigma\,(V_n - V'_n)$. The expression following equation (12) ignores this quantity, but as this is only one year's interest at a very small rate on the difference between the standard and true reserves, the error involved is no doubt small and may be disregarded. The fact that the expression following equation (12) is the basis of Mr. Kilgour's paper, is the reason that attention is drawn to what appears to be a slight discrepancy in the verbal interpretation of this expression.

Since the theoretical accuracy of any method of surplus distribution must depend very largely upon the theoretical accuracy of the allotment of expenses, for which we have as yet no exact method, it seems doubtful whether the labor involved in applying Mr. Kilgour's method would be justified by any difference which would be made in the final results. The general aim at present seems to be to endeavor to maintain a dividend scale which gives substantially equitable results, rather that to attempt any greater refinement which would not be understood, and which would tend to upset the agency forces.

Mr. Kilgour's paper as a theoretical treatise is a useful contribution to the *Transactions* of our Society and brings out an original idea as to the true surplus earned. A student cannot have too many sidelights on the important question of Surplus Distribution.

Mr. Torrey: Mr. Kilgour's paper read at our last meeting, presents a method of surplus apportionment based on the true premium and the true surplus, and at the same time makes the aggregate of the individual dividend agree with the total distributable surplus, according to the Standard Reserve Table. The total distributable surplus, according to the Standard Reserve Table, is, of course, very different from the true surplus which is based on a lower mortality table, and a higher interest rate than the distributable surplus.

Under Mr. Kilgour's method it is necessary for a company to make a careful review of their past experience, and to adopt a hypothetical mortality table and rate of interest, which will approximately reflect their future death rate and interest rate. From this hypothetical table, the true

net premiums are calculated, and also the contribution from the true surplus which each policy must make to maintain the standard reserve. The sum of all these contributions in any given year added to the standard surplus, will give the true surplus according to the hypothetical table, and Mr. Kilgour in his paper shows how these contributions to maintain the standard surplus, can be calculated at the same time as the Standard Reserve is figured.

The true surplus, according to the hypothetical table and interest rate, is apportioned to individual policies, in accordance with the true premium, with such modifications as may best suit individual companies. It is necessary, however, to deduct from the true individual surplus each policy's contribution to maintain the Standard Reserve, as otherwise the aggregate of the individual dividends would exceed the Standard Surplus.

From the above, it would be readily seen that a very considerable amount of additional work is entailed. The calculations of the true premiums are comparatively easy, but the calculations of the contribution to maintain the Standard Reserve would have to be figured from the ground up for every age, kind of policy, and duration, and would take a large amount of time and labor.

Mr. Kilgour's plan would work very equitably, provided the company's actual experience conformed fairly well with the hypothetical table adopted. If, however, the company had a number of lean years with losses largely exceeding those called for by the hypothetical table, it would be necessary for the company to make new assumptions with a new hypothetical table, and a new set of laborious calculations.

I appreciate, of course, that a certain amount of injustice is done where no attention is paid to the true net premium in apportioning surplus, but in view of all the circumstances, I consider that a surplus apportionment based on the general theory of the contribution plan is accurate enough for all practical purposes, and in making up the individual dividends, either the two or three factor plan should be followed, whichever best suits the individual conditions of the company; personally I prefer the two factor plan.

Oral Discussion.

Mr. Henderson: There is a peculiar feature regarding this method which I think may be interesting to the members, although I do not think many have noticed it, and that is, if this method of declaring dividends is analyzed very carefully and followed through to all the results, you will find the final result will be exactly the same as if you had just computed the contribution from the excess interest, which is the second column, and the savings from mortality which is the third column, and distributed the total of those contributions for the existing policies, and declared the balance of profits as a constant, independent of the duration of the policy, but varying according to the definite rule for

the kind of policy and so on. So that it is the contribution method in another shape and arrives at mathematically the same result.

Mr. Kilgour: Mr. President, the purpose of this paper, to elicit discussion, has been amply justified by the opinions expressed.

Mr. Hann and Mr. Torrey have, I think, rather a wrong impression of the effect of the method suggested, and I am very glad Mr. Henderson has spoken as he has. I understood, of course, that it was equivalent to the contribution method, but what I endeavored to do was to reduce it from a three factor contribution method to virtually a one factor contribution method; and my object in working out the tables and suggesting their use was merely to reduce instead of increase the work each year. I thought that a set of tables like these when once worked out would probably last the company very nearly ten years, and that the annual labor entailed would as a result be materially lessened.

The method, as Mr. Henderson has pointed out, is mathematically equivalent to the three factor contribution method, but the annual work would entail the use of only a one factor contribution method. In addition, it was thought that the labor involved in the annual determination of the precise contribution factors would be saved, as the adjustment or corrective function is based upon factors reflecting the experience of a company over a period of years, and would not, it is believed, require frequent alteration.

MORTALITY EXPERIENCE OF YALE GRADUATES, 1792-1901—EDWARD B. MORRIS.

VOL. X, PAGE 230.

WRITTEN DISCUSSION.

MR. COLE: In presenting his paper on the Mortality among Yale Graduates Mr. Morris has furnished us with data concerning a class of lives hitherto uninvestigated, as a separate body, and any new data relating to the mortality experienced among a particular class of persons distinguished from the general population by occupations, extraordinary climatic conditions of the place of residence, or peculiar circumstances of environment is of interest not only from a statistical standpoint in general, but for the members of this Society such data may prove of practical value in furnishing some additional information which may enable us to deal more scientifically and intelligently with certain classes of lives, as insurance risks. It is, nevertheless, somewhat doubtful whether any great reliance can be placed in an experience on college graduates, as such, except in so far as it furnishes an index of the mortality which may be expected among professional men and financiers throughout the country at large. In one respect a body of college graduates is distinguished from others in the same condition of life in that during the three or four years of the college course a comparatively large number of the students engage more or less strenuously in some form of athletic pursuits, which physically build them up and develop the constitution at a formative period. How great a factor in increasing their longevity this would prove to be, it is impossible to determine, although it is highly probable that some favorable effect is produced, for in the Experience on Yale Athletes from 1855 to 1904 presented by Messrs. Gaines and Hunter the ratio of actual to expected deaths by the American Table was 44%, whereas the corresponding ratio on Yale Graduates, exclusive of the graduates of the Medical School, over practically the same time as given by Mr. Morris was 76%, which tends to show that athletics are decidedly advantageous.

In arranging the data for an experience of the character presented by Mr. Morris, which differs from that of a life office or of the population of a given district in that all the lives come under observation at a fixed time of year and remain until death, as the author of the paper points out certain assumptions may be made for determining the ages at which the deaths should be entered which will obviate the necessity for recording the exact dates of birth and death, thus greatly simplifying the preparation of the experience without vitiating the accuracy of the final result.

We are indebted to Mr. Morris for the time and labor he must have expended in obtaining and preparing the data comprised in this Experience of Yale Graduates and for the opportunity afforded to compare with well known tables lives of this class, which, while not actually selected by medical examination, may be expected to show a mortality much more favorable than the average population on account of their superior environment and social condition.

Mr. J. K. Gore: This paper might in a way be considered as the mortality experience of a group of men selected from the great mass of the general population solely for the reason that the fact that these men happen to be graduated from the same college made the compilation of the experience possible.

It cannot be said that the experience as a whole is that of a particular social class, but rather of a group of men so situated financially and socially that the conditions under which they would be apt to live would be better than those under which the majority of the general population lives. This also applies to the academic class group, by far the largest of them all. On the other hand, it is perhaps a safe assumption to make that the majority of those who were graduated from the law and medical schools, and especially this latter, were or are now engaged in the legal and medical professions.

Graduates from the scientific school are probably somewhat more likely to turn to other callings than to those for which they fitted themselves at the school. With this in mind, one would hesitate to rely too much on the experience in this group as a true indication of the probable mortality among engineers, chemists, architects, etc.

Harald Westergaard, in his monograph, "Die Lehre von der Mortalitaet und Morbilitaet," after pointing out the numerous causes that affect the mortality in any group of men, says that the art of the statistician should be to so specialize in his investigations that by a process of filtration, as it were, he would be able to show the effect the fewest possible causes have on the mortality.

The need for this kind of specialization is of particular importance when the practical requirements of a life insurance company are to be the object of an investigation. The investigation of the mortality experience under consideration does not conform with this view to any practical extent with the possible exception of the experience in the medical and legal groups. The former of these is valuable in that it furnishes additional evidence in support of the general impression, based on earlier investigations, that members of the medical profession as a class experience a high mortality. The ratio of actual to expected deaths in the legal group is not as high as in the case of the academical group. This, however, is in accordance with what might be assumed inasmuch as the academic group approximates the general population to a greater extent than a group of members of the legal profession.

It is noteworthy that the scientific group exhibits the most favorable experience of all the groups. If it is safe to assume that the majority of the members are engineers, the reason for the lower mortality might perhaps be that engineers as a rule lead an active out-of-door life, while lawyers as a rule lead sedentary lives.

If it were possible to ascertain accurately the number of those who, graduating for a certain profession, remained in that profession until death or until termination of the experience, there is hardly a doubt but that the experience in the professional groups would gain immensely in

value. So long as such additional knowledge is not forthcoming more or less uncertainty is bound to exist where any conclusions are drawn from the experience in such groups as the legal and scientific.

The most valuable experience with this objection in mind is probably that of the medical group. It is perhaps safe to assume that the percentage of the graduates here who chose other vocations than that of physician or surgeon is so small that for all practical purposes it may be ignored. A return for the investment and time spent in acquiring a medical education can be had in few other ways but by the actual practice of the medical profession. To choose some other occupation would in the majority of cases involve too great a sacrifice. This group, therefore, more truly than any of the others, exhibits the effect of occupation and as such may be considered of practical value for life insurance purposes.

The copy of the card used in arranging the data shows that the occupation of each graduate has been given. This would allow specialization along more practical lines than has been done.

The experience of the academic class cannot be looked upon in any other way than as an experience among men the greater part of whom live under better conditions of life than a group chosen at random from the general population. This experience agrees with others of a similar nature that have been made from time to time, as well in general life insurance experience as among certain social classes where data were readily available.

Interesting data bearing upon the mortality among members of the so-called liberal professions are contained in the Supplement to the Fifty-fifth Annual Report of the Registrar-General of Births, Deaths and Marriages in England, Part II, London, 1897. They are based upon the census of 1891 and comprise the deaths that occurred during 1890 to 1892, inclusive. The following table shows this experience:

Mean Annual Mortality of Males per 1,000 in the Three Years, 1890, 1891, 1892.

Profession.	20-25	25-35	35-45	45-55	55-65	65 and over
Barristers and Solicitors,	2.8	5.3	10.7	17.7	34.5	111.7
Barristers' and Solicitors' Clerks,	5.6	7.9	14.7	24.3	38.5	107.3
Physicians, Surgeons and General Practitioners,	5.8	6.7	14.9	21.0	34.2	112.4
Total Male Population,	5.6	7.8	13.0	21.4	39.0	103.6

The legal profession has a comparatively low mortality as compared with the total male population. Clerks in lawyers' offices show an unexpectedly high mortality at most ages, even exceeding that of the general population. This, as will be seen, is to a certain extent in conformity with the experience among graduates of Yale Law School, which shows a higher mortality than the scientific group and approaches that of the academic group.

The mortality among members of the medical profession is, as in all experiences, comparatively high.

Investigations into the mortality by professions have also been made on the basis of the French census and on the vital statistics of Paris. The results support the deductions made from the English experience.

With all this in mind it would seems that the Yale experience, in the form in which it is presented in the paper under discussion, has its chief claim to consideration in that it further substantiates the correctness of impressions gained and conclusions drawn from earlier and more specialized investigations. From any other point of view one must concur in Mr. Morris' own estimate, that its value is chiefly historical.

I should like to close by repeating the last paragraph of some comments I made in 1905 on a paper presented to the Society in May of that year, by Messrs. Gaines and Arthur Hunter, entitled, "Mortality Among Athletes and Other Graduates of Yale University:"

While a study of the mortality of college graduates is certainly interesting and instructive, it is doubtful whether such a study would be of direct practical value to life insurance companies. It is true that a large body of college graduates might be considered to be representative as to vitality of the so-called better class of lives in general. Nevertheless, they would, in fact, represent only a small part of the risks carried by a life insurance company. Gathered from localities differing greatly as to climate and other important conditions, they form during a brief period of four years a comparatively homogeneous group. For the remainder of their lifetime their environment and general conditions are as various as can be imagined. As compared with all the factors affecting human life from birth (and even before birth) until the time of entering college, and as compared with all the factors affecting human life after graduation, the influences of the four college years upon longevity do not seem important enough to give any special significance to a mortality experience of college graduates as such.

Oral Discussion.

Mr. Moir: The point that Mr. Gore made about the Scientific School was one that impressed me a good deal. For some years back it has seemed to me that the mental characteristics of people have considerable weight in determining the mortality they experience, and men who are of a scientific disposition are usually careful in their mode of life, moderate and thrifty; they look to the future. I think such men have better prospects of longevity than men who have a little more of the gambling instinct in them.

Mr. Morris: The object of this paper has been not only to establish an interesting table of mortality of the graduates of a representative American university, but also to establish data for comparisons of a much broader scope. It has been attempted to determine if possible the experi-

ence of a purely American group of men chosen under conditions somewhat different from those that would be found in a life insurance company. While in such a group there has been no selection from a medical examination standpoint, it is hoped that the results found can be used in the future for comparison with such ultimate tables as may be derived on American lives, especially so when we consider that there is to-day no table of mortality which portrays by itself the mortality of the United States and Canada. We all know how wanting our American Table is in this respect.

MORTALITY TABLE FOR FEMALE BENEFICIARIES IN SURVIVORSHIP ANNUITIES —CHRISTIAN JENSEN.

VOL. X, PAGE 253.

WRITTEN DISCUSSION.

MR. BRADSHAW: We are deeply indebted to Mr. Jensen for the preparation of a novel mortality experience. It has sometimes been hinted, that in view of the wide range of investigations made, the numerous experiences developed, and the frequent and careful consideration of actuarial problems, there is little opportunity for the presentation of papers bearing upon new phases of problems already mooted, or original questions. Mr. Jensen, in his paper, has contributed evidences in support of the view, and made it unmistakably clear, that in the investigation of many important actuarial problems we are merely on the threshold. One who blazes out a road into these fruitful and unexplored provinces and beckons others to follow, has done much to deserve our gratitude.

The author has placed before us a mortality experience, monetary and other tables derived therefrom, based upon the lives of female beneficiaries in reversionary annuities (improperly termed by him *survivorship* annuities) granted by the Danish States' Life Insurance Institution. Only Danish female lives enter into the experience; the period of observation was 1842-1900; only observations above 40 years of age, in which the total number of deaths was 4,041, enter into the graduation; the graduation has been performed by Makeham's formula; lives are observed; the total number of years of risk was 281,151; and the number of actual deaths, 4,900.

The special objects which Mr. Jensen appears to have had in view were (*a*) to secure a table which would exhibit the mortality among female beneficiaries in reversionary annuities, and (*b*) so to graduate it that it might be employed in conjunction with Mr. Hunter's Makehamized American Experience, and so facilitate the determination of premiums and reserves for reversionary annuities.

Several questions naturally arise in a consideration of the subject. How does Danish female mortality compare with like American mortality? If there is a wide difference, then it is only natural to infer that the table would not be of material service on this Continent. In the absence of any definite information, I am inclined to the view that there is not a wide difference in the respective rates of the two classes.

The period of experience, 1842-1900, cannot be regarded as very recent, and consequently does not reflect the mortality now prevailing. It is an established fact, that marked improvement has taken place in mortality within the past sixty years, and this is especially true in respect of female mortality—and even more especially in respect of female annuity mortality. This fact is confirmed by the present investigation of Mr. Jensen; he says, "the mortality in 1842-1880 was much heavier than in 1880-1900. The actual deaths in 1880-1900 agree well enough with the

expected deaths, and the mortality in this period may be expected to be reproduced approximately in the future."

The paucity of the material will also be urged against the experience. The difficulty of obtaining the data respecting any great body of lives upon which to base an experience of the particular class under consideration, must be apparent. The same difficulty has been present in most of the ordinary annuity experiences.

The result of the graduation for ages above 40, when considered from the standpoint of general regularity of the rates deduced; the approximate agreement in the whole number of deaths; the smallness of the accumulated deviations and the frequency of the changes in the signs from positive to negative; the usual tests, must, on the whole, be regarded as satisfactory.

Assuming that there is a close agreement between Danish and American female mortality, does the resultant mortality table reflect a rate of mortality which is likely to be experienced in actual practice? The particular annuity—reversionary annuity—is one which is not commonly purchased on this Continent, either by annual or single payments. The continuous instalment policy, however, involves the reversionary annuity feature, and this form of contract was popular, and is still so, in certain companies. The reversionary annuities granted by the Danish Government presumably provide for the annuity to come to x, the wife, after the death of y, the husband; consequently, two distinct mortality experiences enter into the computation. Such a contract would appeal most strongly to a couple if the husband were a defective life and the wife an unexceptionable one. A medical examination of the husband would, of course, be a necessity. At this point, Table VI may be considered. It sets forth the relative mortality of three classes of Danish female lives in the Danish States' Life Insurance Institution, as follows:

*Insured lives,	98% of expected.
Beneficiaries in reversionary annuities,	88% " "
Ordinary Annuitants,	79% " "

It will be observed that the mortality is heaviest among the insured lives; and that among beneficiaries in reversionary annuities the mortality is heavier than that experienced among ordinary annuitants. These results are what might be expected. It is questionable, however, if just exactly the same ratios would apply on this Continent, or if the mortality among the beneficiaries would not be found to approximate more closely to that among ordinary annuitants. I am strongly inclined to the view that the husband would not elect such a form of contract if he did not believe that

*(It is quite signifieant to note that the actual mortality among female insured lives is practically up to the expected. It would be interesting to know how the actual mortality in respect to male lives in the same iustitution compared with the expected, in order to determine the relative mortality among male and female lives.)

the beneficiary was a first-class life and would likely live to enjoy the advantages of the provision made many years after his death.

The author points out that no previous mortality experience has been developed on the lives of female beneficiaries under reversionary annuities; consequently, it is impossible to make a comparison of the rates of mortality submitted with those deduced from any other similar observations. The nearest approach to such an experience is that relating to female annuitants, and it will therefore be interesting to observe how the representative modern tables compare. Perhaps one of the best methods of making such a comparison is to tabulate the curtate expectation of life. This has been done for the following:—

(*a*) British Government Female Annuitants, 1883.
(*b*) Jensen's Female Beneficiaries in Reversionary Annuities.
(*c*) British Offices L. Annuity Experiences, Female. Old and New.
(*d*) British Offices L. Annuity Experience, Female, New.
(*e*) McClintock and Weeks, Female.

Curtate Expectation of Life according to

Age.	Brit. Gov. 1883.	Jensen.	Brit. Off. O. & N. 1893.	Brit. Off. New, 1893.	McClintock & Weeks.
50	22.77	23.63	23.75	24.04	24.03
60	15.76	16.23	16.48	16.75	16.72
70	9.50	9.85	10.01	10.19	10.50
80	4.96	5.05	5.17	5.25	5.80
90	2.23	2.08	2.48	2.61	2.73

It is generally held that the rates of mortality in the British Government table are too heavy. The table does not reflect present day annuitant mortality because it includes an experience running as far back as 1808; and moreover it includes annuities purchased by trustees and others with very little regard to the health of the subject, a condition which practically never obtains in a life insurance office. It will be noticed that the expectation of life by the Government table is lowest for all the ages given (except Jensen's at 90). The expectations, according to Mr. Jensen's table, lie between those of the British Government and those of the British Life Offices' New and Old combined. They are materially less than the expectations according to the B. L. O. New Experience and McClintock and Weeks' Experience.

While it is hardly likely that quite the same force of selection will obtain in the case of female beneficiaries in reversionary annuities as obtains in the case of ordinary female annuitants, yet in the light of the established fact that both male and female mortality, especially the latter, has greatly improved, I am inclined to the view that it would be safer to adopt the British Life Offices' or McClintock and Weeks' Table for reversionary annuities. In view of the different rates of mortality to which the two lives would be subject, and of the selection which it is

reasonable to assume would follow, I would favor the $O^{[NM]}$ table for y and the $O^{[af]}$ table for x, thus:—

$$a_{[y]\,|\,[x]} = a_{[x]} - a_{[y][x]} \qquad \text{and}$$
$$P\,a_{[y]\,|\,[x]} = \frac{a_{[x]} - a_{[y][x]}}{1 + a_{[y][x]}}$$

Digressing from the immediate subject of the paper, it is well known that the ordinary annuity business of life companies, on this Continent and in Great Britain, has not been altogether satisfactory. The causes are to be found in the superior mortality of annuitants and the decline in interest rates. Within recent years, many of the life companies have revised their rates, adopting the most recent mortality experience and a 3%, and in some cases a 2½%, interest rate. It is questionable if Canadian life companies, perhaps on account of the few annuity transactions entered into, have fully realized the change which has taken place. I have averaged the annuity rates of six representative Canadian companies for decennial ages commencing at 50, and placed beside them for comparison the net rates according to the $O^{[am]}$ and $O^{[af]}$ Tables of Mortality and interest at 3½%, loaded 5%:—

Age.	Average Male annuity rates for 6 Canadian companies.	$O^{[am]}$ 3½% loaded 5%.	Aver. Female annuity rates for 6 Can. companies.	$O^{[af]}$ 3½% loaded 5%
50	1387	1433	1523	1538
60	1072	1096	1191	1227
70	753	758	829	855
80	505	467	552	520

When it is remembered that the only gains which a company is likely to experience in annuity transactions are from interest, the above comparison becomes very significant.

Mr. Strong: The interest of this paper lies both in the graduation of the table and in the experience presented. The graduation is an example of the use of the highly scientific method of least squares to determine the Makeham constants, and we are indebted to Mr. Jensen for presenting in the pages of the *Transactions* an explanation of this method although it had to be in a connection where a rougher determination of the constants, such for instance as is explained in the Text-Book, would have been entirely satisfactory for practical purposes. In the use of so scientific a method the practical side has not been forgotten, for the same value of c was chosen arbitrarily as is used in the Makehamized American Table so that the two tables may be used together for the two classes of lives in the reversionary annuity. This is especially to be commended where, as in this case, such a value of c is sufficiently close to the true value as

to cause no important distortion of the curve, since the use of the table for practical problems depends entirely on the ability to combine it with the Makehamized American. Quite complete derived tables have been prepared both for the Statsanstalten experience alone, and for it in combination with the Makehamized American; it is unfortunate that these are all on the 3½% basis, when so many of the American companies would wish 3% tables for rates in case they should adopt Mr. Jensen's table.

A more important question in discussion than that of graduation and the preparation of derived tables, is whether the lives under observation in this experience were sufficiently similar to American female reversionary annuitants to make the mortality table suitable for adoption here. As there are no data from American companies and as Mr. Cumming's statistics from British companies are both meager and from a country where conditions are different, there is little to rest an opinion on except general considerations. As a starting point we may postulate that no table should be adopted as a standard until at least a positive probability is established that it is a proper one for the class of lives to which it is to be applied. It might be thought that the fact that all lives considered are of the class of female beneficiaries under survivorship annuities establishes such a probability, but this seems to me not sufficient when we take account on the other side of such considerations as:

(*a*) The experience is entirely Danish and there is no reason to believe that Danish and American female mortality in the same class of lives, are approximately the same; on the contrary what we know of Scandinavian mortality indicates the opposite.

(*b*) The effect of a fairly strong selection against the company appears in this experience. Modify the selection and, even though the lives may be from the same class, a different table results. Selection for or against the company in respect to a certain form of policy depends upon the mental attitude of the insuring public towards that policy. This mental attitude may depend upon racial characteristics, upon surroundings and manner of living, and even in a case like the present upon the attitude towards women. From the experience of Statsanstalten we should conclude that the applicants for survivorship annuities had a considerable regard to the prospects of longevity in the beneficiaries, that is to what was to be gotten out of the company, but not as strong a regard as those taking annuities on their own lives. Mr. Hunter's investigation into American annuitants has shown how strong the selection can be here when exercised on what is ordinarily regarded as a business proposition. If the survivorship annuity is regarded in the same way we should expect a strong selection against the companies, probably stronger than the Danish experience shows. If, however, another view of this form of policy is taken, the view that it furnishes a safe living income for the beneficiary who is unable to properly look out for herself in business affairs it may be taken rather for those who are feeble or in poor health, thus turning the selection to favor the companies. I personally

know of a few cases in which this latter view was the one leading to the selection of the survivorship annuity, and I am inclined to think it is rather common. In any case I do not think the same measure of selection will prevail among a population having American ideas as in Denmark, and if this is so a quite different mortality will probably exist.

A convincing determination will be reached only when a sufficiently large American experience has been collected, and I am aware that such an experience may perhaps show the theories I have advanced to be wrong; nevertheless I think they are justified in the present state of our knowledge. I regret coming to such an opinion, for the great service Mr. Jensen has done us in presenting this paper would be much increased in value could we without question adopt the table as a standard for female reversionary annuitants.

I have learned that Mr. Jensen's choice of a c corresponding with Makeham's American table, has been criticised. It seems to me that the most practical point in that table really to make his results useful was the choice of that c, to correspond with the Makeham's experience. If the table is to be useful at all to us to apply to practical problems, it will be in combination with the Makeham's table, and if the c were taken at any other value than that of Makeham's we should have great difficulty in making the combination, whereas we all know that the same value of c, being chosen and the two tables graduated by the Makeham formula, the two can be used together for general revision with very little difficulty.

Mr. Nichols: If properly understood, there seems no good reason why the survivorship annuity should not be in demand for that large class of cases where the sole object of the insurance is to secure a life income to a dependent surviving beneficiary. Such are all those cases where a man, having no children or direct heirs in whom he is concerned, desires simply to protect his widow or some dependent relation in case of his death. It is the cheapest form of insurance for this purpose. The chief reason for its unpopularity is because if the contingencies result unfavorably, little or nothing may be realized from the investment. Hence continuous instalment policies are generally substituted. But the former is both cheaper and more naturally adapts itself to the object in view. Its lack of popularity is a part of that psychologic feature of the business which leads the investor in life insurance to seek an ultimate certain cash return for his venture, rather than a conditional return, even though the latter may be larger in amount and better accomplish his object. Therefore, whatever added knowledge can be secured regarding the value of female life for this purpose, is of material interest and Mr. Jensen has, it seems to me, done a useful service in thus investigating and tabulating the experience of this Danish institution. More than this our knowledge of female insured lives is by no means on a par with that of males. From all the experience that has been obtained, it would seem that selective conditions exert a much stronger influence and without their

knowledge, tabular statistics are much more apt to be unreliable. As Mr. Macaulay once remarked, the most important feature regarding tables of female mortality is apt to concern the question of their safety for the purpose for which they are used and this Danish table seems to me a decidedly valuable addition to those which we possess.

Regarding the method employed on its construction, I am heartily in accord with Mr. Jensen that not only for the particular purpose named, but for any purpose, its adjustment by Makeham's law adds much to its value, provided that law adapts itself to the purpose. That it has done so with sufficient general accuracy here would seem to be shown by the extent and general distribution of the errors. From a purely theoretical standpoint, I am not sure that his method of applying Makeham's law or its results would be deemed the most accurate. He has adopted the Danish principle of determining two of his constants by dealing with the weight of observations on the theory of least squares, rather than the more familiar British methods. Poincairé, the celebrated French mathematician, once said that the ordinary man accepted the arithmetical mean as the true average because he believed it was sanctioned by the mathematicians and the latter, in turn, accepted it because they believed it represented the common sense view of the ordinary man. As a matter of fact, the method of least squares, which is the buttress on which the ordinary arithmetical mean rests, is only one of a number of means whose legitimate use depends on the character of the statistics and the object aimed at. This legitimate use is usually confined to those cases where a single correct value is assumed and variations from such value are to be regarded as accidental fluctuations governed by the ordinary law of error. In such case, it is easy to understand, apart from mathematics, how deviations from the truth may lose their weight in proportion to the square of their departures. The area as we depart from the bull's eye of a target, within which a shot may fall, increases as the square of the departure and we have an equation of the second degree. When these departures are merely lineal in their character, the equation reduces to one of the first degree or the familiar arithmetical mean. But we assume there is a single fixed bull's eye to be determined. On the contrary, when the thing sought for does not require this assumption of a single correct value, the principle does not apply. A mean of annual temperatures gives no correct idea of a local climate. It is a mere ideal from which nothing can be predicated. The statistical deviations of temperature are far too wide for such a grouping. We need to know the characters of their deviations, their frequency and extent. Relative monthly, or even daily averages, with other departures, may be needed. We naturally ask for groupings which will show the various central temperatures about which the observations tend to concentrate and the measure of their departures from such central temperatures in order to a comparison of climates.

So I am disposed to doubt whether in a case like the present, the Danish method of determining the value of the constants by the doctrine

of least squares, is theoretically the best, such might be the case on the assumption that Makeham's Law would accurately represent the experience if the observations were sufficiently numerous. But virtually the assumption here is that the law itself is a mere approximation which, if properly applied, will furnish results that are sufficiently near the truth for practical purposes. In this case, it seems to me that the determination of the constants might perhaps better be made from the construction of equations based on ages so selected according to experiment and judgment as to furnish the best practical approximations and the method of constructing such equations might well be regulated by similar considerations as seems to have been the case among British Actuaries.

In this connection, too, the arbitrary adoption of an American value for log c would seem to mitigate against the theoretical accuracy of the table. I believe I was the first to determine the value of this log in the case of American observations in an analysis of the crude experience of the Mutual Benefit some thirty years ago. I then called attention to the noteworthy fact that while in the case of European observations that value did not differ much from .04, in our own it amounted to .045, showing a distinct modification of the law of life in America, as compared with that of Europe, a modification which can be clearly observed by a comparison of the mortality curves of the two countries, when our own are adjusted without prejudice to preconceived ideas. Log c is the determining factor of the pressure of age on vitality or of the wearing out of life. B is simply a coefficient of that factor, while A is a mere representative of accidental causes common at all ages. It was a decided gratification to me to find that Mr. Hunter had found a corresponding value of this important log in Makehamizing the experience of the American table. My apprehension is that Mr. Jensen would have found the correct value of c to correspond with the ordinary European observations and if so, the deformation of the curve by arbitrarily adopting the American value can only be partially corrected by determining the values of the other constants on this assumption.

All this, however, is merely a theoretical criticism in a case where an approximate value of these annuities was all that was called for. The comparison of actual and expected deaths with the deviations of the latter would seem to show a sufficient correspondence for the purpose in hand. The important question is the reliance which can be placed on the table for practical use. In view of the wide differences that have been observed in female mortality, security should be a prime consideration coupled with a fair degree of accuracy. It seems that in the case of this institution a very marked difference exists at the younger ages between the earlier and later years of the experience. Apparently this must be chargeable to a difference in the selection against the institution which wearing out after middle life the two are in closer agreement. The constants used, as I understand, were derived entirely from the experience above forty on the assumption that they sufficiently expressed the mortality below that age, because the results fairly harmonized with the actual

mortality at younger ages, as shown by the later experience, though the latter was comparatively small, while the earlier and much larger experience showed a much heavier mortality. Evidently this tends to impair confidence in the results for younger ages, and weakens the value of the table at that age for insurance purposes, while it renders it safe for annuities. It is true, that beyond the age of 20, the Makeham formula has been found applicable, but the values of its constants have been modified by the incorporation of the younger ages. But again for practical purposes as an annuity table, this need not in this case be a matter of serious criticism. The comparison of the experience in this office, on whole life policies, survivorship annuities, and ordinary annuities, above the age of fifty, shows a consistency between the three regarding the natural effect of selection which goes far to sustain the table as a fair exponent of Danish life. That it is likely to prove a fair exponent also of life in America, however, is a different proposition. Comparing the mortality in this case with that on French annuitants and that of Mr. McClintock's Table for female annuitants, there is a parallelism between the two former that at once suggests a European mortality curve, differing in some important respects from that of America, as well as being generally in excess of the latter until middle age had passed. This is seen by a comparison with the tables furnished by Mr. Weeks in commenting on the Table of Mr. McClintock in the 6th volume of the *Transactions* and also by Mr. Macaulay in the 4th volume of the *Transactions*. Those essential differences are a much lighter mortality at the younger ages in the case of the American observations, gradually approaching that of the other, then nearly assimilating with it, as the older ages are approached and afterwards again falling below. It is in kind the same divergence that exists between the American and Actuaries Experience, though exceeding it in amount. The presumption seems strong that during those ages, which are most important, the Danish experience differs somewhat widely from that which might be anticipated here. This departure, too, is in the direction of an excess of mortality, making the Table a hazardous one on which to rely, with our present knowledge, for the purpose of granting even survivorship annuities. I entirely concur with Mr. Jensen, that a special table should be used for female beneficiaries, rather than the Makehamized American Experience. All the reliable observations that we have, strongly suggest that the mortality among female annuitants is not only essentially different from that among males, but is liable to be more largely modified by the conditions under which the observations are made. For practical use, I should regard Mr. McClintock's Table in the present state of our knowledge to be not only safer, but presumptively more nearly in accord with the expected facts even in case of survivorship annuities.

But Mr. Jensen's investigations are none the less important in throwing additional light on a subject, which is so imperfectly understood. A comparison of the male and female American annuity experience shows how wide apart are likely to be the conditions regulating the mortality of

the two sexes. It is important that whatever table may be used, should be fair as well as safe and to properly judge of these factors, every experience like that presented by Mr. Jensen, has its value. But I should be disposed to regard the probable differences in the experience to be looked for in the two countries as outweighing any gain resulting from the fact that it has been deduced from survivorship beneficiaries. The paper has a special value, too, as illustrating the method deemed scientifically the most accurate by Danish mathematicians for the determination of the Makeham constants. I have already pointed out why I am disposed to dispute this view as one of universal application, though cases may frequently arise in which the application of the law of Makeham to the statistics is so close that the principle of least squares may be expected to give the most accurate results.

Mr. Welch: Mr. Jensen's paper on "A Mortality Table for Female Beneficiaries and Survivorship Annuities" is a very timely offering to this Society. In recent years the introduction of the "continuous instalment" or "continuous income" policy and of "reversionary annuities" (the latter under the title of "survivorship annuities") has brought home to the actuary the necessity for some tables combining the mortality on an insured life with that on an annuitant.

Having tried to use existing tables, formed from the experience entirely on insured lives or entirely on annuitants, by making various adjustments in the same, and having found them to yield results far from satisfactory, I turned to Mr. Jensen's tables with more than usual interest and that others may judge of their value from the comparisons which I made I give the results of my own investigation.

First of all, I assumed that a company would not care to have a large number of annuitants the premiums and reserves for which were to be formed from the American Experience Table. This would be especially true with annuitants at the older ages, and it would be for the benefit of older men and women that reversionary annuities would usually be taken.

Neither did it seem quite fair to assume the mortality on these annuitants to be as light as had been experienced among those who voluntarily purchased immediate annuities at the upper ages. I was forced, therefore, to look for an annuitant table which would show a mortality something lighter than that of the American experience and something heavier than that of the immediate annuity tables. How far Mr. Jensen's table satisfies this need can be estimated by comparing the premiums for immediate annuities formed on his table with the premiums for such annuities formed on other well known tables. I therefore give a table showing the premiums for an annuity of $100.00 on the American Experience Table and on the British Offices' Annuitant Table both by the select and the ultimate and on the male and the female life.

Single Premium for Life Annuity of $100.00 on 3½ Interest Basis.

Age.	American Experience.	British Offices (Select) Males.	British Offices (Ultimate) Males.
20	2014.	2050.	
40	1645.	1649.	1628.
60	1003.	1044.	1016.
80	344.	445.	401.
85	208.	333.	288.

Age.	British Offices (Select) Females.	British Offices (Ultimate) Females.	Jensen's Beneficiary Annuitant's table.
20	2072.		2147.
40	1704.	1679.	1782.
60	1169.	1138.	1152.
80	495.	451.	434.
85	369.	323.	298.

By the above tables it will be seen that Mr. Jensen's annuity table is one that represents a mortality lying between the extremes of the tables above quoted and one that might fairly be taken as representing a mortality rate more nearly approaching that which American companies will experience on reversionary annuitants if both sexes are to be written on one and the same table.

The value of this table will be in its employment in making the premiums on reversionary annuities of all kinds where an annuitant life and a beneficiary life are to be dealt with in forming premiums and reserves.

Premiums for Revisionary Annuity of $100.00 on 3½ Interest Basis.

Ages		Makehamized American.	British Offices' Annuity Table, insured male, beneficiary female.	Jensen's Table.
Insured.	Benf.			
30	20	23.08	23.12	25.70
30	40	14.00	14.25	15.84
30	60	7.02	6.95	8.12
30	80	2.35	2.25	2.93
50	20	57.65	58.53	63.25
50	40	37.10	39.87	42.31
50	60	15.99	19.09	19.20
50	80	4.43	5.65	5.72

But this discrepancy in the premiums by the various tables is more pronounced when we compare the extra premium for the deferred contingent annuity employed in forming the premium for the ordinary continuous instalment policy.

The following table is for an annuity to the beneficiary deferred twenty years after the death of the nominator and is for $100.00 in amount (this being the extra premium that would be added to the regular premium

for a $2,000 twenty instalment ordinary life policy in order to have the rate for the so-called "continuous instalment" policy.).

Ages Insured.	Ages Benf.	Makehamized American.	British Offices Annuity Table, insured male, beneficiary female.	Jensen's Table.
30	20	6.08	6.78	7.45
30	40	1.97	2.43	2.62
30	60	.13	.28	.26
50	20	17.50	19.64	21.48
50	40	5.04	7.04	7.04
50	60	.27	.68	.56

From the above table it will be readily seen that the use of the American table, which was generally employed at first in forming these continuous instalment premiums, would yield results far below what the company ought to receive, and that the annuitant's table of the British Offices while yielding results that might be in the aggregate safe, could not be used with satisfaction since in one part of the formula for the full premium the insured by assumption would follow one rate of mortality and in another part of the formula quite another rate.

Taking all things into consideration it would seem as if Mr. Jensen's tables, which follow the law of uniform seniority both in the single life table and with the combined lives, would be found of great value to actuaries at this time; for until we have an experience of our own on such risks Mr. Jensen's tables, with adjustments in loading or of increasing or decreasing the annuitant's age, will be found capable of meeting almost any assumed rate of mortality on the insured or beneficiary lives.

Oral Discussion.

Mr. Dawson: Mr. President, I want to say just one word or two about the choice of the *c*. It was done deliberately, but, of course, for a definite purpose, that Mr. Strong has mentioned, namely, to enable the table to be used in connection with the American experience. But those who are inclined to criticise this course, will do well to consider the comparison which Mr. Jensen furnishes to the actual data, showing that his method of graduation did follow the facts very accurately indeed, and, had that not been true he would not have been disposed to insist upon it, but would have tried to find some other way to meet the difficulty.

Concerning the objection that Mr. Strong has just mentioned I would like to say that the female mortality in Denmark is more favorable than in the United States, so far as we know, and also that the things which he mentions in this particular mortality mean that the selection has been more favorable, using the term more favorable in the sense of a lighter mortality. The result is that rates and reserves brought out by the use of this table will be in all probability sufficient; whatever error there is, will not be on the side of danger. So that the tables can be used probably with greater limits of safety on that account instead of less.

STAFF PENSION FUNDS WITH SPECIAL REFERENCE TO A RETIREMENT FUND FOR UNITED STATES CIVIL SERVICE EMPLOYEES—BENEDICT D. FLYNN.

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WRITTEN DISCUSSION.

MR. GROW: Although Mr. Flynn states that the idea of providing for the aged poor of the state, and for the old and faithful employee, has not yet seriously taken hold of the American people, there is no doubt that the subject is attracting considerably more attention than it did, say, ten years ago; quite a number of the large railroad systems, and some of the larger corporations, during the time having made some provision for staff pension funds. Many cities have also adopted some form of old age pension plan in connection with their Police and Fire Departments, and, in some instances for the teachers in the public schools. The daily papers contain frequent news of legislative acts to enable savings banks and other semi-public institutions to pay pensions to old employees whose long service entitles them to some consideration.

Within the past two weeks we have read in the daily newspapers of the presentation, in the British Parliament, of an old age pension plan, but the most notable example at the present time of a pension scheme in active operation is that of the Army and Navy plan of the United States Government. These plans as well as pensions to employees of the Police and Fire Department, Public Schools and other public or semi-public institutions, in most instances do not contemplate payment out of a fund accumulated from the contributions of the employees, but are more in the nature of an annual public tax. The administration of such pension schemes does not therefore require the solution of any complicated actuarial questions.

Pension plans which are to be supported by the contributions of the employees are bound at the start to be more or less of an experimental nature and it would seem that for practical purposes such plans should be as simple as possible. Owing to the fact that very few employees look with favor on any plan that obliges them to part with their contributions to the fund if they sever their connection with their employer before reaching the age at which the pension is payable, the Savings Bank feature would seem to be almost essential to any scheme in order to make it attractive to the employees; but as savings banks already exist in great numbers as separate and distinct institutions, old age pension plans should contain some other attractive and distinctive features of their own.

One of the principal difficulties in the way of inaugurating a successful pension plan is to make some adequate provision for those who have already grown old in the service and who have therefore nearly reached or already reached the age of retirement. If the pension plan depends entirely upon the contributions of its members, there is no practical way of providing for these older members, excepting to charge the younger

members with more than their respective shares of the cost. The younger members, however, will not consider it fair to make them pay, not only the cost of their own pension, but also the heavy excess required to meet the payment of the pensions to the older members who are nearing the age of retirement; and, as the younger members usually far outnumber the older ones, any attempt or plan to assess this extra cost upon the younger members will result in dissatisfaction among the majority of the members.

The effort of the United States Civil Service Retirement Association to find some practical solution of a Staff Pension Plan depending upon its members for support, furnishes a very good example of the difficulty just mentioned. In the early part of 1902 the Executive Committee of the United States Civil Service Retirement Association of Washington wrote me with regard to a plan to enable them to formulate a petition or bill which they were preparing for the consideration of the United States Congress. The matter was taken up, and the work was done after office hours by employees of the Mathematical Department of the New York Life Insurance Company, under the direction of Mr. Arthur Hunter and myself. There were no direct practical results from this work, because of the fact that, after careful consideration of our reports the cost was found to be so great, owing to the large number of old employees, that the plans then under consideration did not meet with the approval of the great body of Government employees.

Since 1902 the United States Civil Service Retirement Association has considered the matter of old age pensions from many standpoints, and has received advice from a number of actuaries regarding the same, but there seems to be no effectual solution of the question unless the Federal Government will consent to make a contribution to the fund sufficient to meet the cost of retirement, when they reach the age of retirement, of the members who are already 45 years of age or more.

The difficulty experienced by the United States Civil Service Retirement Association in finding some solution of the question of providing for those members who are within fifteen years or less of the age of retirement, must also be faced at the inauguration of any staff pension plan, but, in that class of Government pensions where the entire cost of the pension is met by public taxation, the question of providing for the older members does not present such a serious difficulty, as in the other class of cases where it is desired to provide staff pension funds to employees of corporations, commercial houses or banks. If at the inauguration of the plan the provision for the older members must be made by assessing the cost against the younger members, the plan in my opinion is doomed to failure, because it is a well known fact that, owing to the small salaries received by clerks in general it is almost impossible for them to effect any material saving from their income. In case of the death of the employee the employer oftentimes feels called upon to make at least some temporary provision for the employee's family; it therefore follows quite logically that at the outset, at least, employers should do their part

towards meeting the extra cost due to the older members, and if the plan appeals to them in other respects, employers are not likely to raise any serious objection to this, as it is simply another method of providing for those employees who have served them for a long period of years.

Any plan, however, which contemplates nothing more than a provision for maintenance of the employee in old age, after a long period of service, lacks some very essential elements. The plan should also include some provision for the family in case of death while in service, and, if possible, some return to those employees who after having contributed to the fund, leave the service before reaching the pension age.

Actuaries in dealing with this subject are apt to prepare plans which are ideal for the protection of the employer from their standpoint, but experience shows that these plans have to be frequently modified to suit both the employer and employee. We can, therefore, profitably investigate the pension plans which have been actually put into effect in various corporations, in order to get the employer's idea of what is the most suitable kind of plan, instead of the actuaries' idea.

The three types which will be suggested for viewing the subject from this standpoint, are:—

1st. From the standpoint of an insurance company, which, at the request of its agents, prepared a plan supposed to be specially suitable for employers. This plan was prepared after discussing the subject with many employers and with many of its agency men.

2nd. From the standpoint of the Government of Australia.

3rd. From the standpoint of a large New York corporation which has successfully operated the scheme for the last 15 years.

The first thought which occurred in connection with the Life Insurance Company's plan, was that the price should be low. In order to make the price as low as possible, it was thought desirable to eliminate all surrender values and investment elements if the policy should not be carried to maturity.

Policies were not issued at ages over 55. The face amount of the insurance was payable if the insured died at any time before the anniversary when his nearest age would be 61, and the premium was payable in full until the same date. At that date the insurance and the premium were both reduced one-tenth, and were reduced by the same amount each year successively until at the anniversary nearest the age of 69 the insurance for the next succeeding year was only one-tenth of the original amount, and the premium was only one-tenth of the original premium. One year later (that is, when the Insured would reach the anniversary nearest the age of 70) the insurance ceased. No more premiums were payable and the insured thereupon received the first annual payment of an annuity to continue for the rest of his lifetime; the amount of the annuity being derived from the surplus earned by this class of policies was not guaranteed, but the policy provided that the annuity

would be the same per thousand of insurance for all persons reaching the end of their insurance (*i. e.*, the anniversary nearest the age of 70) in the same calendar year, regardless of the calendar year in which they originally insured, or the age of the insured at issue. On the assumption that the rates of mortality, expense, interest and lapse which the company was then experiencing would continue, the amount of such annuity would not be less than $30. for each $1,000. of insurance, but, as stated above, the amount of the annuity was not guaranteed.

As one of the main features of this plan was to make the cost as low as possible in relation to the benefits, it left a rather small margin for expenses, and therefore the commission to agents was small, and in order to provide for the annuity at the age of 70 it was necessary to cut out all surrender values and everything else not necessary to carry out the principal objects of the plan which were insurance for the family in case of death, and annuity for the insured if he should live to age 70. The elimination of the surrender values took away one of the prominent savings bank features, which, as already mentioned, seems to be essential if any plan is to be popular and attractive to the persons who make up the class to whom staff pension fund plans apply; this essential feature being the ability to withdraw the whole or part of their own contributions if they sever their connection with their employer before reaching the pension age.

Our second example shows how a life insurance policy has been used by the Australian Government as a basis of a staff pension fund for government employees. It is well known among those engaged in the business of life insurance that there is a strong leaning among persons of small and moderate means toward an endowment insurance policy, and there is very little doubt that this preference is chiefly because of the large savings bank element which enters into every endowment insurance contract. The advantage of an endowment insurance policy for the purpose of staff pension funds is recognized by the Australian Government plan, which provides that on the confirmation of his appointment every government officer shall effect an insurance on his life, the amount of such insurance being at least equal to one year's salary. Every government employee coming under the provisions of the act holds his appointment subject to the condition that a deduction may from time to time be made from his salary sufficient to cover the premium on his life insurance, and, if the premium be more than two months in arrears, the Chief Officer under the Public Service Act may deduct from the salary of the assured the amount of arrears of premium and fines, if any, payable on account of the assurance and pay the same to the insurance company. The plan also provides that for every increase of one hundred pounds of salary the insurance shall automatically be increased to the amount of the increased salary and the premium shall be in conformity with the rate payable at the age of the insured at the time when the insurances are increased. The policies are all on the Endowment plan and are usually payable when the assured reaches the age of 60 or 65, but

it is optional with the assured to make the policy payable at an earlier date. Policies are not assignable or transferable either at law or in equity, and are wholly exempt from the operations of any law relating to bankruptcy or insolvency, and are not liable to be seized, levied upon, attached or sold, upon, by or under any legal process, or in case of death are not available as are other assets for the payment of the debts of the assured.

If the salary of any person affected by the Public Service Acts is reduced, the assured may agree with the company to reduce the sum assured under the policy to an amount at least equal to the amount corresponding to the reduced salary. If the person is not assurable without a loading of five years or more being made upon his age, a certain sum is deducted quarter-annually from his salary and paid into a trust fund to be invested by the Government and accumulated in the interest of such person from whose salary the amounts have been deducted and to be repaid with interest on his leaving the service or if he should die while in the employ of the Government. These Acts require that all policies shall be deposited with the Chief Officer under the Public Service Act so long as the assured remains in the Public Service. The Acts also provide that, if the assured retires or is dismissed or otherwise leaves the Public Service, he shall have delivered to him all his policies of insurance which are in the possession of such Chief Officer and shall notify the insurance company.

It will be seen from the above that the Australian Government plan depends entirely upon the contributions of the members, that it makes a provision for the family in case of the death of the employee, that it provides a fund for him in case he should leave the service, and that if he lives to age 60 or 65 the policy will mature as an Endowment and thus make some provision for his old age.

Having considered the Life Insurance Company's Plan and the Australian Government Plan, it will be interesting to consider a third plan, which for the sake of convenience we will call the Commercial Company's Plan.

This plan was adopted in 1893 by a New York corporation and consisted in an arrangement with a life insurance company to issue straight fifteen and twenty year endowment policies on the lives of the employees of the Commercial Company. The employees were divided into three groups.

1. Consisting of Superintendents who were allowed to insure up to $5,000.

2. Consisting of Supervisors, Chief Clerks, etc., who were allowed to insure up to $4,000.

3. Consisting of the balance of the employees of the Company who were allowed to insure up to $3,000.

A minimum of $1,000. was fixed in every case.

No medical examination was required, as all the beneficiaries had recently been examined by the Commercial Company's own physicians and the Commercial Company furnished the life insurance company with a

certificate signed by the applicant and witnessed by two officers of the Commercial Company that the applicant was in good health and actively engaged in his duties.

The annual premium was paid by the Commercial Company at the beginning of the policy year, but there was an understanding between the company and its employees that one-half the total annual premium should be deducted from their salaries, such deduction being made in monthly instalments; the remaining half of the premium was contributed by the Commercial Company.

In case of the death of the employee, the amount insured under the policy was paid direct to the beneficiary selected by the insured. It was recommended that married men should make their wives the beneficiaries, in order that the wife would be sure to receive the whole amount of the insurance which she might not do if the policy were made payable to the insured's estate, and at the time of his death there should be any claim against the estate on account of death.

When a policy matured the employee was required to leave the proceeds of the same under the control of the Commercial Company. In its announcement to its employees at the inception of the plan, the Commercial Company stated that "after contributing one-half of the premiums on the policies, the company cannot permit the amounts of the policies to be drawn at maturity and at the same time allow the beneficiaries to continue in the company's employment during a period when, through unfortunate investment or speculation, the money may be dissipated and the company eventually find itself with men on its rolls whom it is unable to keep and who may be unable to earn a livelihood elsewhere."

The Commercial Company also called the attention of its employees to the fact that, if the money were applied to the purchase of an annuity, at, say age 50, it would yield a handsome return on the sum actually invested and that at older ages the return would be proportionately greater. The policies were deposited with the Commercial Company and each employee was under agreement with the Commercial Company not to assign, sell or in any way encumber his policy during his service with the company without their consent. Each employee received a copy of the policy. In case the employee severed his connection with the Commercial Company, the life insurance company was notified, and he was given his policy to do with it as he pleased. The sum, substance and object of the plan were embodied in a single sentence in a letter written in 1895 by the Vice-President of the Commercial Company, which reads as follows:—

> "My aim and desire is to make it certain that men after years of service, and when perhaps I may not have a voice in the company's affairs, shall not under stress of circumstances have to leave the 'Commercial' empty handed."

The transaction has been a very satisfactory one to the insurance com-

pany, owing to the small number of lapses and the low death rate. The Commercial Company and their employees have also been very well pleased with the arrangement, as the result during the past fifteen years has been to increase the amicable relations between the company and its employees, and the proceeds of the policies now represent a very handsome return to the employee for the amount of money which he has been obliged to contribute personally.

The subject of staff pension funds is one which has received much thought from actuaries for a great many years and it has been considered from almost every conceivable point of view, an immense amount of work and thought having been devoted to this subject, but notwithstanding all this the tendency among the governments seems to be toward some simple plan either in the form of taxation, or as in Australia, by means of obligatory endowment insurance.

These three examples which we have cited have been selected because they illustrate in a simple and practical manner how the subject of staff pension funds has been handled in some particular instances, and with the hope that they may also suggest a ready means at hand of avoiding in a measure the difficulties which Mr. Manly mentions arising from the fact that "employers are prone to change the rules of the plan about every five years, oftentimes without the knowledge or advice of the actuary who made the original calculations."

As the interest of the commercial employer in staff pensions is the same as that of the Government employer, namely, to secure continued efficient service, it would appear that a plan somewhat like that of the Commercial Company, described above, would be eminently satisfactory for the purpose, as it leaves plenty of room for the employer to change the rules for the *administration* of the plan without in any way changing the basic mathematical and actuarial principles on which the plan depends for its continuance.

Mr. S. S. Hall: Mr. Flynn in his paper concerning "Staff Pensions" has called the attention of the Society to an actuarial problem which, as yet, has received but little attention in this country, but which must, as the country becomes more thickly settled, come into greater prominence. There appears, also, to be a general socialistic tendency which will undoubtedly, in due time, cause the people to adopt some method of providing for aged employees. In fact, we find in our larger financial institutions, that the old employees are being taken care of, usually through the method of continuing these employees on the salary list for a reduced amount.

The particular point to which Mr. Flynn calls our attention is the recent discussion endeavoring to provide for some systematic pensioning of government employees appointed under the Civil Service rules. There has gradually been a growing feeling among such employees that the salaries paid by the government are not sufficient to enable them to save for their old age and that, in consideration of their having spent the

best years of their lives in the service of the government, some means should be adopted whereby a living salary should be secured to them after they are incapable of working. In the method which has been proposed, there appears to be two serious errors which no doubt have been caused by the popular dislike to an annuity system, pure and simple. The rate of interest at which the government is to accumulate the savings of its employees was taken at 4 per cent., whereas in all the recent insurance laws a maximum of 3½ per cent. has been fixed; in fact, 3½ per cent. would appear to me to be a high figure, since the rate of interest which the government is compelled to pay, where it desires to borrow money, is less than this rate. Any charge above the rate which the government can borrow money for, becomes, in reality, a pension contribution by the government itself. The rate of 4 per cent. which was adopted, therefore, in reality, becomes a heavy pension charge on the government. Doubtless this rate has been assumed, since 4 per cent. may be obtained from many of the larger savings banks. An examination of other features of the scheme appears to warrant the assumption that a compromise has been made, in order that the employee may always feel that the money contributed by him is being accumulated as in a savings bank and is under his control.

If we examine the various options of settlement at the age of maturity, we find that the employee has the right to withdraw the accumulated sum contributed by him if he may so elect, or he may use the accumulation to buy an annuity for a specific term of years. This feature of giving the employee absolute control of his savings makes it feasible for anyone who is in poor health at the age of retirement to withdraw from the service of the government an amount equal to that which he has contributed, together with 4 per cent. compound interest, and the result will be that only those in good physical condition will take the annuity out. Manifestly, this will make a selection against the government and I believe this selection will be exercised in case the scheme is adopted to such a degree as to make the contributions by the government heavier than the mortality table used will provide for.

Probably these points have been carefully gone over by Mr. Flynn, and I notice he states that he was consulted, on various occasions, in regard to the scheme adopted. It would be interesting to have a statement from him covering the points which I have brought in question, as, doubtless, the Society will be able thereby to obtain additional information in regard to the attitude of the public towards the scheme, particularly of those whom the scheme was intended to benefit.

Mr. A. Hunter: My experience with pension schemes has been that the employers are enthusiastic in regard to putting in force a plan to superannuate their employees until the actuarial cost of it is known, when their enthusiasm is apt to vanish. Their humanitarian motives are excellent, but their business instincts rebel at the *apparent* cost. The

result is that a comparatively small percentage of employers, who investigate the subject, eventually put into effect a pension scheme. This may be due in some cases to their actuarial adviser looking at the subject solely from the standpoint of the apparent cost of the pensions, and not from the true point of view of the improved efficiency of the force which may entirely offset the apparent cost. Let us take an actual case and assume that the actuary informs the employer that the pension scheme would be equivalent to increasing the aggregate salaries by 5%. He should not, however, stop there, but should bring forcibly before the management that an increase of 5% in the effective work of the employees would counterbalance the cost of the pension scheme. The matter, therefore, for the employer to determine is, whether the morale of the force will be so improved by the offer of old age pensions that 5% more work will result. In my judgment, that is probable, because the employees do not then have to worry about their ability to earn a livelihood in old age and their loyalty towards the employer is increased. Another advantage to the employer is that the employee becomes more satisfied with his position and does not have his attention distracted by looking elsewhere to improve his standing. This is particularly valuable in the case of the type of men who are earning all they are worth and who are of more value to their present employer than to anyone else on account of their knowledge of his business affairs. Another practical phase of the problem, which should be considered as an offset to the cost of the pensions, is, that a better grade of people can be obtained when provision is made for their faithful service.

A point of view which the actuary should place before the employer is, that the pension itself is not all outlay, as more effective service is obtained from the young man who takes the place of the old employee. Let us consider as an illustration the case of a man who has been 35 years in the service, and has attained the age of 65, his salary for the past few years having averaged $1,200. per annum. Such a man could probably live comfortably on $720. a year, or $60. per month. If he is retired at age 65, instead of at a later age, it means that his powers have declined and that he is not up to his old standard of efficiency. It is therefore probable that for $60. a month a young man could be obtained to do as good work as the pensioner, so that the combined cost of the pension and the salary of the young man would be $120. a month, or $240. per annum more than the salary of the old employee. At a cost, therefore, of $240. a year the employer has been generous to a faithful worker, has given peace of mind to many other employees, and is getting the work as well, if not better, done than formerly. Of course from another and brutal standpoint, the employee could be discharged when his usefulness had ceased, but a firm that carries out such ideas at the present time is apt to be injured thereby. For example, a wealthy firm recently discharged an employee after forty years of service, simply because he sympathized with his men during a strike, altho not rendering them any assistance. The effect of that action was the cancellation of the

accounts of many customers, as those who knew the facts were sufficiently indignant to make a point of telling their neighbors. The effect on the employees was greater, as they were discouraged from doing more work than was absolutely necessary to hold their positions, many of the best men left the service and a feeling of animosity and distrust was created which interfered with proper conduct of the business.

In the minds of many legislators there is an objection to pension schemes, but such objection is gradually dying out. Twenty-five years ago in this country pension schemes were very little known and when mentioned usually called forth adverse comment; but to-day the tide seems to have turned, and there is a general disposition on the part of large corporations to reward those who give faithful service. This change has probably come thro recognition of the fact that it is good business judgment to look after the employees. In the older established countries, like France and Britain, it is unusual to find a large insurance company or bank which does not make provision for its employees. In many cases pensions are given without any cost to the employee, while, in others, a small monthly payment is exacted,—as low as 10% of the actual cost of the pensions.

As a body of humanitarians, as well as actuaries, we should do our best to have pension schemes introduced wherever possible, and we should therefore bring before those who employ us or consult us the strongest arguments on behalf of the pension scheme from the humanitarian standpoint and from the far-seeing business standpoint, dwelling strongly upon the good effect of the scheme on the morale and quality of the force, which partly, if not altogether, balances the monetary cost.

Mr. Flynn (*Note in reply to Written Discussion*): The Superannuation plan mentioned in my paper has not yet been adopted by the Federal Government. A bill proposing this plan was introduced in the last congress and taken up in committee but was not acted upon before adjournment. There was considerable discussion in committee with regard to the proper rate of interest to use, and it was finally decided that 3½ per cent. would be a better rate than 4 per cent.—which had been originally proposed. In making this decision the fact that most insurance companies are guaranteeing this rate as a maximum was given considerable weight. The idea originally in mind when four per cent. was suggested, however, was that although the government might be able to borrow money at two or two and one-half per cent., that as a help to the Civil Service Superannuation plan it could afford to grant four per cent. upon deposits. In order to relieve further the government of the apparent pension contribution in the shape of a higher rate of interest than it would have to pay as a borrower, the bill has been amended so that the money may be placed in savings banks at interest, or invested in securities described in the bill.

With regard to Mr. Hall's second point that in view of the fact that the employee upon reaching the age of retirement could either withdraw the total of his accumulated savings or purchase an annuity, and that

therefore the selection against the government would be very severe, I would say that this point was taken into consideration and is mentioned in the paper. In figuring the amount of annuity purchasable by these funds the selection was considered as high or even higher than that against insurance companies, and a table—British Offices' Annuitant table—was used which is fully as severe as that used by most American companies for determining annuity rates. It might be that Mr. Hall had in mind the American Experience table which is mentioned as the basis of the annuities contributed by the government. In view of the fact that this government aid can only be taken in annuities and cannot be commuted, it can be seen that there would be practically no selection against the government. Everyone who reaches the age of retirement whether in good or bad health is offered an annuity, and he must take this annual payment with no privilege of commutation.

AN INSTRUCTIVE MORTALITY EXPERIENCE—M. M. DAWSON.

VOL. X, PAGE 289.

ORAL DISCUSSION.

MR. HENDERSON: I have to apologize, Mr. President, for not preparing a written discussion on this paper. When I received your note I studied this paper somewhat and studied some other experiences, and had some figures prepared. Unfortunately the sheet which contained my figures, did not contain the source of them, and the day before yesterday when I came to prepare my remarks, I found that I could not discover the source of my figures, so that I cannot give you the references for the general impression which I desired, that this delayed minimum regarding which Mr. Dawson speaks in the rate of mortality was not one depending exactly on the age attained, but rather arose from a poorer selection of those who enter at ages under twenty-five. In that general experience, and especially in the particular one which I have tabulated, which I cannot tell you where it came from now, the minimum, combining the first five insurance years was at ages twenty-five to thirty. The minimum containing six to ten, or ages thirty to thirty-five; and the minimum ten to fourteen, ages thirty-five to forty, seemed to be exactly at ages entering between twenty--five and thirty, and it seemed to me to indicate that it was a matter of entrance selection rather than a feature of a general population experience from which the class was drawn.

MR. DAWSON: Mr. President. The suggestion just made by Mr. Henderson is a novel one to me. Of course, being employed in similar investigations, presumably we did the same work as to the application, and it is not impossible that that may have had some influence.

One thing that troubled me most about ages is that while it be true concerning the first four calendar years and while it proved to be true in the ultimate conclusion of this table, it should not have proven to be true in the year of admission, when a person is on the average, about six months. If the same tendency had been found there I should have felt a little more certain of my ground.

I would like to call attention to the fact that in a paper read by Mr. Hunter yesterday, the Tropical mortality shown exhibits the same peculiarity, and if some theories which I have mentioned as a possible explanation should apply in the one case, it evidently would not apply there. The theory which I offered being that it was where there were superior surroundings, where the environment was particularly good this result might be expected. And the fact that where the environment is causing the highest mortality it appears to be true that men in the prime of life, somewhere from thirty-five to forty, have the greatest resisting power, and they are exhibiting the smallest mortality, proved to be true in that practical experience, leads me to feel that the whole subject needs to be reconsidered in the light of that experience.

Mr. J. M. Craig: There is one point in Mr. Dawson's paper that I thought I would like to make a remark on. He says:

"As to mortality among women, all investigations or nearly all show that, whatever the relation to death-rate among men at the same age, there is a marked increase during the ages when maternity is most common and a gradual subsidence. And as maternity is possible and not infrequent before complete maturity is attained, it follows that the decrease may yield to an increase prematurely, because of the intrusion of this extra hazard."

I would like to say that, so far as the great mass of industrial risks is concerned, that this statement would not hold true; that the rate of mortality is very materially lower among women than among men, and perhaps it would not be amiss to say that in the company with which I am connected the rate of mortality is not higher among women than it is among the men in its branch of Ordinary risks as distinguished from the Industrial, and that the experience takes in about a hundred thousand years of risk.

Mr. Dawson: Mr. President, I am very glad that Mr. Craig brought up this matter, as it leads me to think that perhaps there is some misconception of what I at least intended to say.

I am quite well aware that in many groups at least, the mortality among women is continuously lower than among men. I have personally investigated the experience of the largest Society, composed entirely of women, in the United States, worked it out on the select basis and accounted for the result.

What I meant to say was this, this curve which I have shown shows the mortality diminishing from about age eighteen to roundly age thirty-five, then gradually increasing. Now, it may be that the rate of mortality among women at some place I have never had the opportunity to investigate, women of the same class exactly as in the Minnesota Lodge, it may be that the mortality would be lower continuously than this curve shows, but, per contra, it would probably show an increasing mortality from age eighteen to thirty-five, or that neighborhood and gradually subside. The whole of it might lie below the other curve, in my opinion very likely would lie below the other curve, but it would be a different situation entirely from the one which I found here, and I may say that in the analysis of the experience of the Ladies of the Maccabees, embracing, I think, more than three hundred thousand lives, although I am not quite positive, at least two hundred thousand lives were found to be of an extremely low mortality, but we also found just the peculiarity of the curve to which I have referred, and I think it has been found in every investigation of female mortality.

NOTE ON AN APPROXIMATE METHOD OF MAKING MORTALITY INVESTIGATIONS —ARTHUR HUNTER.

VOL. X, PAGE 361.

WRITTEN DISCUSSION.

MR. JOHNSTON: Mr. Arthur Hunter's note on an approximate method of making mortality investigations calls for very little comment inasmuch as it is a statement of facts concerning the method followed by him. The system described by him is very simple and there are some points which will bear emphasis.

Referring to the use of cards, Mr. Hunter says, "It is not true economy to put these same cards to too many uses." This is a very practical statement and one borne out by our own experience, as we sometimes have one set of cards tied up in one investigation when it becomes desirable to make some other form of investigation, in which case another set of cards is used. On the other hand, it is not desirable to multiply the number of sets, as the work of keeping each set up to date is considerable. It is our practice to write several cards from each application after a policy is issued, and each set of cards so written is put to a distinctive use, such as, for valuation purposes, renewals and various investigations. With regard to the question raised by Mr. Hunter as to whether it is better to write the cards after the policies have been paid for or from the application papers at the time the policy is written, we have found it of advantage to write the cards directly from the application. By comparing the several sets of cards with each other a cross check is obtained on the accuracy of the work.

The method adopted by Mr. Hunter in finding the duration under lapsed policies is very simple and advantageous in that it gives an integral number of years' duration in each case. As shown by Mr. Hunter, the difference between his method and the "exact" duration is a negligible quantity, especially when comparing investigations which have all been made on the same basis.

Mr. Hunter's method of grouping the ages of the exposed to risk and applying a mortality factor for the central age of the group is undoubtedly a time saver, and where several experiences are prepared on the same basis the results by this method are comparable, but I would take it that there would be considerable variation at the higher ages where the numbers exposed to risk are constantly diminishing while at the same time the mortality factor is rapidly increasing.

Another interesting point brought out by Mr. Hunter is the fact that the multiplication was done more rapidly by means of Meech's or Cogsworth's Tables than by means of calculating machines. Personally, I have always been in favor of the calculating machines and have always felt that even in such cases where Meech's Tables were applicable a clerk after working any length of time with such tables got tired or stale and was very liable to make an error in copying from the tables.

Mr. Hunter's statement that it is not necessary to spend money and labor on great accuracy in obtaining results which are to be used approximately is a good point. In comparing experiences of one class with another it is more necessary that the methods followed in taking out these experiences should correspond than that extreme accuracy should be attempted if the methods differ. Sometimes a refinement of method is something like trying to measure a distance by pacing a part of it and measuring the rest of it with a foot-rule.

It may not be out of place to mention briefly a method of approximating a mortality experience which we use in our office. In making the usual valuations the business at the end of the various periods is arranged on sheets according to years of issue, kinds of insurance and ages at entry, and it is a simple matter to find the exposures by taking the mean between the existing at the beginning and end of any year. Taking the mean of these amounts and adding one-half of the deaths, figures very close to the true exposures can be obtained, not only by calendar years of issue but by various kinds of policies, such as life forms and endowments. Slight adjustments have to be made, however, in the case of business in its first and second calendar years, where the lapse rate is heaviest. These adjustments are made by taking the mean between the business in force at the end of June and the mean in force for the year. The result is that we get each year a mortality experience by calendar years of issue and by kinds. While the results might differ very slightly from those obtained by a more refined method, as these experiences are all made up on the same basis they give an excellent means of comparison of one year's issue with another. While the amount of clerical labor is considerable, it is nothing like as great as it would be if a mortality experience had to be taken out in the usual way, and we have the further satisfaction of knowing very closely from year to year our mortality experience.

Mr. H. Moir: I have read this note with great interest in connection with the paper which appeared in the May, 1907, number of our *Transactions*; it gives a completeness and finish to that paper which is exceedingly satisfactory, although most of us are prepared to accept figures compiled by Mr. Hunter even without explanations showing exactly how the results are derived. The general outlines of Mr. Hunter's method follow those which have been recognized as the most suitable by the leading actuaries of the world. But a good many special points had to be covered where originality was necessary, and the modifications are admirably adapted to the peculiar circumstances and features of this particular investigation.

Mr. Hunter directs particular attention to the use of policies and not lives, also to the fact that duplicates were not eliminated; and he is careful to show that in a small experience this might possibly bring about a disturbance of the results. It has seemed to me that too much prominence was given to the fact that the Actuaries' or Combined Ex-

perience Table (Seventeen Offices' Experience Table) was prepared from a record of policies and not from lives. This has been a favorite criticism of that Table; yet I have not been able to see that the results were appreciably affected because of that method of deducing the experience. There is perhaps in theory a disturbing influence caused by the fact that at young ages duplicate policies scarcely exist; while those who remain healthy and prosperous effect more insurance as they advance in age until a prosperous man of from forty to fifty probably holds eight, ten or more life insurance policies, and under the policy system would be ranked as eight or ten deaths; nevertheless, if the experience is a large one he also counts as eight or ten exposures during all the time that he carries those policies, and this affects the denominator of the mortality fraction in about the same relative degree that the numerator is affected.

It must be remembered that Mr. Hunter's recommendations were made principally for a large—a very large—company, his methods having been evolved for a company with nearly two billions of insurance outstanding. When this is kept in mind, we must all entirely agree with his remark—"The card was prepared solely for mortality purposes as experience has shown that it is not true economy to put the same card to too many uses, such as for mortality, valuation, and record of dividends." But lately I have had the opportunity of seeing one or two small companies, each with a few hundred policyholders, starting card systems and having different cards for all kinds of information, maintaining several complete sets of separate cards. In a case of that kind it is doubtful whether cards are an advantage at all, except for future developments, since the policies are carefully recorded in a policy register.

I call attention to this particular feature because young men are so prone to copy what may be an excellent method for one company but poor and unsatisfactory for another. One of the most important qualifications an actuary should have is originality of thought as evidenced by adaptability to his surroundings.

This note of Mr. Hunter's will prove of much value to students, who generally have no occasion to work out for themselves the details which have to be considered in framing a mortality table, and who therefore need careful explanations regarding the processes. The note was prepared by request, and we are all much indebted to Mr. Hunter for having furnished it.

Mr. Rhodes: Mr. Hunter advocates grouping the cards by quinquennial ages and claims that the results obtained will not differ materially from the more lengthy process of grouping each age separately. In order to test this, the following table is given, which shows (1) the "Exposed to Risk" during the year 1906, arising from policies issued by the Mutual Benefit between 1846 and 1906; (2) the "Expected Loss" by the American Experience Table according to Actual Ages; (3) the same according to "Central Ages;" and (4) the percentage of Expected loss by Central Ages to that by Actual Ages. In order to see how far these

percentages might hold true for other years, the percentages derived from the Expected Loss for the year 1901 are added in the right-hand column. The table is as follows:—

EXPERIENCE 1906—ISSUES 1846–1906.

Central Age.	Exposed to Risk.	Expected Loss by American Experience Table		Percentage expected Loss by Central Ages to that by Actual Ages.	
		Actual Ages.	Central Ages.	(1906)	(1901)
18	$ 3,372,700.	$ 26,188.	$ 25,970.	99.2	99.1
23	16,729,700.	133,823.	133,838.	100.0	99.9
28	32,367,400.	267,882.	268,649.	100.3	100.3
33	48,931,700.	427,349.	425,706.	99.6	99.7
38	61,778,003.	582,064.	580,713.	99.8	99.5
43	62,556,639.	661,388.	656,845.	99.3	99.6
48	57,818,992.	723,707.	722,737.	99.9	99.4
53	39,431,190.	643,110.	642,728.	99.9	99.9
58	29,480,150.	672,618.	675,095.	100.4	100.6
63	19,305,051.	646,027.	654,441.	101.3	100.3
68	12,556,800.	646,050.	652,954.	101.1	101.1
73	7,266,716.	573,985.	582,791.	101.5	101.8
78	3,723,042.	440,348.	449,743.	102.1	101.4
83	1,371,636.	252,312.	262,805.	104.2	102.7
88	409,814.	133,941.	142,083.	106.1	104.8
93	48,345.	32,172.	35,495.	110.3	
Totals.	$397,312,303.	6,862,964.	6,912,593.	100.7	100.6
			6,862,964.		
		Difference,	$ 49,629		

It will be seen that for all ages combined the use of central ages involves an error of very little more than the ½ of 1% mentioned by Mr. Hunter, but if it is desired to compare the mortality at different ages it would appear that some considerable error might easily be introduced by the use of central ages.

It would appear that Mr. Hunter's method is not well adapted to keeping track of mortality records from year to year, being intended principally for making one investigation only of a particular class. There are many companies which have not sufficient business on their books to warrant them in undertaking an investigation of special classes, but if a method were employed which kept a running account of the "Exposed" and "Died" from year to year, the labor involved each year would not be very great and year by year the data becomes more valuable. The following method has been well tested by the Mutual Benefit:

A running account is kept to show the General Mortality Experience each year and similar accounts are kept of the Mortality Experience in each State in which the Company has done business.

The General Mortality Experience is based on amounts and is kept by ages and calendar years of issue.

Card (A) known as the General Mortality Card, is written for each policy issued. Where the insurance is continued under extended insurance or paid up policy. Card (B) is written which contains the full record of the policy and replaces card (A).

A

No. of Policy .....

Am't Ins'd - - $.....

In 19..... Reduced to $.....

Prms. Pble.	Day	Mo.	Year
Date of Issue,	.....	.....	19
Date of Exit,	.....	.....	19

Age at Issue.....	.....
Age Jan. 1st, 19.....	.....
Mode of Exit.....	.....

Occupation,.....

Residence,.....

Term Pol. to..... 19.....

Cards (A) representing the new issues for the year are sorted by ages and the amounts at risk are footed and carried to column 1 on Card (C), there being one such card for each age and for each year of issue. A policy exposed for a fraction of a year is treated as if exposed for a fraction of the amount for a full year.

The Cards (A) corresponding to policies ceased during the year are picked out and the total amount represented is entered in column 2 of Card (C). The difference between columns 1 and 2 will give the amount

in force at the beginning of the following year. The fractional amount exposed under each ceased policy is entered in pencil on cards (A) and the total of these fractional amounts added to the amount remaining in force throughout the entire year will give the amount at risk, which is entered in column 3 of Card (C). The amounts cancelled by death are

B

No. ORIGINAL, _____

AM'T OF ORIGINAL, $_____

No. { EXTENSION, / PAID UP, _____

AM'T { EXTENSION, / PAID UP. $_____

	DAY	MONTH	YEAR
DATE OF ORIGINAL,			
DATE { EXTENSION, / PAID UP,			
DATE OF EXIT,			

AGE OF ORIGINAL POLICY,	
AGE JAN. 1, 19 ORIGINAL	
AGE { EXTENSION, / PAID UP,	
AGE JAN. 1, 19 { EXTENSION, / PAID UP,	
MODE OF EXIT,	

OCCUPATION,

STATE,

entered at the same time in column 4 and the expected loss is then computed and entered in column 5. It will be noticed that when the General Mortality Footings' Cards (C) have been prepared for any year of issue, the only individual Cards (A) that need be handled each sub-

sequent year are those corresponding to the ceased, restored and changed policies.

The State Mortality Experience is also based on amounts and is kept by risk ages but irrespective of years of issue, for to do so would very

C **GENERAL MORTALITY FOOTINGS.**

Year of Issue _____

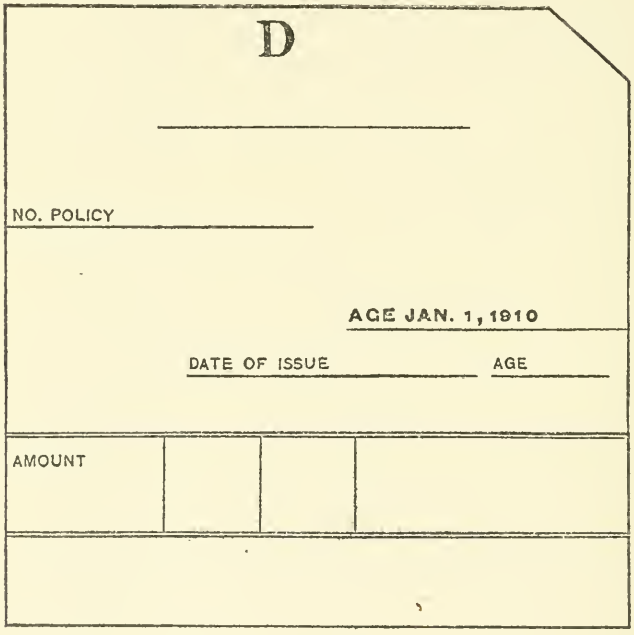
Age Jan. 1 _____→

	IN FORCE JAN.	TOTAL CEASED	AT RISK	ACTUAL LOSS	EXPECTED LOSS
1907	1	2	3	4	5
1908					
1909					
1910					
1911					
1912					
1913					
1914					
1915					
1916					
1917					
1918					
1919					
1920					

materially increase the number of summary cards to be handled. It has been found to be more convenient to have a separate State Mortality Card (D) written for each policy, rather than to attempt to use cards

(A) for both General and State Mortality. Cards (D) are written from Cards (A) so that a comparison of the former ensures the accuracy of the latter.

In calculating the amount at risk for the State Mortality the duration is taken to the nearest half year. The summary cards are used for all years of issue combined so that provision must be made for entering the



new issues each year. A section of a card which may be used for this purpose is shown (E). The card may be made to contain the records for as many years as is desired and the heading of the card will show the State and the age. It will be seen that the amount at risk each year is shown: and a subtraction of the item "Ceased 2nd ½ year" from the item immediately above it gives the amount in force at the beginning of the next year.

It is suggested that in place of keeping the records of ages as at a fixed date, *i. e.*, 1910, it might be preferable to keep the records according to year of birth.

E

State..... Age 1910.....

	19.....	19.....
In force 1st Jan..		
New 1st ½ Yr.....		
Total.		
Ceased 1st ½ Yr..		
At Risk.....		
New 2d ½ Yr.....		
Total.....		
Ceased 2d ½ Yr...		
Expected Loss...		
Dead.....		

The method described by Mr. Hunter seems well adapted to the particular purpose for which it was designed, and we are indebted to him for making it known to us. Having once obtained the "Exposed" and "Died" some method such as I have endeavored to describe may then be found very useful in keeping the experience up to date.

Oral Discussion.

Mr. Hutcheson: I would like to say a few words about the system which is used sometimes in the office with which I am connected. I do not say that it is a good method for all purposes, but it is a very useful method in particular cases. It is in relation to calculating the expected death losses by the "select" method. We write a card for each policy, and have a table prepared of the sum values of the "select" q's for all ages and durations. We put on each card the product of the sum insured and the sum values of the q's for whatever duration the policy may have been or be in force. Once we have these products on the cards, we sort the cards in any way we want them, and take out the expected and the actual losses. For example, if we want to divide them according

to the years of issue, we can. Then if we want to divide them according to localities, etc., we can do so. We can divide them in many different numbers of ways, and we have all the facts on the cards and anyone can get the details there.

As to using policies, instead of lives, there is no objection when you want to get the expected deaths or death losses by the "select" method, because if a man takes out a second policy one or two years after he has taken out his first policy, the expected death rate on the second is different from that on the first, and you must keep each policy year separate.

MR. A. HUNTER: The main object in preparing this "Note" was to assist the younger men by giving them a practical method for conducting mortality investigations. I have had many requests to see the working of the method in use in the New York Life, from those who thoroughly understood the methods employed in carrying on the Specialized Investigation, and the recent Institute Investigation, but who were not familiar with any satisfactory approximate methods. The discussion on the Note has resulted in the actuaries of three large companies stating their methods of conducting mortality investigations, and these methods should be of great use to all the members of the Society, as we can thereby obtain suggestions for the betterment of the system in use in our own companies.

It cannot be said that any one of the four systems submitted, viz., that in the Note, and the three in the discussion thereon, is any better than the others, but that each system is best suited to the purpose for which it is intended. Any actuary, therefore, who has not heretofore had a continuous process of conducting mortality investigations would do well to read the discussions on this subject before laying down a plan.

With regard to having a separate card for mortality investigations, I do not wish to be considered as laying down that as an absolute rule; in fact, in another paper I suggested one card for mortality and for valuation purposes. The use of a card for more than one purpose depends largely on the size of the company. For a good sized company it seems to me most essential to have a card solely for mortality purposes, while for a small company a single card might be used for mortality, valuation, and dividend purposes.

Abstract from the Minutes of the Annual Meeting of the Actuarial Society of America, held in New York, on Thursday, May 21st, and Friday, May 22nd, 1908.

College Room, Hotel Astor,
New York, *May 21st*, 1908.

The meeting was called to order by the President, Mr. Wells, at eleven o'clock, A. M.

FELLOWS PRESENT.

Alsop,
Carpenter,
Cole,
Craig, J. D.
Craig, J. M.
Dawson,
Dow,
Fackler, D. P.
Fackler, E. B.
File,
Flynn,
Gaylord,
Gore,
Graham, W. J.
Grow,
Hall, S. S.
Hallman,
Hann,
Hardcastle,
Henderson,
Hunter, A.
Hunter, R. G.
Hutcheson,
Ireland,
Joffe,
Johnston,
Kilgour,
Kirkpatrick,
Lee,
McClintock,
Macaulay,
Macdonald,
Messenger,
Moir,
Nichols,
Papps,
Paterson,
Peiler,
Pierson,
Plumley,
Robertson,
Rhodes,
Sheppard,
Smith, W. S.
Strong,
Tatlock,
Thompson,
Torrey,
Van Cise,
Weeks,
Welch,
Wells,
Wood,
Woodward, J. H.
Wright, W. C.
Young.

ASSOCIATES ATTENDING.

Allstrom,
Angell,
Blehl,
Bliss,
Breiby,
Brown,
Cathles,
Davenport, J. S.
Davenport, I.
Davis,
Dickenson,
Earle,
Forbes,
Forster,
Gould,
Graham,
Hammond,
Homans,
Hughes,
Jackson,
Kaufman,
McKechnie,
Morris, E. B.
Morris, W. O.
Mowbray,
Niven,
Perrin,
Phillips,
Rice,
Rose,
Ryan,
Smith, E. H.
Washburne, J. H.
Washburn, A. C.
White,
Whitney.

After calling the roll, a quorum of Fellows being present, the minutes of the Semi-Annual Meeting held in Toronto, Canada, on October 10th and 11th, 1907, were approved, as printed in the *Transactions*. The Secretary then read an abstract of the proceedings of the Council since the last meeting. This included:—

(*a*) The adoption of symbols to cover the discounted profit from mortality, that is, the assumed mortality gains (see page vii).

(*b*) The consideration of matters regarding the Sixth International Congress, including the appointment of the Secretary of the Actuarial Society as the correspondent for the Congress and the request that all papers for the Congress be submitted to the Council of the Actuarial Society for approval in the same way as papers submitted for presentation to our Society.

(*c*) A change in the method of passing upon papers submitted to the Society, a special committee of four members of the Council having been appointed with authority to deal with the matter.

(*d*) The proposed investigation into the mortality under Term Insurance, which will be taken up with companies individually by a Committee of the Council.

The Council also reported that the following were the winners of the Associates' Triennial Prizes, presented by Mr. D. P. Fackler:—

1st, Christian Jensen, for his paper on "Mortality Table for Female Beneficiaries in Survivorship Annuities." (Vol. X, p. 253).

2nd, J. M. Langstaff, for his paper on "Misstatements that avoid the Policy." (Vol. X, p. 452).

It was also reported that Mr. D. P. Fackler had renewed his offer of Associates' Triennial Prizes, and that the offer had been accepted by the Council with hearty thanks.

On motion, the proceedings of the Council were duly approved.

The Secretary then announced the names of those who had passed the recent examinations, as follows:—

Examination for qualification as Associates:—

Sinclair E. Allison,
Samuel Beatty,
Edmund Ernest Cammack,
Isaac Davenport,
Milton P. Langstaff,
Franklin Bush Mead,
Harry I. B. Rice,
William A. Sinclair,
Albert W. Whitney.

Examination for qualification as Fellows:—

MILTON D. GRANT, JOHN S. THOMPSON.
ALBERT G. PORTCH,

In addition to the foregoing, H. E. Vineberg passed the examination as Associate but cannot be enrolled as such until he attains the age of 21.

In accordance with Article IX of the Constitution, Harold Worthington Curjel was admitted as Associate without examination.

The existing membership was reported as follows:—

Fellows	131
Associates	87
Total	218

The Treasurer then submitted his report duly audited, which was accepted and ordered to be filed.

The proposed addition to Article VII of the Constitution, reading as follows, "Only Officers and ex-Vice-Presidents shall be eligible to the Office of President," was, on the recommendation of the Council, disapproved.

The following amendment to the Constitution was duly adopted by the Society:—

ARTICLE IV.—The words "a first and a second Vice-President" were changed to "two Vice-Presidents."

The following amendment was made to Article III of the By-Laws:— The words,

"The next Vice-President in order" were changed to "one of the Vice Presidents."

An election was then held for Officers, and for four members of the Council (one to fill the unexpired term of Mr. Moir, elected a Vice-President), which resulted as follows:—

President	JOHN K. GORE.
Vice-President	ARCHIBALD A. WELCH.
Vice-President	HENRY MOIR.
Secretary	ARTHUR HUNTER.
Treasurer	DAVID G. ALSOP.

Editor of the Transactions.....CLAYTON C. HALL.

Members of Council to serve until 1911..... { JAMES M. CRAIG, FREDERICK H. JOHNSTON, PERCY C. H. PAPPS.

Member of Council to serve until 1909 (to fill an unexpired term)..... .....JOEL G. VAN CISE.

At one o'clock the Society took a recess until two o'clock.

AFTERNOON SESSION, *May 21st*, 1908.

Upon the reassembling of the Society at two o'clock P. M., the presentation of papers prepared for this meeting was begun.

The presentation of original papers having been concluded, the Society adjourned to meet again for business at ten o'clock the following morning.

SECOND DAY.

FRIDAY, *May 22nd*, 1908.

The meeting was called to order at ten o'clock A. M., and the discussion of papers presented at the Semi-Annual Meeting held at Toronto in October, 1907, was begun.

Upon the conclusion of this discussion, the meeting was finally adjourned.

ARTHUR HUNTER,
Secretary.

William Thomas Standen.

William Thomas Standen died on August 26, 1907. Mr. Standen was born in London, England, in 1852, and came to America at an early age. After following various occupations a chance remark led him to turn his mind to the actuarial profession, and in 1886 he became Actuary of the United States Life Insurance Company. In 1906 he joined the Capitol Life Insurance Company of Colorado, and after nearly two years of invaluable service to that company died of typhoid fever in Denver.

Mr. Standen was well known to his professional brethren, being a charter member of the Actuarial Society and the author of luminous papers which appeared in its *Transactions*. He was for many years Consulting Actuary for several American and Canadian companies.

He was a man gifted far beyond the ordinary with facility of composition. Besides contributing to insurance periodicals he edited his company's magazine and contributed monthly most instructive and valuable editorials touching upon almost every phase of insurance work. A comprehensive volume of his writings was published some years ago under the title of "The Ideal Protection,"—a book which is justly valued by every student of life insurance.

He was a man of great amiability of temperament, extremely kind-hearted, a genial companion and an affectionate friend.

His death, which occurred in the very maturity of his powers, has caused a distinct loss to his profession.

Vol. X No. 40

ACTUARIAL SOCIETY OF AMERICA

"The work of science is to substitute facts for appearances and demonstrations for impressions."—RUSKIN.

TRANSACTIONS

October 15th and 16th, 1908

NOTICE.

The Society is not responsible for statements made or opinions expressed in the articles, criticisms and discussions published in these *Transactions.*

CONTENTS OF NO. 40.

ACTUARIAL SOCIETY OF AMERICA.

October 15th, 1908.

THE COUNCIL.

Officers: JOHN K. GORE, President.
ARCHIBALD A. WELCH, Vice-President.
HENRY MOIR, Vice-President.
ARTHUR HUNTER, Secretary.
DAVID G. ALSOP, Treasurer.
CLAYTON C. HALL, Editor of the Transactions.

Ex-Presidents: DAVID PARKS FACKLER,
HOWELL W. St. JOHN,
EMORY McCLINTOCK,
THOMAS B. MACAULAY,
OSCAR B. IRELAND,
RUFUS W. WEEKS,
DANIEL H. WELLS.

		Term Expires.
Elected:	WILLIAM C. MACDONALD,	May, 1909
	JOEL G. VAN CISE,	May, 1909
	WILLIAM A. MARSHALL,	May, 1909
	WILLIAM A. HUTCHESON,	May, 1910
	FRANK SANDERSON,	May, 1910
	EDWARD J. SARTELLE,	May, 1910
	JAMES M. CRAIG,	May, 1911
	FREDERICK H. JOHNSTON,	May, 1911
	PERCY C. H. PAPPS,	May, 1911

FELLOWS.

Thomas Gans Ackland,
David Griscom Alsop,
Jesse John Barker,
Amédée Bégault,
Alfred Kimball Blackadar,
Samuel Sweet Boyden,
Thomas Bradshaw,
John Jones Brinkerhoff,
David Carment,
Raymond Van Arsdale Carpenter,
James Chisholm,
Hubert Cillis,
Henry Cockburn,
Richard Huntington Cole,
James Douglas Craig,
James McIntosh Craig,
James Camp Crawford,
Emma Warren Cushman,
Adolph Davidson,
Miles Menander Dawson,
Joseph Arend De Boer,
William Roadley Dovey,
Herbert Beeman Dow,
David Parks Fackler,
Edward Bathurst Fackler,
Colin Campbell Ferguson,
Robert Patterson Field,
Lorne Kenelm File,

Benedict Devine Flynn,
Morris Fox,
Frederick William Frankland,
John Marshall Gaines,
William Standish Gaylord,
James Howard Gore,
John Kinsey Gore,
William Joseph Graham,
Milton Daniel Grant,
Arthur R. Grow,
Clayton Colman Hall,
Samuel Stickney Hall,
Menno Snyder Hallman,
Robert George Hann,
Edward Edgington Hardcastle,
Francis Henry Hemperley,
Robert Henderson,
William Hendry,
Charles Daniel Higham,
Charles Hildebrand,
William Richmond Hitchins,
John Marshall Holcombe,
George William Hubbell,
William Hughes,
Arthur Hunter,
Robertson Gilbert Hunter,
Robert Watkinson Huntington, Jr.
William Anderson Hutcheson,
Oscar Brown Ireland,
Solomon Achillovich Joffe,
Frederick Henry Johnston,
David Ewitt Kilgour,
George King,
George Halsey Kirkpatrick,
Christopher Kyle,
James Morgan Lee,
James Miles Langstaff,
Omer Lepreux,
George Leslie,
Louis Linzmeyer,
Charles Alvin Loveland,
John Bodine Lunger,
James Baldwin McKechnie,
Emory McClintock,
Thomas Bassett Macaulay,
William Campbell Macdonald,
Henry William Manley,
Léon Marie,
Elbert Pike Marshall,
William Andrew Marshall,
Hiram John Messenger,
Henry Moir,
Edward Bontecou Morris,
Walter Smith Nichols,
Joseph Howard Nitchie,
Percy Charles Herbert Papps,
John Sherman Paterson,
Maximilian Heinrich Peiler
Gardner Ladd Plumley,
Albert Garfield Portsch,
J. C. Albert Quiquet,
Charles Grant Reiter,
Edward E. Rhodes,
Josephus Hargreaves Richardson
John George Richter,
Hugh Wilfred Robertson,
John Francis Roche,
Gerald Hemmington Ryan,
Howell Williams St. John
George Ferry Salter,
George White Sanders,
Frank Sanderson,
Edward James Sartelle,
Frederick Schooling,
Ernst Willem Scott,
Herbert Norman Sheppard,
William Simeon Smith,
Edward Lincoln Stabler,
Samuel Edgar Stilwell,
Wendell Melville Strong,
John Tatlock,
Richard Teece,
Herbert Cecil Thiselton,
John Spencer Thompson,
Morris Whittemore Torrey,
Joel Garretson Van Cise,
James Douglas Watson,
Rufus Wells Weeks,
George Wegenast,
Archibald Ashley Welch,
Daniel Halsey Wells,
William Joseph Hutchings
Whittall,
Asa Shove Wing,
Leonard Joan Wolterbeek,
Arthur Barton Wood,
Ernest Woods,
George Badger Woodward,

Joseph Hooker Woodward,
Peter Troth Wright,
Walter Channing Wright,
Frank Bertrand Wyatt,
Tsuneta Yano,
Thomas Emley Young,
William Young.

ASSOCIATES.

Sinclair E. Allison,
Henry William Allstrom,
Charles Hart Angell,
Charles Henry Armstrong,
Walter Crosby Baber,
William Algernon Bain,
Samuel Beatty,
Edward Gordon Blackadar,
Ernest Mar Blehl,
George Isaac Bliss,
O. William Breiby,
Franklin Brough,
James Cornelius Brown,
George Edward Bulkley,
Henry Wright Buttolph,
Edmund Ernest Cammack,
John Randolph Leigh Carrington,
Lawrence Maclagan Cathles,
William Chubb,
Horton Woods Cochnower,
John James Cooper,
Harold Worthington Curjel,
Thomas Arthur Dark,
Isaac Davenport,
John Sydney Davenport,
Mervyn Davis,
David Stephen Dickenson,
Arthur Percival Earle,
John Maynard Emery,
John William Fisher,
Charles Reginald Fitzgerald,
William George Fitzgerald,
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Charles Hughes,
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Christian Jensen,
Henry Nicholas Kaufman,
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John Morrison Laing,
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Alfred McDougald,
James Allen Macfarlane,
John Archibald McKellar,
Michael Alexander Mackenzie,
Frank Douglas Macorquodale,
Donald Matheson,
Franklin Bush Mead,
G. Cecil Moore,
William Oscar Morris,
Albert Henry Mowbray,
Charles Park Muckle,
John Ballantine Niven,
John Gowans Parker,
Oliver Winfred Perrin,
Thomas Ashley Phillips,
Sidney Herbert Pipe,
Edward Ernest Reid,
Harry I. B. Rice,
Douglas Hall Rose,
Harwood E. Ryan,
William A. Sinclair,
Charles Gordon Smith,
Edwin Henry Smith,
Walter Harold Sommerville,
Walter Newell Stanley,
Allan Wilmot Strong,
Abel Travassos,
Dwight A. Walker,
James Herman Washburn,
Alva Courtenay Washburne,
Arthur William Watt,
Wilfred Clare White,
Albert W. Whitney,
Frederick Alfred Williams,
William Archibald Porter Wood.

NOTATION.

ADOPTED OCTOBER 23D, 1890.

RESOLVED, that in the presentation of papers the Symbols of the Text-Book of the Institute of Actuaries, with the additions noted below, be followed when convenient, and that in any event the use of such symbols in other senses be avoided; that the letters denoting commutation columns, according to the system of Dr. Farr, customarily followed by American Actuaries, be printed in plain "full-face" type; and that otherwise there be entire liberty as regards notation.

$k_x = \frac{d_x}{l_{x+1}} = \frac{C_x}{D_{x+1}}$.

$u_x = \frac{l_x}{l_{x+1}}(1+i) = \frac{D_x}{D_{x+1}}$.

S = Sum insured.

K = Cost of insurance = $q(S - V)$.

Π = Single premium (alternative for A).

The following notation for the discounted value of the future profits from mortality was recommended for adoption May 21st, 1908.

G_x = the present value at date of issue of the assumed mortality gains on a Single Premium Whole Life Insurance of 1 issued at age x.

$G^2_{x\,\overline{n}|}$ = the value at the beginning of the second policy year of the assumed mortality gains on a Single Premium Endowment Insurance of 1 for n years issued at age x.

${}_n g_x$ = the present value at date of issue of the assumed mortality gains on a Limited Payment Life Insurance of 1 issued at age x by n annual payments.

$g^2_{x\,\overline{n}|}$ = the value at the beginning of the second policy year of the assumed mortality gains on an Annual Premium Term Insurance of 1 for n years issued at age x.

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TRANSACTIONS

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The Mortality Experience of the Mutual Benefit Life Insurance Company Under Extended Insurance.

BY E. E. RHODES.

Owing to recent legislative enactments and other causes, the practice of granting extended insurance in case of lapsed policies, either automatically or as an option, has recently become very general. The general non-forfeiture system of The Mutual Benefit Life Insurance Company, adopted in 1879, provided that in event of default in premium payments after two years' premiums had been paid the insurance would be extended as term insurance for the full amount of the original policy, without any action by the owner thereof. The full American Experience four per cent. reserve, less any outstanding indebtedness, was to be used as a single premium for term insurance under a table of rates based upon the American Experience Table of Mortality and 4% interest, with a loading of twenty per cent. of the net premium, except for ages under thirty-five, where the loading was somewhat less than twenty per cent. In event of death during the first three years of the period of extended insurance the overdue premiums were deducted from the amount insured. The original non-forfeiture System also provided that proofs of loss must be filed within one year after the death of the Insured. This last provision, however, was found to be of no practical value and was

never enforced, very few cases having arisen where it would have been of any effect.

In 1895 the Company began the practice of making policy loans up to the limit secured by the cash surrender value. Loans up to that time had been limited to one-half of the policy reserve. At the same time the non-forfeiture provisions were changed so as to provide that if death should occur during the *first* year of extended insurance there would be deducted from the amount insured the overdue premium together with a sum equal to the indebtedness outstanding at time of lapse. Such a provision was not necessary when the indebtedness did not exceed one-half the policy reserve; but when the Company's practice in regard to policy loans was made more liberal it became necessary to protect the Company against a moribund policyholder who, knowing that he had only a short time to live, could procure a policy loan, leaving only sufficient value to extend his insurance through his probable lifetime. It is clear that without a provision for the deduction of the indebtedness, such a policyholder might be said to "eat his cake and have it, too." If there was an indebtedness, the amount to be applied to the purchase of extended insurance was equal to the American Experience 4% reserve reduced in the ratio of the indebtedness to the cash surrender value.

In 1900 the Company changed its reserve basis to 3 per cent., and at the same time adopted new non-forfeiture provisions applicable only to policies thereafter issued, under which the *cash surrender value,* less any outstanding indebtedness, was to be applied to the purchase of extended insurance at net rates based upon the American Experience Table of Mortality and 3 per cent. interest. The cash surrender value was in all cases equal to the American Experience 3 per cent. reserve, less a surrender charge of one per cent. of the amount insured. The provision for the deduction of the overdue premium and a sum equal to the outstanding indebtedness at time of lapse, if death occurred during the first year of the extended insurance, was retained.

The Company's Non-forfeiture System was again changed in January, 1908. Under the present system, which is applicable to policies issued since 1907, the insurance is extended for the amount of the original policy, less any outstanding indebtedness, and in event of death there is no deduction on account of either

overdue premiums or indebtedness. The surrender charge of one per cent. mentioned above is reduced one-tenth of one per cent. each year beginning with the sixth year, so that at the end of the fifteenth year and thereafter there is no surrender charge. A value is allowed whenever, at time of lapse, the reserve is in excess of $10.00 per $1,000 insured.

The original Non-forfeiture System of 1879 was made applicable to all outstanding policies as well as to policies thereafter issued. At the end of the year 1879 there were 28,819 premium-paying Life and Endowment policies in force, insuring $97,436,037, to which the automatic extended insurance provision had become applicable. On December 31, 1906, there were 162,077 premium paying Life and Endowment policies in force, insuring $383,497,544. It is undoubtedly true that no other company may be said to have had as much experience in the matter of extended insurance as The Mutual Benefit. The policies actually extended have been sufficiently numerous to give the requisite data for a mortality investigation, and the results of such an investigation are presented herewith with the hope that they will prove sufficiently valuable to justify the time and labor which have been spent. At the outset I desire to say that after the plan and scope of the investigation had been determined, I was unavoidably absent from the office during the greater part of the time, and credit for the work is largely due my associate, Mr. Papps.

Objects of Investigation.

The general mortality experienced by the Company on extended insurances has been carefully watched from year to year; but it was deemed interesting and valuable to investigate the mortality according to

(*a*) Years elapsing between dates of entry and extension;
(*b*) Years elapsing since extension, irrespective of the time elapsed between entry and extension;
(*c*) Age of Insured at time of extension;

The most important question to be determined was whether there was any adverse selection by the Insured through failure to pay premiums. The investigation was made on the bases of "policies" and "amounts."

Bases of Comparison.

To obtain a proper basis of comparison it was necessary to calculate the expected deaths according to a select mortality table. It was decided to use three select tables, viz:—

(*a*) Modified Healthy English;
(*b*) $O^{[M]}$;
(*c*) Compound Progressive.

It will be remembered that (*a*) was the table used as a basis of comparison in the Specialized Mortality Investigation. It shows separate rates of mortality for each of the first five years after entry, and the ultimate rate for the sixth and subsequent years is that shown in Farr's Healthy English Male Table, except for certain modifications for ages 15-21 inclusive and for ages 52-61 inclusive.

The $O^{[M]}$ table gives the mortality experience on healthy male lives according to the last investigation of the British offices. Separate rates of mortality are shown for each of the first ten years, and the ultimate rate applies to the eleventh and subsequent years.

The Compound Progressive Table is based upon the experience on Deferred-dividend policies issued by the New York Life Insurance Company, and appears on pages 100-101 of Volume 9 of the Society's *Transactions*. The period of selection extends through eleven years, the ultimate rate becoming effective in the twelfth year.

The expected deaths according to the American Experience Table are also calculated for the entire experience, irrespective of durations, and for the exposures grouped according to years elapsed between entry and extension.

Preparation of Data.

The ages at dates of extension were nearest ages. In cases of extended insurances terminated by expiry the duration was taken to the nearest one-tenth of a year. The tenths were determined

according to months, days being disregarded. By disregarding days there is a possible error of one month more or less; but the errors will tend to counterbalance one another. In reducing months to tenths of a year, three months and nine months are equivalent to two and one-half and seven and one-half tenths, respectively. Three months were considered as three-tenths of a year, and nine months as seven tenths. For the months one to three the *tenths are identical.* From four to eight months the tenths are *one less* than the number of months, and from nine to eleven months *two less.* In cases of extended insurances canceled by the reinstatement of the original policies or by surrender, the durations were calculated in the same manner as for the expired. In cases of death the durations were taken to the anniversaries of the dates of extension following the actual dates of death. The Policy Year method was followed in the investigation, the observations being carried in case of all policies existing December 31, 1906, to the policy anniversaries in 1907. In cases where insurances were extended and the original policies were afterwards reinstated and there was a second lapse followed by a second extension, *the duration under the second extension was computed from the date of the medical examination for reinstatement.*

The shortest duration of entry to extension was two years, except in case of 256 policies insuring $651,400 which, after having lapsed and been extended and afterwards reinstated, were allowed to lapse and be extended for the second time within less than two years after reinstatement. These cases were so few in number that they were included in the two-year group. This group, with the exceptions referred to, includes all policies which paid two years' premiums. The second group, aside from cases arising from reinstatement and subsequent lapse, included all policies which paid more than two, but not more than three, years' premiums. The other groups were treated in the same way.

The cards were first sorted according to duration from entry to extension; then by ages at dates of extension; and then by duration under exposure. The results of this third sorting were entered upon sheets designated as "Table A," of which the following is a specimen:

TABLE A.

YEAR EXTENDED 2. (*Specimen Sheet.*) EXTENDED AT AGE 35.

Year of Exposure	EXPIRED		CANCELED		EXISTING		DIED		YEARLY DECREMENT		AT RISK	
	No.	Amount	No.	Amount	No.	Amount	No.	Amount	No.	Amount	No.	Amount
0	25	$51,600	5	$ 8,700					30	$60,300	314	$693,600
1	53	130,700	5	9,3								
	25	57,7	3	3,1	10	$10,500	2	$ 3,500	98	214,8	284	633,3
2	37	80,5	1	1,5								
	42	96,3	1	3,1	8	13	1	1	90	195,4	186	418,5
3	9	19.2	0	2								
	15	37	1	2	8	12			33	72,2	96	223,1
4	20	46,5	0	2								
	14	30,7			3	3,1			37	80,5	63	150,9
5	4	8,3										
	3	8,9							7	17,2	26	70,4
6	2	5,6										
	1	2,2			1	5			4	12,8	19	53,2
7	4	14,3										
	4	8,6							8	22,9	15	40,4
8	2	3,5										
	2	6							4	9,5	7	17,5
9											3	8
10	0	3							0	3	3	8
11	1	7										
					1	2			2	2,7	3	7,7
12	0	5							0	5	1	5
13	1	4,5							1	4,5	1	4,5

NOTE :—The columns showing the Expected deaths are omitted from the above table.

There were 78 policies insuring $182,300 in the first group (Year Extended 2) the age at time of extension being thirty-five years. The nearest whole number of the total durations, integral and fractional, is entered upon the top line, opposite Year 1, and the difference between the entire number, 78, and this number, 53, or 25, is entered opposite Year 0. The amounts at risk were treated in a similar manner, being taken to the nearest hundreds of dollars, except where the age at time of exposure was sufficiently high to bring out a large value of q_x. In the second year there were altogether 62 policies. The total exposures were equivalent to 37 policies exposed to risk for the full year, so that the remaining 25 policies were entered on the lower line opposite year 1, not being included in the observations of the second year.

The expected deaths according to policies and amounts were calculated by three select mortality tables as has been stated. Preliminary sheets were first prepared, showing for each policy year from the third to the eleventh inclusive, the rates of mortality by each of the three tables, and the difference in the rates for each age at entry. The multiplications were then performed by means of the "Millionaire" calculating machine. The number or amount at risk was set up on the machine and multiplied by the q_x from the table showing the lowest value. The result was tabulated and left on the machine, and the same number or amount at risk was then multiplied by the difference between the lowest and the next higher value of q_x. This gave the same result as if a separate multiplication by the higher value of q_x had been made. The results of the multiplications by the middle and highest values of q_x were also obtained by means of differences. The method resulted in a considerable saving of time, as the machine had to be cleared only after every third calculation.

The following table summarizes the footings of the sheets comprising Table A:—

Mode of Exit.	Year "0."		Other than Year "0."	
	Policies.	Amount.	Policies.	Amount.
Expired	1,979	$5,409,704	19,379	$46,603,217
Canceled	1,156	2,954,950	2,259	5,993,250
Existing			3,849	6,608,763
Died			1,069	2,789,136
Total Decrements . .	3,135	$8,364,654	26,556	$61,994,366
			3,135	8,364,654
		Total Data,	29,691	$70,359,020

It will be observed that 29,691 policies insuring $70,359,020 were extended. Of this number, 3,135 policies, insuring $8,364,654, were counted as not exposed to risk, leaving 26,556, insuring $61,994,366, which actually entered into the investigation. The total exposed to risk was equivalent to 97,012 policies, for $224,429,217, at risk for one year. As 26,556 policies, insuring $61,994,366, entered into the investigation, the average duration of exposure was 3.65 years by policies, and 3.62 years by amounts. A noteworthy feature is the large proportion of the data terminating naturally. The percentage for each mode of exit is as follows:—

Mode of Exit.	Percentages by	
	Policies.	Amounts.
Expired	72.97	75.17
Canceled	8.51	9.67
Existing	14.49	10.66
Died	4.03	4.50
	100.00	100.00

Table B.

The policies and amounts at risk, the actual deaths and the expected deaths, were summarized so as to show the results for each "Year Extended" for all ages combined. The results are shown in Table B, which gives the experience according to years elapsed between the dates of entry and extension.

Table B shows that the ratio of actual to expected deaths is generally higher by amounts than by policies. According to policies the actual deaths were 92.0 per cent. of the expected by the Modified English; 88.9 per cent. by the $O^{[M]}$; 99.6 by the Compound Progressive; and 85.4 by the American Table. By amounts the percentages of actual to expected deaths were 97.5, 92.7, 103.3, and 90.3 respectively. It will be noticed that the actual deaths are considerably less than the expected in case of policies in force two or three years before the insurance was extended. For policies having a greater duration between the time of entry and time of extension, the actual deaths are generally more than the expected. This is more easily seen in Table B^1, which shows the percentages of actual to expected deaths for insurances extended after the original policies have been more than two, three,

etc., years in force. This table was compiled by summing the data used in Table B for two years and over, three years and over, etc. A column is added to Table B^1, showing the percentages of actual to expected deaths (based upon amounts) by the American Table for the *entire general mortality experience of the Company*. It will be observed that with the wearing off of the effect of medical selection, the mortality under extended insurance becomes increasingly greater than under the Company's general experience.

Table C.

The policies and amounts at risk, the actual deaths and the expected deaths were also summarized by ages at date of extension. The summary was first made for each age, but the data was not sufficient to give smooth results, and Table C was accordingly prepared to show the results for ten-age groups. It will be seen that the percentages of actual to expected deaths by the Modified English Table tend to increase with age, the percentages based on policies showing a steady increase with the exception of ages over seventy-five, where there were only seven policies ceased by death. The percentages based on the $O^{[M]}$ and Compound Progressive, which are true select tables, do not show this tendency to the same extent. In fact it would appear from a careful study of the percentages shown that any tendency towards higher percentages at the older ages is sufficiently explained by the fact that at the older ages the policies will naturally be those which have had the longest durations between entry and extension, and which have a higher mortality.

Table D.

The policies and amounts at risk, the actual deaths, and the percentages of actual to expected deaths for each year of exposure under extension, are shown in Table D. For the first year of exposure the actual deaths exceed the expected deaths according to each of the three bases of comparison for both policies and amounts. The second year of exposure shows lower percentages than the first, and the third lower than the second. The percentages for the fifth year of exposure are higher than those of

the fourth and sixth years, due probably to the lack of sufficient data to show smooth results. The figures for the fifth-ninth years and the tenth-twentieth years have been grouped.

The insurance under many policies is extended by reason of carelessness or financial inability on the part of the policyholder. In some cases extended insurances may have been in force at the death of the Insured and the Company not have been informed of the death. This would apply more particularly to those early policies which contained no non-forfeiture provisions. If there be any such cases, the mortality has been understated.

287 policyholders died during the first year of extended insurance. The causes of death were as follows:—

Tuberculosis,	25
Cancer,	13
Bright's Disease,	18
Apoplexy, or Brain Disease,	47
Heart Disease,	24
Suicide,	22
Alcoholism,	8
Diabetes,	2
Unclassified,	
Chronic,	13
Acute,	115
Total,	287

The loss papers in the above cases were carefully examined in an effort to ascertain the proportion of cases in which it might be reasonable to assume that the Insured was so far aware of his physical condition as to justify him in allowing his insurance to be extended. To illustrate:—Mr. F. B. W. died of tuberculosis eight months after his policy had lapsed and the insurance been extended for 5 years and 76 days. The loss papers show that he was aware of his condition when the policy lapsed, and the Company's correspondence shows that he desired the insurance to be extended. We are safe in assuming that in such a case the granting of extended insurance was a determining factor in the lapse of the policy.

The following tabulation shows the number of cases in which death occurred the first year and in which it is either certain or

probable that at time of lapse the Insured knew that he had only a short time to live and that the term of the extended insurance would cover his probable lifetime:—

Tuberculosis,	19
Cancer,	11
Bright's Disease,	11
Apoplexy, or Brain Disease,	22
Heart Disease,	14
Suicide,	5
Alcoholism,	0
Diabetes,	2
Unclassified,	
Chronic,	11
Acute,	0
Total,	95

On the other hand, the loss papers show that in many cases the Insured were not in any physical condition at time of lapse to attend to business matters. It is, of course, in its application to such cases that the system of extended insurance finds its greatest justification.

There have been 57 cases of suicide under extended insurance. Of these, 13 have occurred within less than one year from the date of expiry; 8 between one and two years from the date of expiry; 9 between two years and three years; 7 between three years and four years; 3 between four years and five years; 3 between five years and six years; 6 between six years and seven years; 2 between seven years and eight years; 4 between eight years and nine years; 1 between nine years and ten years; and 1 between ten years and eleven years. There have been proportionately five times as many cases of suicide under extended insurance as there have been in the Company's general business (excluding extended insurance) since the adoption of the non-forfeiture system in 1879.

Table E.

This table shows the Company's experience on extended insurances by calendar years according to the American Table, compared with the Company's general experience during the same time, excluding extended insurance.

Conclusions.

I believe that the results of the investigation warrant the following conclusions:—

The Company's system of automatic extended insurance on the whole has been entirely satisfactory. It has not proved burdensome to the Company, and experience has demonstrated its attractiveness to the insuring public.

Any system of extended insurance requires safeguards if the company is to be protected from loss. There is undoubtedly an adverse selection on the part of members taking extended insurance, the full effect of which is not revealed by the present investigation.

Inasmuch as under automatic extended insurance many policies are extended without thought by the Insured as to their disposition, the adverse selection would be greater with a company which allowed extended insurance as an option instead of automatically.

There should be either a surrender charge during the years in which lapses are most frequent, or the extended insurance should be bought at loaded rates, if the persistent policyholders are to be protected against adverse selection.

The mortality savings of any favorable years, and any excess interest earnings, should be held by the company to meet the unfavorable mortality of other years. In other words, the extended insurance should be on a non-participating basis.

The sufficiency of the American Experience Table of Mortality, as a measure of the mortality on American insured lives, is again demonstrated.

TABLE B.

Year Extended.	At Risk.		Actual Deaths.		Percentage of Actual to Expected by Mod. Eng.		O[M].		Comp. Prog.		American.	
	Policies.	Amounts.	Policies.	Amounts.	Policies.	Amounts.	Policies.	Amounts.	Policies.	Amounts.	Policies.	Amounts.
2	18,041	37,933,745	111	253,354	68.5	71.6	75.4	77.9	82.8	85.2	59.5	62.0
3	17,832	38,186,000	121	242,460	69.5	63.3	75.5	68.1	85.1	76.7	64.5	58.7
4	13,405	29,508,418	124	321,036	88.0	101.0	91.3	103.7	104.0	118.3	83.6	95.8
5	9,959	22,281,640	104	250,484	93.9	98.7	92.6	96.3	106.2	110.4	89.7	94.0
6	7,039	16,092,750	85	197,380	105.9	105.0	100.2	98.7	116.0	114.1	100.7	99.8
7	5,417	12,244,100	57	129,845	84.5	82.6	77.7	75.5	88.9	86.1	79.9	78.0
8	4,130	9,443,800	44	127,663	86.2	107.3	77.8	96.6	89.6	110.9	81.5	101.5
9	3,065	7,078,716	40	91,871	99.5	96.3	88.0	84.7	100.9	96.9	93.9	90.3
10	2,849	6,827,248	49	135,696	126.6	144.4	111.4	126.8	126.9	144.2	118.6	136.3
11	2,295	5,830,450	32	92,483	97.9	108.2	85.7	94.4	96.8	106.2	92.0	101.6
12	1,783	4,635,290	35	101,453	131.6	141.2	114.5	122.4	128.8	137.0	123.3	131.8
13	1,317	3,459,850	21	50,711	103.9	97.3	90.5	84.8	101.3	95.0	97.4	91.3
14	1,599	4,569,250	21	69,000	82.1	93.0	70.9	79.9	79.2	89.3	76.7	86.8
15	1,089	2,938,580	23	61,000	122.9	123.3	105.7	105.8	117.1	117.6	114.5	115.0
16–20	4,405	14,247,230	103	318,550	116.2	110.2	99.6	94.4	108.3	102.5	107.6	101.9
21–25	1,875	6,296,150	58	214,200	119.0	128.3	102.7	110.6	108.2	116.3	109.0	117.3
26–58	912	2,856,000	41	131,950	116.3	120.6	102.3	106.1	103.1	107.0	104.3	108.3
Total,	97,012	224,429,217	1,069	2,789,136	92.0	97.5	88.9	92.7	99.6	103.3	85.4	90.3

TABLE B.[1]

Extended after being in force over	Percentage of Actual to Expected by								Percentage of Actual to Expected by American Table. Company's General Experience (Amounts)
	Mod. Eng.		O[M].		Comp. Prog.		American.		
	Policies.	Amounts.	Policies.	Amounts.	Policies.	Amounts.	Policies.	Amounts.	
2 Years	92.0	97.5	88.9	92.7	99.6	103.3	85.4	90.3	89.7
3 "	95.8	101.1	90.8	94.4	101.9	105.5	89.9	94.6	90.9
4 "	101.4	108.0	93.5	98.5	104.9	109.8	95.3	101.2	91.9
5 "	104.1	109.2	93.9	97.6	105.1	108.6	97.7	102.1	92.8
6 "	106.1	110.9	94.2	97.8	104.9	108.3	99.2	103.4	93.7
7 "	106.1	111.8	93.3	97.7	103.3	107.6	98.9	103.9	94.5
8 "	109.5	115.5	95.6	100.5	105.4	110.2	101.9	107.3	95.2
9 "	112.7	116.5	97.9	100.9	107.4	110.1	104.6	107.9	96.0
10 "	114.3	118.4	99.1	102.4	108.1	111.3	105.9	109.6	96.8
11 "	112.7	115.7	97.5	99.9	105.8	108.1	104.2	106.8	97.2
12 "	114.5	116.5	99.0	100.5	106.9	108.3	105.7	107.4	97.7
13 "	112.6	114.1	97.3	98.4	104.6	105.6	103.8	105.0	98.0
14 "	113.4	115.3	97.9	99.4	104.9	106.4	104.3	106.0	98.2
15 "	117.6	118.0	101.5	101.8	108.1	108.3	108.0	108.3	98.5
16 "	117.0	117.6	101.0	101.4	107.2	107.6	107.3	107.7	98.7
21 "	117.8	125.3	102.5	108.9	106.0	112.6	107.0	113.7	99.4
26 "	116.3	120.6	102.3	106.1	103.1	107.0	104.3	108.3	98.3

TABLE C.

Year Extended.	Actual Deaths Policies.	Actual Deaths Amount.	Percentage Actual to Expected Deaths according to Mod. Eng. Policies.	Mod. Eng. Amount.	O[M] Policies.	O[M] Amount.	Comp. Prog. Policies.	Comp. Prog. Amount.
Ages (at date of extension) 16–25.								
2	6	$ 9,055	40.8	41.0	49.4	49.6	54.3	54.5
3	7	8,500	58.2	49.1	70.4	59.5	78.8	66.6
4	0	0	0	0	0	0	0	0
5	3	4,061	134.5	139.8	156.4	162.5	177.8	184.4
2–5	16	21,616	47.0	43.9	56.6	52.9	63.0	58.8
6–10	2	3,026	137.2	126.8	154.7	141.6	176.5	162.3
11–15	0	0	0	0	0	0	0	0
16–20	0	0	0	0	0	0	0	0
21–25	0	0	0	0	0	0	0	0
26–58	0	0	0	0	0	0	0	0
Total	18	24,642	50.7	47.7	60.9	57.3	67.8	63.8
Ages (at date of extension) 26–35.								
2	33	$ 53,883	57.2	49.2	67.4	57.9	75.3	64.6
3	41	79,938	68.0	67.2	79.5	78.7	90.6	89.7
4	41	106,648	87.0	113.6	97.5	127.4	112.7	147.2
5	30	71,652	96.7	115.4	105.0	124.9	122.3	145.5
2–5	145	312,121	73.9	81.2	84.7	92.9	96.6	106.1
6–10	58	117,646	122.0	124.9	126.6	129.5	148.9	152.4
11–15	4	10,819	92.4	110.2	92.2	109.6	108.6	129.5
16–20	0	0	0	0	0	0	0	0
21–25	0	0	0	0	0	0	0	0
26–58	0	0	0	0	0	0	0	0
Total	207	440,586	83.5	90.2	93.5	100.9	107.4	116.0
Ages (at date of extension) 36–45.								
2	42	$ 128,775	82.5	105.6	88.0	112.5	99.2	126.7
3	42	89,350	69.1	62.5	72.9	65.8	84.1	75.8
4	42	96,203	88.0	84.0	89.7	85.5	104.7	99.8
5	33	81,936	79.8	85.3	77.8	83.0	91.5	97.6
2–5	159	396,264	79.2	83.3	81.7	85.9	94.4	99.1
6–10	102	271,094	94.3	109.5	86.7	100.7	102.9	119.4
11–15	43	102,179	106.0	106.2	94.7	94.9	112.1	112.4
16–20	14	54,950	115.7	155.5	101.0	135.6	118.8	159.9
21–25	1	2,000	65.4	73.3	56.1	63.5	65.1	74.0
26–58	0	0	0	0	0	0	0	0
Total	319	826,487	87.8	96.3	85.4	93.6	99.8	109.4
Ages (at date of extension) 46–55.								
2	18	$ 38,019	66.4	55.5	66.2	55.3	71.9	60.0
3	22	49,100	74.2	65.2	73.4	64.6	81.9	72.0
4	32	101,003	105.3	129.6	99.9	122.8	113.1	138.8
5	26	66,606	103.3	95.6	93.6	86.4	106.8	98.4
2–5	98	254,728	87.2	87.4	83.8	83.8	93.7	93.7
6–10	81	205,893	98.0	95.7	84.0	82.1	96.3	94.1
11–15	51	166,324	111.1	121.7	93.5	102.5	105.9	116.1
16–20	47	130,000	119.8	100.6	100.6	84.5	113.1	95.0
21–25	15	46,500	75.5	64.7	63.6	54.5	70.3	60.0
26–58	5	14,750	108.2	111.2	91.2	93.3	100.4	103.1
Total	297	818,195	97.5	95.4	86.4	84.1	97.5	94.9

TABLE C— *Continued.*

Year Extended.	Actual Deaths Policies.	Actual Deaths Amount.	Percentage Actual to Expected Deaths According to Mod. Eng. Policies.	Mod. Eng. Amount.	O[M] Policies.	O[M] Amount.	Comp. Prog. Policies.	Comp. Prog. Amount.
Ages (at date of extension) 56–65.								
2	11	$ 22,622	117.0	99.3	118.8	101.3	117.2	99.8
3	9	15,572	91.5	59.6	92.3	60.1	93.6	61.3
4	5	9,082	58.7	44.2	57.5	43.1	59.5	44.7
5	10	22,229	104.2	108.3	97.2	100.8	101.3	105.3
2–5	35	69,505	93.7	77.2	92.1	76.1	93.9	77.6
6–10	25	61,263	80.7	82.8	70.4	72.3	74.2	76.2
11–15	25	81,825	96.5	111.4	82.5	95.2	86.4	99.8
16–20	37	121,100	129.3	131.1	111.4	112.7	115.6	117.2
21–25	23	102,000	122.8	160.1	106.5	139.0	109.4	142.8
26–58	16	56,100	94.4	103.4	82.4	90.1	84.1	92.1
Total	161	491,793	101.5	109.8	90.4	97.4	93.6	100.9
Ages (at date of extension) 66–75.								
2	1	$1,000	47.2	113.4	50.5	121.8	43.5	103.1
3	0	0	0	0	0	0	0	0
4	4	8,100	198.3	201.6	203.5	199.3	184.0	189.7
5	2	4,000	231.5	211.3	236.1	217.0	215.1	196.5
2–5	7	13,100	107.3	75.6	113.2	79.6	100.7	70.3
6–10	7	23,533	102.0	118.0	94.3	109.4	91.4	106.0
11–15	9	13,500	134.4	85.9	120.6	77.1	119.4	76.3
16–20	3	4,500	44.1	187.8	39.7	169.2	39.2	167.0
21–25	17	59,200	228.0	253.1	205.9	228.5	203.0	225.3
26–58	17	50,600	200.7	185.5	181.4	167.9	178.7	164.9
Total	60	164,433	140.1	128.8	129.7	119.1	125.8	115.4
Ages (at date of extension) above 75.								
2	0	$ 0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
2–5	0	0	0	0	0	0	0	0
6–10	0	0	0	0	0	0	0	0
11–15	0	0	0	0	0	0	0	0
16–20	2	8,000	108.0	98.0	98.9	89.6	95.7	86.5
21–25	2	4,500	172.0	84.6	156.6	76.8	149.7	72.2
26–58	3	10,500	59.1	76.2	53.4	68.8	49.1	63.2
Total	7	23,000	81.1	81.3	74.0	74.1	69.9	70.1

TABLE D.

Year of Exposure.	At Risk. Policies.	At Risk. Amounts.	Actual Deaths. Policies.	Actual Deaths. Amounts.	Percentage of Actual to Expected by Mod. Eng. Policies.	Mod. Eng. Amount.	O[M]. Policies.	O[M]. Amount.	Comp. Prog. Policies.	Comp. Prog. Amount.
1	26,556	$61,993,929	306	$795,055	107.5	112.9	110.4	113.8	121.4	124.8
2	19,388	44,889,622	198	491,412	90.6	92.1	92.1	91.9	102.9	102.5
3	14,123	32,638,691	136	321,688	81.8	79.0	80.5	76.3	90.8	85.7
4	10,435	24,083,789	107	269,240	83.9	86.0	79.7	80.1	90.3	90.3
5	7,565	17,602,878	92	264,358	95.3	110.0	87.6	99.5	99.4	112.2
6	5,623	13,002,940	59	164,029	78.7	88.1	71.0	78.4	80.3	88.2
7	4,164	9,576,418	56	137,780	97.6	97.2	87.0	85.7	98.2	96.0
8	3,010	6,857,900	38	95,576	88.4	90.0	78.2	78.9	88.0	88.0
9	2,194	4,953,200	24	74,957	74.5	95.1	65.6	82.9	73.5	92.2
5–9	22,556	51,993,336	269	736,700	88.4	97.8	79.7	86.9	90.0	97.6
10	1,543	3,462,400	23	58,341	99.1	102.2	86.9	89.0	97.1	98.5
11	982	2,202,100	11	49,000	73.2	131.8	64.1	114.8	71.6	126.5
12	625	1,383,250	7	20,700	72.8	88.5	63.6	76.9	71.0	84.9
13	388	843,950	6	24,000	97.9	161.5	85.6	139.8	95.2	154.0
14	214	458,050	2	4,000	60.5	49.4	52.7	42.9	58.8	47.0
15	118	265,300	2	7,000	107.1	149.7	93.0	129.4	103.5	142.1
16	50	128,100	2	12,000	249.1	548.5	215.1	475.3	240.1	524.7
17	19	54,400	–	———						
18	9	21,100	–	———						
19	5	9,200	–	———						
20	1	2,000	–	———						
10–20	3,954	8,829,850	53	175,041	87.4	117.3	76.5	102.0	85.3	112.5
Total	97,012	224,429,270	1069	2,789,136	92.0	97.5	88.9	92.7	99.6	103.3

TABLE E.

Mortality Experience—1879–1906.

By Years of Membership.

Years of Membership.	Amount at Risk. (Extended Insurance.)	Actual Deaths. (Extended Insurance.)	Percentage Actual to Expected Deaths, American Experience. (Extended Insurance.)	Company's General Experience Excl. Ext. Insurance.)
3	$ 8,241,573	$ 49,519	57.1	55.8
4	18,978,863	127,528	63.6	62.3
5	21,127,747	176,250	78.3	64.6
6	20,050,971	168,951	78.0	65.2
7	18,032,306	167,328	84.0	71.1
8	15,852,687	123,051	68.6	73.2
9	14,087,541	142,598	86.9	73.0
10	12,345,012	150,606	102.6	71.0
3–10	128,716,700	1,105,831	77.9	66.0
11–20	66,400,222	993,594	106.1	91.6
21–30	20,677,182	488,200	103.8	101.2
31–60	4,489,330	167,500	96.5	97.5
Total	220,283,434	2,755,125	91.9	88.0

Note:—The total amount at risk and actual loss shown above do not agree with similar items shown in the other tables. In compiling the above table the observations ceased on 31st December, 1906, while in the special investigation of the mortality under extended insurances the observations were carried to the anniversary of the dates of extension in the year 1907.

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On a System of Valuation by Movement and Recurrence.

G. Bohlmann.

SUMMARY.

I. Introduction.

1. *Object of the paper.* Under the present laws and regulations several states in Europe require companies doing business in such countries to make an annual valuation of policies issued on applications which have been signed therein. For a large company it may not be feasible to provide in the valuation of the total business, for subdivisions according to countries. In such case special records are kept for states requiring special valuations, and companies so proceeding may welcome a method by which the labor caused by additional valuations is reduced, without weakening the trustworthiness of the result. To distribute the work of valuation equally over the business year, to have the work as near complete as possible at the actual end of the calendar year, to adapt it to a comparatively small clerical force, are other points which, in this connection, may be regarded as worthy of consideration.

The object of this paper is to set forth a system of valuation which has been satisfactorily carried out in practice for the purposes designated above, and which, furthermore, involves some theoretical interest.

2. *General character of the system, formula of recurrence.* The system which will be described in the following paper is based on a formula of recurrence for mean reserves published by the author in 1905, in the German insurance journal, *Zeitschrift für die gesamte Versicherungswissenschaft,* Vol. 5, Berlin, 1905, p. 63. The method can briefly be described as a valuation by movement and recurrence, because an amplified movement of insurance, completed by net premiums and reserves, keeps track of the increases and decreases occurring during the year, and the formula of recurrence brings the reserves valued as for the beginning of the business year, to the reserves valued as for the end of the business year.

The system does not require any subdivisions according to plans or years of issue (except that the new business of the year

must be valued apart); it is only necessary to separate according to ages attained for each valuation basis of mortality and interest. This means in many cases a considerable saving of labor, if we take for instance the ordinary group valuation method as standard of comparison. In detail the advantages and conditions of applicability of the system can only be discussed (Par. 10, 11) after the plan has been thoroughly explained. But two points may be stated here at the beginning: (1) We deal only with the case of the full level net premium reserve; (2) We assume that said premium reserve is valued on the basis of an aggregate table. The method may not be practicable for a company which bases its valuation on a select table.

Any process of accumulation, however—and the system is such a process,—is liable to accumulate from year to year clerical errors once committed. Therefore the method should not be used, as pointed out in my German paper, unless an independent valuation is made say every five years in order to check the results and to arrive at a new starting point. With this provision it appears to me that the system is at least as reliable as any other valuation method known to me. As a matter of fact the practical experience of the system has so far been very favorable.

3. *Approximate element involved in formula of recurrence.* After having emphasized the last statement, it is important to point out that the formula of recurrence, which is the backbone of the whole system, is not exact, but only an approximation. The approximation, however, is so very close that the committed error can as a rule be neglected. In Par. 12 of this paper the result of a practical valuation is given taken from a certain section of the German business of the New York Life Insurance Company, in which there was an amount of insurance in force on December 31, 1906, of about 150 million marks. The reserve amounted to about 21 million marks, and the reserve obtained by the formula of recurrence was only 270 marks less than the reserve derived by an independent group valuation. Such satisfactory agreement was confirmed on every new test made.

Nevertheless the approximate character of the formula of re-

currence is a serious point. The practical man should not, and the mathematician will not, be satisfied by a simple reference to a certain number of tests in practice. The mathematician who sees an approximate equation will ask what its error is, and he will require that at least the sign and an upper limit of the error be ascertained. The practical man needs this information as well, because he must know the conditions and restrictions of the applicability of the formula.

In order to attain this end I have given in the theoretical part of this paper (Part III) a new derivation of the formula of recurrence which is entirely different from that which I set forth in the above mentioned German article. The new method discloses a very elementary principle (Par. 14, theorem I) which is the basis of the approximation. It gives a simple algebraic expression for the error (Par. 15), from which an upper limit is derived which can be easily calculated numerically in practice (Par. 17). The sign of the error can also be determined without difficulty, and it thus becomes possible to calculate close limits between which the reserve must be contained and to state the conditions under which the formula can be used (Par. 16 and 17). The practical result of this investigation may be expressed in the statement that the formula is inapplicable only to cases in which extreme ages have an appreciable influence on the total. Such cases will rarely occur in practice, so that under normal conditions the error of the formula can be entirely disregarded or, when the utmost accuracy is desired, the error can be estimated by the methods of this paper and added to the reserve value obtained by the formula.

4. *Literature.* Before speaking of the literature, it must be stated that the whole plan as here described has been put into practice not by the author, but by Mr. A. Davidson, Actuary of the New York Life, to whom the author is greatly indebted for the keen interest he has taken in the subject. His arrangement has been substantially adopted by the author, and essentially such way of operating has been described in §§ 2 and 3 of Part II. The idea of the fictitious reserve and its introduction in the system (Par. 9), is also due to Mr. Davidson. Besides the theoretical interest in-

volved in this idea, its use means a considerable facilitation of the office work.

Now, as regards the literature, the reader will himself notice the close relations existing between the present investigation and other papers published in this journal. The discussions on such papers have, therefore, to a large extent, a bearing on the present subject as well. We must here limit ourselves to a few references.

The formula of recurrence for terminal reserves (Par. 15, formula (5)) from which we derive our formula for mean reserves is, according to Mr. M. H. Peiler (Volume III of these *Transactions,* p. 25), known to the readers of this journal as "Emory McClintock's accumulation formula."

In Volume IX of these *Transactions,* p. 22, Mr. M. H. Peiler describes how by a set of similar accumulation formulæ based on terminal reserves, a valuation system can be established that we may properly call, like the method described in this paper,—"a valuation by movement and recurrence." Incidentally it may be pointed out, that Mr. M. H. Peiler's paper just mentioned, also deals with the separation of increases and decreases according to source, with which problem any system of valuation by movement and recurrence is intimately connected.

In the French insurance journal, *Bulletin de l'Institut des actuaires français,* Paris, Mr. G. Fouret has made use of Mr. McClintock's accumulation formula for the purpose of valuation (see said *Bulletin,* vol. II, 1891, p. 35, p. 62; vol. V, 1894, p. 60), by reducing the balance sheet reserves to terminal reserves. He arrives at this by placing the anniversary of every policy—for the purpose of valuation—on the first of January of that calendar year which is nearest to the actual anniversary of the policy. Mr. Fouret then fully develops a system of valuation by movement and recurrence which is based on Mr. McClintock's formula, extending it not only to joint lives, but even to sickness insurance. Occasionally, in a brief way, Mr. Fouret refers to the modifications to be applied to his method, in case the first of July of the business year be taken as the average due date of premiums.

II. Practical Phase.

§ 1. Statement and application of formula of recurrence.

5. *Statement of formula.* Using the symbols of the Actuarial Society, the formula of recurrence may be written as follows:

$$(1) \quad V_{+\frac{1}{2}} = \tfrac{1}{2}(S_n + S_{n+1}) + \tfrac{1}{2}P - \{\tfrac{1}{2}(S_n + S_{n+1})v - (V_{-\frac{1}{2}} + \tfrac{1}{2}P)\}u_{x+n-\frac{1}{2}}$$

where for every index z :

$$(2) \quad u_z = \frac{1+i}{p_z}$$

and for $z = x + n - \frac{1}{2}$

$$(3) \quad p_z = p_{x+n-\frac{1}{2}} = \tfrac{1}{2}(p_{x+n-1} + p_{x+n})$$

This formula applies to an individual policy in force at the beginning of the business year and remaining in force at the end of such business year ; x denotes the age at entry, n the years of insurance completed at the anniversary of the policy in the business year. S_n and S_{n+1} are the amounts insured in event of death in the n^{th} and $n + 1^{st}$ insurance years respectively. $V_{-\frac{1}{2}}$ and $V_{+\frac{1}{2}}$ are the mean reserves at the beginning and end of the business year respectively, so that for instance, $V_{-\frac{1}{2}}$ is given for an ordinary life policy by :

$$(4) \quad {}_{n-\frac{1}{2}}V_x = \tfrac{1}{2}({}_{n-1}V_x + {}_nV_x) + \tfrac{1}{2}P_x$$

P is the annual net premium payable at the anniversary in the business year. For insurances in event of death without premium return, such as the ordinary endowment and life policies, S_n, S_{n+1} and $\frac{1}{2}(S_n + S_{n+1})$ are simply the face amount of the policy and P is 0, when no premium is payable in the business year, as is the case for paid-up and single premium policies. For pure endowments S_n and S_{n+1} are the premium return, when there is any.

An extension of the formula to annuities is given in my above quoted German article, but omitted here because annuities are not treated in this paper. The formula applies to joint life insurances, as has been observed by Mr. A. Davidson. Taking for instance, instead of the one life (x), two joint lives (x, y), we have simply to replace:

$$u_{x+n-\frac{1}{2}} = \frac{1+i}{p_{x+n-\frac{1}{2}}} \text{ by } u_{x+n-\frac{1}{2},\, y+n-\frac{1}{2}} = \frac{1+i}{p_{x+n-\frac{1}{2},\, y+n-\frac{1}{2}}}$$

In this article, however, we shall confine ourselves to life and endowment insurances on single lives.

A glance at the formula (1) shows that, assuming an aggregate of policies which are all in force at the beginning as well as at the end of the business year, we can at once apply the formula to any part of that aggregate in which the age attained at the beginning of the business year is the same for all policies. We shall illustrate the working scheme by a mass of this last kind.

6. *Numerical application.* The numerical data of our illustration are those of the two tables I and I′ (page 621). For the business year we take 1908; it may be the first year in which the formula of recurrence is used. We start the process of recurrence with the reserves of December 31st, 1907, ascertained by an ordinary group valuation, which reserves appear in column (9) of the two tables, I and I′, and we proceed to ascertain how near the process of recurrence will bring us to the reserves of December 31st, 1908, as obtained by an independent valuation and shown in column (11) of tables I and I′. The basis is the American 3% table.

As is conveniently done in practice, we have here excluded the premium return element. But among the 50 twenty-payment life policies issued in 1906 at age 34 for $2000 each, there are, let us suppose, 40 which were issued with full return of the premiums paid within twenty years, the annual gross premium rate being $42.86. The corresponding annual increment to the amount insured of $85.72 is stated at an even $86. for each policy in column (4) of table I′ which contains the premium return element.

7. *Process of recurrence.* The tables, II and II′ (pages 623, 624), show how the formula of recurrence (1) applies to the mass of policies assumed in Par. 6. The scheme of the table has been reproduced as it is actually used in practice, so that one line is provided for each age attained at the beginning of the business year. For the sake of illustration 15½ has been assumed as the youngest, and 90½ as the oldest age attained on January 1st, 1908. As all policies of our mass belong to the same age attained on January 1st, 1908, they can be valued as a unit and require only the filling in of the one line "age 35½." The figures appear

Table I. Illustration Data.

Assumed insurance in force both at Dec. 31st, 1907, and at Dec. 31st, 1908, excluding premium return element. *Basis American 3%.*

Amount insured on each policy, $2,000.

(1) Year of issue.	(2) Age at issue.	(3) Plan.	(4) Number of policies.	(5) Face amount $	(6) Annual net premium rate 0/00	(7) Annual net premium amount $	(8) Mean reserve Dec. 31, 1907 rate 0/00	(9) Mean reserve Dec. 31, 1907 amount $	(10) Mean reserve Dec. 31, 1908 rate 0/00	(11) Mean reserve Dec. 31, 1908 amount $
1904	32	Life, single premium.	1	2,000		0	423.62	847	431.20	862
1905	33	Ordinary life.	250	500,000	19.87	9,935	40.28	20,140	53.03	26,515
1906	34	Life, 20 premiums.	50	100,000	29.27	2,927	47.27	4,727	69.88	6,988
1907	35	20 year endowment.	500	1,000,000	41.97	41,970	38.28	38,280	73.48	73,480
		Total:	801	1,602,000		54,832		63,994		107,845

Table I′. Illustration Data.

Assumed premium return insurance in force both at December 31, 1907, and at December 31, 1908.

Basis: American 3%

(1) Year of issue	(2) Age at issue	(3) Plan	(4) Yearly increment $	(5) Total increment in force Dec. 31, 1907 $	(6) Additional net premium rate 0/00 of (4)	(7) Additional net premium amount $	(8) Additional reserve Dec. 31,1907 rate 0/00 of (4)	(9) Additional reserve Dec. 31,1907 amount $	(10) Additional reserve Dec. 31,1908 rate 0/00 of (4)	(11) Additional reserve Dec. 31,1908 amount $
1906	34	P. R. in 20 years.	3,440	6,880	107.50	370	205.41	707	300.39	1,033
		Total:	3,440	6,880		370		707		1,033

again as "totals" in the last line of the tables where the footings resulting from all ages would appear in an actual valuation.

For the convenience of the reader we have added in the last line of the headings of tables II and II′ brief references to the corresponding columns of tables I and I′.

As in table II face amounts only appear, the formula of recurrence reads for this table:

$$V_{+\frac{1}{2}} = S + \tfrac{1}{2}P - \left\{Sv - (V_{-\frac{1}{2}} + \tfrac{1}{2}P)\right\} u_{x+n-\frac{1}{2}}$$

where S denotes the face amount.

For table II′, which refers to the premium return element, formula (1) has been written in the following form:

$$V_{+\frac{1}{2}} = S_n + \tfrac{1}{2}S + \tfrac{1}{2}P - \left\{(S_n + \tfrac{1}{2}S)v - (V_{-\frac{1}{2}} + \tfrac{1}{2}P)\right\} u_{x+n-\frac{1}{2}}$$

S_n being the total increment and S the yearly increment of premium return insurance in force December 31st, 1907. P is the *additional* net premium and $V_{-\frac{1}{2}}$ and $V_{+\frac{1}{2}}$ are the *additional* reserves at December 31st, 1907, and at December 31st, 1908, respectively.

Now comparing the reserve values in tables I and I′ obtained by an ordinary group valuation, with those in tables II and II′ obtained by the formula of recurrence, we find that in this particular case the formula of recurrence has worked as an absolutely correct formula.

§ 2. The amplified movement of insurance.

8. *Process of movement.* In case of an actual company the process of recurrence alone does not suffice for the valuation of the policies. In reality we have not simply to deal with a mass of policies which are all in force both at the beginning and at the end of the business year. Among the policies in force at the beginning of the year, there are terminations of all kinds and decreases by change during the year: there are also increases by change and reinstatements. Furthermore we have the new business paid for during the year and again in the new business increases and decreases of all sorts. By its nature the formula of recurrence is limited to the old business. The valuation of the new business must therefore be made by some other method, such as the usual group valuation system. But even for the old business the process

TABLE II. ILLUSTRATION OF FORMULA OF RECURRENCE.

Valuation of insurance in force December 31, 1907, (as) for December 31, 1908, excluding premium return element.

Basis: American 3%. $v = 0.970874$.

(1) Age at beginning of 1908 $x+n-\frac{1}{2}$	(2) Amount in force Dec. 31, '07 I (5) $	(3) Half annual net premium $\frac{1}{2}\times I$ (7) $	(4) Reserve Dec. 31, 1907 I (9) $	(5) (3) + (4) $	(6) (2) $\times v$ $	(7) (6) − (5) $	(8) ${}^{u}x+n-\frac{1}{2}$	(9) (7) × (8) $	(10) (2) + (3) $	(11) (10)—(9) Reserve Dec. 31, 1908 $
$15\frac{1}{2}$										
$16\frac{1}{2}$										
- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
$35\frac{1}{2}$	1,602,000	27,416	63,994	91,410	1,555,340	1,463,930	1.039374	1,521,571	1,629,416	107,845
- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
$90\frac{1}{2}$										
Total,	1,602,000	27,416	63,994							107,845

TABLE II′. ILLUSTRATION OF FORMULA OF RECURRENCE.

Valuation of premium return insurance in force December 31st, 1907, (as) for December 31, 1908.

Basis: American 3%.

$v = 0.970874.$

(1) Age at beginning of 1908 $x+n-\frac{1}{2}$	(2a) P. R. force Dec. 31, 1907 ½×yearly amount ½×I'(4) $	(2b) Increment in force Dec. 31, 1907 total amount I' (5a) $	(2c) (2a)+(2b) $	(3) Half additional net premium ½×I' (7) $	(4) Additional reserve Dec. 31, 1907 I' (9) $	(5) (3)+(4) $	(6) (2c)×v $	(7) (6)−(5) $	(8) $u_{x+n-\frac{1}{2}}$	(9) (7)×(8) $	(10) (2c)+(3) $	(11) (10)−(9) Additional reserve Dec. 31, '08 $
$15\frac{1}{2}$												
$16\frac{1}{2}$												
- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -
$35\frac{1}{2}$	1,720	6,880	8,600	185	707	892	8,350	7,458	1.039374	7,752	8,785	1,033
- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -
$90\frac{1}{2}$												
Total,	1,720	6,880	8,600	185	707							1,033

of recurrence alone does not furnish us with the reserve on the insurance in force at the end of the year, unless the reserves on the increases and decreases are known to us. Such reserves may be found by an individual valuation of each policy entering and of each policy ceasing. For each such increase or decrease we must also ascertain the annual net premium which would have been payable in the business year, if the policy had been in force both at the beginning and at the end of the year. Instead of the simple movement of insurance which takes into account:

(1). Number of policies,

(2). Amounts,

we therefore need an *amplified movement of insurance* in which we record, for every increase and decrease:

(3). The net annual premium,

and (4). Mean reserve, valued as for the end of the business year.

Assuming that we know for each age attained and for each one of these four elements the totals which are in force at the beginning of the business year, the valuation work to be done for the business year will consist of the following:

I. Process of Recurrence.

(*a*) From the valuation of the insurance in force at the end of the preceding business year, we draw recapitulations according to ages attained, giving the totals of each one of the four elements. If the valuation by movement and recurrence was already in use in the preceding year, these totals are immediately furnished by the records kept for such preceding year.

(*b*) By the process of recurrence we derive from the mean reserves valued as for the beginning of the business year, the mean reserves valued as for the end of the year on the insurance in force at the beginning of the business year.

II. Process of Movement.

(*c*) Daily, during the year, we record the values of the four elements for every increase and decrease occuring during the year, including the increases and decreases on the new paid business, and also including the increase due to the fact that a policy dated back to a previous year is paid for in the business year, but ex-

cluding increases accruing when a policy dated in the business year is paid for in that year, these last being the normal new issues.

(*d*) After the books have been closed for the business year, the increases and decreases referred to under (*c*) are footed for each one of the four elements and for each age attained.

(*e*) By the usual group valuation method we ascertain the reserves on the new business, dated in the business year and valued as of the end of the year, taking all these policies as they stood at the moment when they were paid for. We draw recapitulations indicating the totals for each one of the four elements and for each age attained.

(*f*) We then determine, by the process of movement, for each age attained and for each one of the four elements, the totals in force at the end of the business year, starting in case of the reserves with their values as for the end of the year on the insurance in force at the beginning of the year, which values were obtained by the process of recurrence referred to under (*b*).

The final totals thus obtained for each age attained and for each one of our four elements will be the figures to start off with in the next business year, which figures will exactly correspond to the amounts that were the initial values referred to under (*a*) for the current business year.

It has been assumed that all policies are valued on the same mortality table and at the same rate of interest. If the case is otherwise, the entire business must be divided into as many sections as there are different combinations of mortality tables and rates of interest, and the processes above described applied to each section. The number of policies is not needed for the valuation, but this element comes in naturally and is convenient for checking purposes. The new paid business which is dated back to previous years, can be properly treated along with the increases in the old business. A balance of the system with the ordinary movement of insurance can be easily obtained for the number of policies and amounts of insurance.

9. *Fictitious reserves.* The analysis of the work required to be done under the system of valuation by movement and recurrence is completely given in items (*a*)-(*f*) of the preceding paragraphs, with the exception of one point which remains to be explained.

Policies like maturing endowments and expiring term insurances which end by their terms in the business year, have no reserve values at the end of the business year. To take out such policies at the beginning of the year, would be to disturb or at least complicate the machinery. But if we apply to such a policy the formula of recurrence mechanically, starting with the correct reserve value $V_{-\frac{1}{2}}$ at the beginning of the year, we arrive at a *fictitious mean reserve* of our policy which may be designated by $V'_{+\frac{1}{2}}$ and which is exactly the contribution of our policy to the reserve value obtained for our mass, by mechanically applying to the mass the formula of recurrence without taking out those policies which end by their terms in the business year. At the time of termination of such a policy we shall then record among the decreases of the day, as described in (*c*) of Par. 8, the values of the four elements of this policy, but entering as reserve the fictitious mean reserve of the policy calculated for the end of the business year.

It is easy to derive exact formulæ for these fictitious reserves by which they can be calculated without knowing the mean reserves of the preceding year of insurance. By the aid of these formulæ the fictitious mean reserve rates can be calculated and tabulated for every age at issue and all plans for which they come into play.

It would be possible to avoid the fictitious reserves in the system by applying the process of recurrence referred to under (*b*) of Par 8, *after* the process of movement described under (*c*) and (*d*) had been completed. We would then confine the process of movement to the increases and decreases on the insurance in force at the beginning of the year, recording for every such increase or decrease the mean reserve valued as for the beginning of the year, instead of treating the mean reserves valued as for the end of the year as described in Par. 8. We should thus, by the process of movement, arrive at the reserves as for the beginning of the year on the old insurance in force at the end of the year, and would obtain therefrom, by the process of recurrence, the reserves for the end of the year on the old business in force at the end of the year. We should then have to add the reserves on the new business in force at the end of the year as ascertained by a direct group valuation. This course was described by the author in his German paper quoted here in Par. 2. The suggestion of treating the reserves

for increases and decreases valued as for the end of the year, and the use of the fictitious reserves involved in the proposition, are both due to Mr. Davidson. The author has adopted this scheme, which appears to him far preferable, because—among other advantages—it makes it possible to start the valuation work for the current business year with the process of recurrence, instead of deferring said process—as the other system does—to a time after the books have been closed for the business year.

§ 3. The system in practice.

10. *The office work.* The office work required to be done under the system as described in Par. 5-9 can, for a comparatively small and steady business, be arranged as follows:

(*a*) A company which keeps policy-cards for increases and decreases can enter daily, at a proper place on such card, the annual net premium and mean reserve, valued as for the end of the year, both for the face amount and for the premium return insurance, if any. Then, say every month, the cards for all increases and decreases of the month can be arranged according to ages attained and the necessary data copied for every case on "increase sheets," where there is an increase, and on "decrease sheets," where the case is a decrease. Proper columns of the increase sheets provide for the record of the mode of increase, the policy number, the face amount, the annual net premium and the mean reserve, and also for the indication of the additional insurance, the additional net premium and the additional reserve on premium return, where there is any. All policies belonging to the same age attained are entered in chronological order under the same heading: "Age attained....." The decrease sheets are arranged in exactly the same manner.

After the books have been closed for the end of the year, the various columns of the increase sheets and decrease sheets are footed for each age attained, and thus the data referred to under (*d*) of Par. 8 are obtained as well for the face amount of insurance as for the premium return insurance. The totals are then carried to "movement sheets," using one sheet for each one of the four elements:

(a) Number of policies.
(b) Face amounts.
(c) Annual net premiums on face amounts.
(d) Mean reserves on face amounts valued as for the end of the business year.

There are three additional sheets corresponding to (b), (c) and (d) for the premium return element.

The scheme of the movement sheets (a)-(d) is the same. Each line corresponds to an age attained printed in the first column of the sheet. The second column of each movement sheet (a)-(d) is headed "In force Dec. 31st, of preceding business year." The figures for this column are, in case of elements (a)-(c), known from last year's valuation and, in case of element (d), they are derived from the figures of the last year by the process of recurrence. The figures are to be filled in when the valuation work for the current business year is started. The third column takes up the totals which come from the increase sheets, and the fourth column the totals which come from the business dated and paid for in the year—see (c) of Par. 8. A fifth column may be conveniently added, showing the sums of columns (2-4). In the sixth column we copy the totals which come from the decrease sheets, and in the seventh column we obtain as difference of columns 5 and 6, the totals corresponding to the insurance in force at the end of the year. This last column, when footed, must, for number of policies and amounts, balance with the policy exhibit, and it gives for the premium reserve sheet the total reserve at the end of the year, as resulting from our system of valuation by movement and recurrence. The figures appearing in each movement sheet in the various lines of column (7) are, in case of elements (a)-(c), at the same time the totals to be carried forward to column (2) of the corresponding movement sheet for the following business year. In case of element (d) they are derived from said totals by the process of recurrence.

The arrangement of the movement sheets for the premium return element is the same, except that the "movement of amounts" has to be a little amplified, because it has to provide both for the yearly increment and for the total premium return in force.

11. *Possible modifications.* We do not, of course, mean to say that the system of valuation by movement and recurrence should,

when applied in practice, be literally carried out, step by step, as it has been described in the preceding section. Each company will, in applying the system, suit its own convenience. The scheme explained in § 2 has proved satisfactory and safe in its application to a comparatively small and steady business. But the author thinks that the system, when properly adapted to the conditions existing in each case, may be also used with success under other circumstances and on a larger scale. On the other hand governmental requirements or the number of different valuation bases used by the company may necessitate the introduction of so many subdivisions of "categories," that it would not pay to apply the system here described. In regard to possible modifications of the system the following remarks may be made:

(*a*) We have assumed an individual valuation of the policies coming in during the year by increase and of those going out by decrease. When we have to handle say 1200 terminations a year, including decreases by change, we have to value, on an average, 4 terminations a day. This is an amount of work that is really not felt at all. But for a larger business and a business with many more terminations, the work might be reduced by applying to the terminations of say the last four years of issue the ordinary group valuation system, recording on the decrease sheets individually only the decreases for earlier years of issue.

(*b*) Changes have been treated in the increase and decrease sheets on the basis that the old policy with all its elements goes out entirely in the decrease sheets and the new plan comes in as a new case in the increase sheets. This is a safe disposition applicable to a small business. But, when we have to face a larger volume of business with many changes it may require less work to record only those elements or those parts of the various elements which have actually changed their values. A glance at the formula of recurrence shows, how far it permits of simplification in such cases. The changes by non-payment of premium to term extension or to fractional paid-up form the most frequent kind in practice. Policies becoming paid-up by their terms, because the premium-paying period expires, can be treated, as is done by Mr. A. Davidson, by providing for a special column in the movement of net premiums in which special column such expiring net premiums are collected.

(*c*) Incidentally it may be mentioned that in the scheme as described in § 2 it has been found very convenient to indicate the mode of increase and the mode of termination for the individual entries in the increase and decrease sheets. This facilitates balancing with the ordinary movement of insurance and helps to localize the errors in case of any discrepancy.

12. *Statement of the result of an actual valuation.* The system of valuation by movement and recurrence was applied to the German business of the New York Life Insurance Company for the first time in 1906 substantially in the form described in § 2. The result was compared with the figures obtained by an independent group valuation of the insurances in force December 31st, 1906. In the following table, III, we exhibit the results obtained for a certain section valued on the American 3% basis, condensing the ages in quinquennial groups. The reserve value arrived at by the direct valuation is a little higher than that which is found by the formula of recurrence. This confirms what was expected according to theory (theorem II of Par. 16). But the difference of Marks 270 is only 0.00018% of the amount insured.

13. *Example of a poor approximation.* From the practical case above exhibited the reader may have got the impression that the formula of recurrence is not an approximation, but is exact. In order to convince him of the contrary we will now give—in the form of a transition to the theoretical phase of this subject—a numerical example in which the approximate character of the formula becomes very evident. For that purpose we must have recourse to very extreme cases in which the probabilities of dying and the reserves vary rapidly with the duration of the policy. We therefore consider a 5 year endowment policy, issued at age 90 for $1,000. with annual payment of premiums. The problem is to derive by the formula of recurrence the mean reserve of the fourth year of insurance from the mean reserve of the third year of insurance, the basis of valuation being the American 3% table. The data found upon calculation are:

$$P = P_{90,\overline{5}|} = 503.54\ ;\quad V_{-\frac{1}{2}} = {}_{2\frac{1}{2}}V_{90,\overline{5}|} = 531.39\ ;\quad u_{x+n-\frac{1}{2}} = u_{92\frac{1}{2}} = 3.26174$$

Table III.

An American 3% section of the German business of the New York Life Insurance Company, in force December 31st, 1906. *Currency: Marks.*

(1) Age at beginning of 1906	(2) Reserves December 31, 1906, valued by systems of movement and recurrence	(3) Reserves December 31, 1906, valued directly	(4) = (3)−(2) Difference between the two results: plus	(5) = (2) − (3) Difference between the two results: minus	(6) Difference between the two results: net	(7) Amounts in force Dec. 31, 1906
14½–18½	57,119	57,118		1		532,500
19½–23½	240,880	240,881	1			2;447,202
24½–28½	1;130,579	1;130,465		114		11;845,108
29½–33½	2;982,287	2;982,354	67			27;194,712
34½–38½	4;609,923	4;609,859		64		34;764,373
39½–43½	3;961,323	3;961,430	107			27;013,400
44½–48½	3;887,492	3;887,561	69			23;394,128
49½–53½	2;428,568	2;428,695	127			14;151,466
54½–58½	1;210,191	1;210,265	74			6;387,403
59½–63½	431,504	431,501		3		1;833,403
64½–68½	28,341	28,348	7			142,000
Total, -	20;968,207	20;968,477	452	182	+270	149;705,695

The formula of recurrence gives therefore :

$$
\begin{aligned}
{}_{3\frac{1}{2}}V_{90} &= (1{,}000 + 251.77) - \{970.874 - (531.39 + 251.77)\} \\
&\qquad \times 3.26174 \\
&= 1{,}251.77 - [970.874 - 783.16] \times 3.26174 \\
&= 1{,}251.77 - 612.27 = \$639.50.
\end{aligned}
$$

The correct value of this reserve found by an independent calculation is, however, \$650.37. The value given by the formula of recurrence is therefore too small by \$10.87.

Incidentally we may state that the approximation would become still worse, if we had calculated $u_{92\frac{1}{2}}$ as the arithmetical mean of u_{92} and u_{93}, instead of using equations (2) and (3) of Par. 5. We should have then obtained $u_{92\frac{1}{2}} = 3.34548$ which leads to a value of ${}_{3\frac{1}{2}}V_{90} = \$623.78$, being smaller than the correct value by \$26.59.

But, returning to our original definition of the u, the one illustration given just now, with its error of \$10.87, shows the insufficiency of mere empirical tests and the necessity of a complete theoretical investigation, which we will therefore now undertake.

III. Theoretical Phase.

§ 1. Derivation of formula of recurrence.

14. *A simple property of the arithmetical mean.* We designate the arithmetical mean $\frac{1}{2}(a_0 + a_1)$ of two numbers a_0 and a_1 by Mean (a_0, a_1) so that :

$$\text{Mean } (a_0, a_1) = \tfrac{1}{2}(a_0 + a_1)$$

The principle of approximation on which our formula of recurrence for mean reserves is based, is given by the following theorem :

Theorem I. Let a_0, a_1 be two numbers which do not differ very much and let β_0, β_1 be another such pair of numbers, then the arithmetical mean

$$\textit{Mean } (a_0\beta_0, a_1\beta_1)$$

of the products $a_0\beta_0$, $a_1\beta_1$ is approximately equal to the product

$$\textit{Mean } (a_0, a_1) \times \textit{Mean } (\beta_0, \beta_1)$$

of the arithmetical means of a_0, a_1 and of β_0, β_1. The error η involved in this approximation is given by:

$$\eta = \tfrac{1}{4}(a_1 - a_0)(\beta_1 - \beta_0)$$

so that we have for all values of a_0, a_1, β_0, β_1 the identity:

$$Mean\ (\alpha_0\beta_0,\ \alpha_1\beta_1) = Mean\ (\alpha_0,\ \alpha_1) \times Mean\ (\beta_0,\ \beta_1) + \eta$$

In fact the assertion is :

$$\tfrac{1}{2}(\alpha_0\beta_0 + \alpha_1\beta_1) = \tfrac{1}{2}(\alpha_0 + \alpha_1) \cdot \tfrac{1}{2}(\beta_0 + \beta_1) + \tfrac{1}{4}(\alpha_1 - \alpha_0)(\beta_1 - \beta_0)$$

and this relation is proved by carrying out the multiplications.

15. *Demonstration of formula of recurrence for mean reserves.* Starting with *McClintock's* formula of recurrence for terminal reserves, and writing V for the reserve at the end of the n^{th} year and V_{+1} at the end of the $(n+1)^{\text{th}}$ year, and P for the uniform net annual premium, we have :

$$(5) \qquad V_{+1} = S_{n+1} - \{v \cdot S_{n+1} - (V + P)\}\ u_{x+n}$$

Since vp_{x+n} is the reciprocal of u_{x+n}, formula (5) can be written in the following form :

$$(6) \qquad (S_{n+1} - V_{+1})vp_{x+n} = \{v \cdot S_{n+1} - (V + P)\}$$

Replacing $\overline{n+1}$ by n, we also have :

$$(6') \qquad (S_n - V)v \cdot p_{x+n-1} = \{vS_n - (V_{-1} + P)\}$$

Now combining equations (6) and (6') and dividing by 2, and putting

$$(7) \qquad \gamma = \text{Mean}\ [(S_n - V)v \cdot p_{x+n-1},\ (S_{n+1} - V_{+1})v \cdot p_{x+n}]$$

we obtain

$$\gamma = \frac{v}{2}(S_n + S_{n+1}) - \tfrac{1}{2}(V_{-1} + V + 2P) :$$

applying formula (4) of Par. 5 this becomes

$$(8) \qquad \gamma = \frac{v}{2}(S_n + S_{n+1}) - (V_{-\frac{1}{2}} + \tfrac{1}{2}P)$$

We now apply to equation (7) theorem I of the preceding paragraph, putting :

$$\alpha_0 = S_n - V,\ \beta_0 = vp_{x+n-1}$$
$$\alpha_1 = S_{n+1} - V_{+1},\ \beta_1 = vp_{x+n}$$

so that

$$(7') \qquad \gamma = \text{Mean}\ (\alpha_0\beta_0,\ \alpha_1\beta_1)$$

We then find :

$$\text{Mean}\ (\alpha_0,\ \alpha_1) = \tfrac{1}{2}(S_n + S_{n+1}) - \tfrac{1}{2}(V + V_{+1})$$

or

$$(9) \qquad \text{Mean}\ (\alpha_0,\ \alpha_1) = \tfrac{1}{2}(S_n + S_{n+1}) + \tfrac{1}{2}P - V_{+\frac{1}{2}}$$

Furthermore :

$$\text{Mean}\ (\beta_0,\ \beta_1) = v \cdot \tfrac{1}{2}(p_{x+n-1} + p_{x+n})$$

or (according to equation (3) of Par. 5) :

$$(10) \qquad \text{Mean } (\beta_0, \beta_1) = vp_{x+n-\frac{1}{2}}$$

Therefore theorem *I* of the preceding number gives :

$$(11) \quad \gamma = \text{Mean } (\alpha_0\beta_0, \alpha_1\beta_1) = \text{Mean } (\alpha_0, \alpha_1) \text{ . Mean } (\beta_0, \beta_1) + \eta$$

where the correction η is given by

$$\eta = \tfrac{1}{4}(\alpha_1 - \alpha_0)(\beta_1 - \beta_0) = \tfrac{1}{4}[S_{n+1} - S_n - (V_{+1} - V)]v(p_{x+n} - p_{x+n-1})$$

or :

$$(12) \qquad \eta = \tfrac{1}{4}[(V_{+1} - V) - (S_{n+1} - S_n)]v(p_{x+n-1} - p_{x+n\cdot})$$

We substitute in (11) the values of γ, Mean (α_0, α_1) and Mean (β_0, β_1), drawn from equations (8), (9) and (10) respectively and find :

$$\tfrac{1}{2}(S_n + S_{n+1})v - (V_{-\frac{1}{2}} + \tfrac{1}{2}P) = \{\tfrac{1}{2}(S_n + S_{n+1}) + \tfrac{1}{2}P - V_{+\frac{1}{2}}\}vp_{x+n-\frac{1}{2}} + \eta.$$

According to equations (2) and (3) of Par. 5 the quantity $vp_{x+n-\frac{1}{2}}$ is the reciprocal of $u_{x+n-\frac{1}{2}}$. Therefore we calculate from the last equation the following expression for $V_{+\frac{1}{2}}$:

$$(1') \qquad V_{+\frac{1}{2}} = \tfrac{1}{2}(S_n + S_{n+1}) + \tfrac{1}{2}P - \{\tfrac{1}{2}(S_n + S_{n+1})v - (V_{-\frac{1}{2}} + \tfrac{1}{2}P)\}u_{x+n-\frac{1}{2}} + \epsilon$$

where, according to (12) :

$$(13) \quad \epsilon = \eta u_{x+n-\frac{1}{2}} = \tfrac{1}{4}[(V_{+1} - V) - (S_{n+1} - S_n)]v(p_{x+n-1} - p_{x+n})u_{x+n-\frac{1}{2}}$$

The equation last but one above and which is designated by (1′) is our formula of recurrence (1) for mean reserves (Par. 5), but completed by the error ϵ for which we derive from the last equation (13) the following two equivalent expressions:

$$(1'') \qquad \epsilon = \{(V_{+1} - V) - (S_{n+1} - S_n)\} \cdot \tfrac{1}{4}\frac{p_{x+n-1} - p_{x+n}}{p_{x+n-\frac{1}{2}}}$$

$$= \{(V_{+1} - V) - (S_{n+1} - S_n)\} \cdot \tfrac{1}{4}\frac{u_{x+n} - u_{x+n-1}}{\frac{1}{2}(u_{x+n-1} + u_{x+n})}.$$

§ 2. Discussion of the error.

16. *Theorems and illustrations.* For the usual life and endowment policies without premium return, we have in formula (1′) of Par. 15 :

$$S_{n+1} - S_n = 0$$

and therefore [see (1″)] :

$$\epsilon = (V_{+1} - V) \cdot \tfrac{1}{4} \frac{p_{x+n-1} - p_{x+n}}{p_{x+n-\frac{1}{2}}}$$

Confining ourselves to ages at issue x, for which the probabilities of dying q_{x+n} increase with the duration n of the policy, we know that the reserves of life and endowment policies without premium return increase with the duration of the insurance, whatever the mortality table or the rate of interest may be. For such policies, which form quite the majority of all practical cases, we can therefore assert that ϵ is positive and we have the remarkable:

Theorem II. When we have a life or endowment policy issued for a level sum of insurance and at an age from which the probabilities of dying increase with the duration of the policy, then the formula of recurrence:

$$V_{+\frac{1}{2}} = S + \tfrac{1}{2}P - \{Sv - (V_{-\frac{1}{2}} + \tfrac{1}{2}P)\}$$

gives too small reserves. The correction to be added is given by

$$\epsilon = (V_{+1} - V)\tfrac{1}{4} \frac{p_{x+n-1} - p_{x+n}}{p_{x+n-\frac{1}{2}}}$$

By this theorem we know the sign of the error. We also see from it, that the error is the smaller, the less the probabilities of dying and reserves vary with the duration of the insurance. The error is therefore higher at older ages, than at younger ages, and it is higher for endowment policies than for life policies, and smaller for single premium policies than for annual premium policies, if the conditions are otherwise the same. The theorem also shows how to calculate the error in any individual case.

As an illustration we calculate ϵ for the four plans contained in table I (Par. 6), assuming only $1,000. insurance on every policy, and add as a fifth case a 20 payment life policy issued at age 34 with full premium return in twenty years, which case corresponds to table I′.

As, for all the five examples, $x + n$ is 36, we have:

$$\tfrac{1}{4} \frac{p_{x+n-1} - p_{x+n}}{p_{x+n-\frac{1}{2}}} = \tfrac{1}{4} \frac{p_{35} - p_{36}}{p_{35\frac{1}{2}}} = 0.000036$$

and formula (1″) becomes:

$$\epsilon = \{(V_{+1} - V) - (S_{n+1} - S_n)\} \times 0.000036$$

The calculation is given in extenso in table IV :

This table discloses the reason why, in case of our illustration (Par. 6), the error of the formula was not felt; it also shows that for premium return policies (which are excluded in theorem II) ϵ may become negative.

On the other hand, if we had taken our 5 year endowment policy of Par. 13, issued at age 90, our formula (1″) would have given us the value of ϵ \$10.87, already found, as it must be. This extreme illustration should, however, not disturb our judgment as to the practical accuracy of the formula of recurrence. In practice, cases like those of tables I and I′ are near the average, and practically an extreme proposition will be a 20 year endowment policy issued at age 50 for which the mean reserve of the eleventh year is to be derived. At least the relative weight with which cases like this will come into calculation will be light. And for this example formula (1″) gives an error ϵ of 2.4 cents for each \$1,000. of insurance.

17. *A convenient upper limit of the error.* The disadvantage of formula (1″) for the error of ϵ is that it contains terminal reserves which in case of an actual valuation are, as a rule, not known. But by some simple mathematical considerations which are omitted here, for want of space, we arrive at the following:

Theorem III. The probabilities of dying may show an accelerated increase with the age, for all ages attained entering into the calculation. The insurances to be valued may be all insurances in event of death issued for a level amount and the reserves on every individual plan may show an accelerated increase with the duration of the policy. Then the error ϵ *involved in the formula of recurrence is always contained between the limits*:

$$0 < \epsilon < \sum_{(x+n-\frac{1}{2})} (V_{+\frac{1}{2}} - V_{-\frac{1}{2}}) \tfrac{1}{4} (u_{x+n+\frac{1}{2}} - u_{x+n-\frac{1}{2}})$$

where the summation indicated by Σ *has to be extended over all groups* $(x + n - \frac{1}{2})$ *of ages attained into which the insurance had to be subdivided for the purpose of valuation.*

TABLE IV.

Error ϵ of formula of recurrence, calculated for five typical cases taken from Tables I and I′.

Face amount \$1,000 :

Basis: American 3%

(1) Plan	(2) Age at issue x	(3) Duration n	(4) V_{+1}	(5) V	(6)=(4)−(5) $V_{+1}-V$	(7) $S_{n+1}-S_n$	(8)=(6)−(7)	(9)=(8)×0.000036 =ϵ
			\$	\$	\$	\$	\$	\$
Ordinary life	33	3	49.56	36.63	12.93	0	12.93	0.00047
Life, 20 premiums	34	2	66.74	43.75	22.99	0	22.99	0.00083
20 year endowment	35	1	70.40	34.59	35.81	0	35.81	0.00129
Life, single premium	32	4	435.04	427.36	7.68	0	7.68	0.00028
Life, 20 premiums with full return in 20 years	34	2	79.32	52.37	26.95	43.00	−16.05	−0.00058

This theorem has, for the sake of an illustration, been applied to that part of the business of the New York Life Insurance Company which has been referred to in Par. 12, with this result:

$$o < \epsilon <$$ Marks 556.

The actual deviation was, according to Par. 12, equal to 270 marks.

The Incontestable Clause in Life Insurance Policies.

BY WENDELL M. STRONG.

Most modern policies contain an incontestable clause which states that the policy shall be incontestable after a certain date, which is ordinarily either the date of issue or one or two years after the date of issue. Usually there is a condition "provided the premiums have been duly paid," and occasionally some further condition or exception is incorporated; these conditions and exceptions are of no importance to us, since the question we shall consider is the legal effect of such a clause with regard to defences not specifically excepted by it from its operation—whether this effect is to bar these defences absolutely or if not, what defences are still admissible. This question is of moment both to the policyholder and the company. The policyholder wishes when insuring to be certain he is leaving his family a sum of money rather than a lawsuit in his life insurance. To the company it is essential to the determination of its methods to know whether, even if there is fraud in the inception of a policy, that policy nevertheless must be paid, unless steps are taken to contest it within the limit of time allowed by the incontestability clause. Unfortunately the whole question of "incontestability" has been most unsatisfactorily treated in the text-books so that with one or two exceptions little reliance can be placed on what they give as the law.

It is practically without question that a clause stating that the policy is incontestable covers all cases of misrepresentations and incorrect statements innocently made,[1] even though the application

[1] An extreme case showing how far the law will go where neither fraud nor public policy is involved is Wood *v.* Dwarris, 11 Exch., page 493. In this case the policy contained no incontestable clause, but a prospectus had been published, stating that the policies would be indisputable, except for fraud. It was held that this statement in the prospectus barred the contest of a policy for a misrepresentation not fraudulent.

is so worded as to make the statements of the insured contained therein warranties. Hence it is only to the cases involving other elements that we need to give our attention.

There are four different cases, not exhaustive, which must be distinguished, as they involve different legal considerations and principles; they are:

1. Where the incontestable clause states that the policy is incontestable from date.
2. Where the incontestable clause states that the policy is incontestable after a certain period of time, as one year from date.
3. Where public policy is involved, as in a "wager" policy.
4. Where there is substitution in the examination.

In the discussion of 1 and 2 it will be tacitly assumed that the defences which form cases 3 and 4 are not involved.

I.

If a policy reads that it is "incontestable from date" or simply that it is "incontestable," the question arises whether the legal effect of these words is to bar all defences including fraud. There is a well known principle of law that fraud in the inception of a contract vitiates that contract. Evidently an agreement as part of a contract that fraud, if it has existed in the inception of that contract, shall nevertheless not be a defence to that contract, is directly contrary to this principle of law and must consequently be of no effect. As both Bliss and Bunyon in their treatises say, "An agreement that the insurer will not raise any objection, even in the case of direct personal fraud, is a void condition."

Practically the only case in which this point has been directly decided is that of Reagan *v.* The Union Mutual Life Insurance Company (189 Mass., 555), but this case is very clear and convincing and will undoubtedly be generally followed in the future when a decision turns upon the same point. In this case the court says, "This is not like the numerous cases in which the policy provides that it shall be incontestable for fraud after the expiration of a specified time which is not unreasonably short. It has often been held that a provision of that kind is valid, because it is in the nature of a limitation of the time within which the defendant may avoid the policy for this cause. Such a provision is

reasonable and proper, as it gives the insured a guarantee against possible expensive litigation to defeat his claim after the lapse of many years, and at the same time gives the company time and an opportunity for investigation, to ascertain whether the contract should remain in force. It is not against public policy as tending to put fraud on a par with honesty."

A very similar point came up later in Massachusetts in reference to the approval by the Insurance Commissioner of policy forms for use in that State. The Commissioner refused to approve a form which made the policy incontestable from date, and the court upheld the Commissioner, stating that "such a provision is not in accordance with public policy." [1]

There are a number of very decided dicta upon the same point. In the Massachusetts Benefit Life Association *v.* Robinson (104 Ga.. p. 256), the court says, "If however the policy stipulated that it should be incontestable from its date and the insurer would not be allowed any defences, whether originating in fraud or otherwise or if it were clear from the terms of the contract that it was the intention of the parties that fraud should not be a defence, then such a contract would be void as being opposed to the policy of the law."

In Welch *v.* The Union Central Life Insurance Company (108 Iowa, 256), the court says, in referring to a clause making the policy incontestable from date, "In these cases then fraud, if not mentioned, must be assumed to be excluded, since that construction is always to be preferred which will support a contract, and it is never to be supposed that the parties to it intend an illegal stipulation where a lawful meaning can be given to their words. Of course this construction cannot make the policy really indisputable, for it leaves open the question whether the statement or omission complained of was fraudulent or not, and also what is the true meaning or construction of the policy itself."

On the other side there is the case of Insurance Co. *v.* Fox (106 Tenn. 347), which decides that a clause making the policy incontestable from date, with certain specific exceptions, of which fraud was not one, would bar the defence of fraud. The opinion

[1] New York Life Insurance Company *v.* Hardison (Massachusetts Supreme Court, September, 1908).

in this case does not seem entitled to great weight as it stands practically without support beyond that of a dictum in Patterson *v.* Natural Premium Mutual Life Insurance Company (100 Wis., 118), and it will probably not be followed by other courts.

The standard Policy Form of New York State and recent laws of several other States explicitly recognize that the provision that a policy shall be incontestable from date is a proper provision to be incorporated in the policy contract. While it is possible that this may make a difference in the attitude of the courts of those States towards such a provision, as regards the defence of fraud, the legal principle that the right to dispute for fraud cannot be absolutely contracted away is so well settled that it seems probable the courts will consider that the defence of fraud is impliedly excepted from this provision, as they have in general done heretofore.

II.

There has been some tendency among writers, which fortunately has not extended to the courts to any great degree, to confuse the case where the incontestable clause sets a limit to the time within which the policy can be contested with the case just discussed where the clause states that the policy is incontestable from date. It is in essence entirely different; while the question with a policy made by its terms incontestable from date is whether the right to contest for fraud can be absolutely contracted away, the question here is not at all that of contracting away a right to contest for fraud, but whether a limit of time can be set within which any contest to be made shall be made. Setting thus a limit of time for contest is in effect, as has been said in the opinions of many courts, a short statute of limitation. Statutes of limitation are much favored in law, and many exist in every State—nearly every legal right must be asserted within a proper time to have a standing in court; if not it will be barred by a statute of limitation. This is not contrary to justice as might appear at first sight, but is rather in the interest of justice and serves the purpose of avoiding fraud. If one has a legal right, there is no hardship in requiring that it be asserted within some reasonable time, for it is often very difficult to establish the rights and wrongs of a matter if the question is left till many years have passed; with the lapse

of time the witnesses and evidence which are necessary to establish the real facts may become difficult to obtain, so that the decision may as a matter of legal necessity go to the party whose evidence has been the more enduring, rather than to the one who really had the rights of the case and who would have had at an earlier date the evidence to establish those rights. From the above considerations it may be seen that the effect of a clause which limits the time within which a policy may be contested by a company for any cause, including fraud, is in no sense a condoning of fraud, for it leaves a reasonable time for the company to investigate and discover if there is fraud, and if fraud is found, for it to bring an action to cancel the policy. On the other hand there may be the same advantage in such a clause as a prevention of fraud as in the statutes of limitation referred to just now. If a policy is allowed to run for a considerable term of years and then at death the validity of the policy is contested on the ground of fraud, it is apparent that satisfactory evidence will be much more difficult to obtain, particularly as the contest is made after the death of the insured, who would naturally be the one to know the facts of the case and what evidence existed. It may be urged that reputable insurance companies do not contest policies without good occasion and real fraud, but it must be borne in mind that not all companies are reputable, and that there are some which would use any possible means to avoid paying death claims.

That the view expressed above is generally accepted is evident when we consider both the decisions and the dicta of the various courts. The decisions are, I think, without exception in accordance with the views that have been expressed, and the dicta are in great preponderance on the same side. The case of Wright *v.* The Mutual Benefit Life Association of America, before the Court of Appeals of New York State in 1890 (118 N. Y. 237), is usually regarded as the leading case on this subject. The incontestable clause in the policy read "That no question as to the validity of an application or certificate shall be raised, unless such question be raised within the first two years from and after the date of such certificate of membership and during the life of the member named therein." The Court held that inasmuch as more than two years had elapsed the defence of fraud could not be availed of; part of the opinion was as follows: "It (the clause

quoted above) is not a stipulation absolute to waive all defences and to condone fraud. On the contrary, it recognizes fraud and all other defences but it provides ample time and opportunity within which they may be, but beyond which they may not be established. It is in the nature of and serves a similar purpose as statutes of limitations and repose, the wisdom of which is apparent to all reasonable minds. It is exemplified in the statute giving a certain period after the discovery of a fraud in which to apply for redress on account of it and in the law requiring prompt application after its discovery, if one would be relieved from a contract infected with fraud. The parties to a contract may provide for a shorter limitation thereon than that fixed by law and such an agreement is in accord with the policy of statutes of that character."

In the case of Massachusetts Benefit Life Association *v.* Robinson (104 Ga., 256), the court says, "As the law may prescribe such a limitation (of time) in which actions shall be brought by the party to be affected, it is also within the power of the contracting parties to agree among themselves upon a period which would amount to a statute of limitations, either greater or less than the period fixed by the law."

The court says, as a dictum, in Royal Circle *v.* Achterrath (204 Ill., 558). "Stipulations to the effect that a policy or certificate shall become incontestable for fraud in procuring the same, after the lapse of a specified period from the date of its issue, have been held valid as creating a short statute of limitations in favor of the insured, and as giving the insurer a limited period for the purpose of testing the validity of the policy."

In R. H. Clement *v.* New York Life Insurance Company (101 Tenn., 22), the court in speaking of a clause making the policy incontestable after a certain length of time says, "Such a stipulation is neither unreasonable nor contrary to public policy . . . . Fraud is always required to be set up promptly when discovered, or it may be treated as waived; and the effect of this stipulation is that the insurer must exercise due diligence to discover such fraud within the year, and, if it fails to do so, it will treat it as waived and no inquiry will be made or allowed into such matters." Although the court spoke as above in regard to the question of incontestability for fraud and in other passages which are too long

to quote spoke even more strongly the case was decided in favor of the company on the ground that the contract was a gambling contract in its inception and, therefore, its enforcement would be against public policy.

In Canada we have the case of "The North American Life Assurance Company *v.* Elson (33 C. S. C. Repts., 383): "I am of the opinion that the Supreme Court of British Columbia was right in holding that the object of the above incontestable clause was to provide an automatic cutting off at the end of the triennium . . . . and I would add further, of all defences arising out of any untrue or incorrect material statement made in the application for the policy." While this is not sufficient to commit this Court, it would indicate that it probably held the same view of the law as the American Courts, and would have decided as they have, had a similar case come before it.

Brady *v.* The Prudential Insurance Company (168 Pa., 645) is a somewhat peculiar case. The policy was incontestable by the terms of an incontestable clause, but contained a further clause that any suit must be brought within six months. This latter clause was held to be enforceable as not in any way contrary to the incontestable clause. Besides its direct result, the case is of interest to us as showing how far the courts go in favoring a clause of limitation.

It sometimes happens that a policy may state that suicide is not a risk assumed or that the policy is not payable in the case of suicide and, at the same time, have a clause making it incontestable after a given length of time. In general, it is held in such a case that the incontestable clause after the limit of time has expired makes the policy incontestable for suicide, notwithstanding the clause against suicide. Mareck *v.* The Mutual Reserve Fund Life Association (62 Minn., 39). Goodwin *v.* The Provident Savings Life Association Society (97 Ia., 226). Starck Adm'r *v.* Union Central Life Insurance Company (134 Pa., 45) is sometimes quoted *contra* but incorrectly since the wording of the clause made the policy incontestable only as to statements in the application.

The case of Anctil *v.* The Manufacturers' Life Insurance Company ([1899] A. C., 604) is sometimes quoted as being contrary to the views we have expressed. This is probably not so, although the Court (the British Privy Council) used the expression "that

the one year clause was contrary to public law and order," since this case was decided not on the ground of fraud but of public policy and of the Civil Code which required an insurable interest; the case consequently will be cited in the sub-division which relates to the question of "public policy" as a defence.

III.

There are some situations where a question of "public policy" is involved in the payment of a policy. This is particularly true in the so-called "wager" or "gambling" policies. These policies are ordinarily obtained only because the company does not know the full circumstances, for a company does not intentionally issue a policy such that its issuance is contrary to the spirit of the law. The cases which arise out of such policies are frequently complicated by fraud in the inception of the policy. Another class of cases where the payment of the policy comes under the question of public policy is where the insured is executed for a crime. Also the principle of public policy has been applied in some cases where the insured has committed suicide. It is not for our purpose, however, so important to consider what defences come under the head of public policy as how the courts deal with the incontestable clause when they decide that a question of public policy is involved in the payment of the insurance. There is little difference of opinion as to whether an incontestable clause setting a limit of time for contest would bar disputing a policy on the ground of public policy. The courts favor the principle that where the interest of the public is so essentially concerned, the right to contest can never be barred by the contract. This seems to be the better view, for while it may be the company instead of the state which makes the contest, the interests involved are more than private interests.

The cases upon this point are not altogether satisfactory, as in some of the most important ones other points enter into the question of the decision. One of the most noted of these cases is an English case, known as Fauntleroy's case, or Amicable Society *v.* Bolland (4 Bligh, N. S. 194, p. 211). In this case the insured was convicted of forgery and executed. The court, after speaking of the particulars of the case, says "Is it not void upon the plainest

principles of public policy?" The Supreme Court of the United States—Ritter *v.* Mutual Life Insurance Company (169 U. S., 139)—in speaking of this case, says "The doctrine asserted in Fauntleroy's case, that death by the hands of public justice, the punishment for the commission of crime, avoids a contract of life insurance, though it is not so expressed in the contract, has not, so far as we have examined, been questioned." The same Court says, in another part of the same case, "A contract, the tendency of which is to endanger the public interests or injuriously affect the public good, or which is subversive of sound morality, ought never to receive the sanction of a court of justice or be made the foundation of its judgment." While the case of which the Court was speaking was a case of suicide where the defence of public policy is not usually considered good, nevertheless, what the Court says upon the general subject of a defence of public policy has the same bearing.

In the case of Anctil *v.* Manufacturers' Life Insurance Company ([1899] A. C. 604), the judgment of the Supreme Court of Canada is upheld in sustaining the defence against a policy having a one year incontestable clause and which had been in force more than one year. The ground of defence was no insurable interest. The case is somewhat obscured, so far as the question of whether the decision was on the ground of public policy, by the fact that the Civil Code of Lower Canada specifically requires an insurable interest in the life upon which the insurance is effected, but, inasmuch as this provision of the Code is itself on the ground of public policy, rather than of any question of the private rights of the parties, I think the case may fairly be taken as indicating that had no such statute existed, the Court would nevertheless have given the same decision upon the general ground.

In Clement *v.* Insurance Company (101 Tennessee, 22), the case is that of concealing a gambling contract under an assignment made immediately after issue. The court holds that the incontestable clause does not estop the Company from contesting, apparently taking the ground largely that the incontestable clause did not run to the benefit of an assignee, so far as his assignment was concerned, but the question of public policy evidently is also kept in mind in the opinion, as the following quotation shows: "It can see no reason why the like advantage and benefit should be extended to

one who has no insurable interest in the insured, who does not take or claim in good faith, and whose entire connection with the matter is shown to have been for a speculative and fraudulent purpose, and no sound public policy can be subserved by so holding, but on the contrary, such holding would sanction wagering insurance contracts, to the greatest detriment of the public morals and public good."

In the New York Mutual Life Insurance Company *v.* Armstrong (117 U. S., 591), the insured had been murdered by the assignee of the policy for the sake of obtaining the insurance. The court says "But, independently of any proof of the motives of Hunter in obtaining the policy, and even assuming that they were just and proper, he forfeited all rights under it when, to secure its immediate payment, he murdered the assured. It would be a reproach to the jurisprudence of the country, if one could recover insurance money payable on the death of a party whose life he had feloniously taken. As well might he recover insurance money upon a building that he had willfully fired."

IV.

An entirely different situation arises when the person examined —purporting to be the applicant,—is really some one else impersonating the applicant. This is, of course, fraud, but as we have seen above, fraud is not sufficient to render a policy contestable which contains a clause making it incontestable after a reasonable time from date. Hence, fraud alone would not be sufficient cause to contest in this case. It is, however, evident that there is not only fraud here, but something more, which goes to the very essence of the contract. A policy has been written naming one person as the insured, while there has appeared before the company and, ordinarily also, has signed a part of the application, a different person, so that there is a real question as to who the person insured would be. The company, in issuing the contract, undoubtedly issued it with the intention of insuring the person who appeared before its medical examiner under a false name, and it was justified in regarding such person as the one whom it was insuring, whereas the person really named in the policy and who claims to be the insured is a different person. In other words, there has

been no agreement as to the subject matter and no meeting of the minds, and according to a most elementary principle of law, this means that no contract has been made, because the two parties have not had the same subject matter in view. Under such a condition of affairs, the incontestable clause could hardly be invoked, because no contract has been made. The side of a claimant is also certainly not helped by the fact that the failure to have the same subject matter in mind was due to deliberate deceit and fraud, practised by the one procuring the policy, on the basis of whose rights any claim must be made.

While there are no cases, and, so far as I know no dicta, there is little question as to how the courts will decide the law when such a case comes before them.

The Nature of the Contribution Principle Involved in Insurance Contracts.

BY WALTER S. NICHOLS.

This paper has been suggested by the ingenious mathematical analysis made by Professor Whitney of the fundamental principles of a contract of indemnity, in his able paper presented at our last meeting and is intended, in a measure, to supplement his discussion of that subject. The problem of apportionment of losses, which he presented, while it is one which primarily concerns the fire underwriter, is also involved in every branch of insurance whose contracts are of the nature of an indemnity. Whenever a definite moneyed loss is to be made good by more than one insurer under their contracts, the question arises in what proportion shall they contribute? We meet with it in marine, and tornado, and employers' liability, and it would be involved in both life and accident policies, if they were strictly contracts of indemnity. Indeed, it is easy to see how the question of contribution may be raised in life insurance through policies on the life of a debtor. Such policies are frequently like those of fire insurance contracts of indemnity, whose liability is limited by the amount of the debt and should this happen to be covered by more than one policy, whose aggregate amounts exceed the debt, the question may at once arise as to how they shall contribute. Indeed, it is easy to see how such insurances may be taken out in one company for the protection of combined interests, and in another for the protection merely of one of these interests, thus presenting the case of non-concurrent policies so familiar in fire underwriting.

Two fundamental characteristics, however, distinguish such life insurance policies as contracts of indemnity from the fire policy. The loss in the former is always total in so far as concerns the life which is the subject of insurance; it can only be partial as concerns the amount of debt which may be owing at the date of death. The loss, too, is instantaneous at the moment of death,

while in case of the fire policy it is progressive as the fire continues to burn. It is this latter feature of the fire policy which was taken advantage of by Professor Whitney, as the basis on which he has applied the principle of fluxions to the solution of the non-concurrent apportionment problem. This problem in contracts of indemnity, whether fire or marine, is almost as old as insurance itself, and has been dealt with in many different ways in the different countries, where fire and marine insurance has been practised. The earliest condition regulating the question of contribution was what is known as the principle of average, which is universally incorporated in marine policies, and in the early part of the last century found its way into various English fire policies, and has borne fruit in the familiar co-insurance clauses and the different forms of excess insurance often written by our American fire offices. Under this average provision, which is still in use among British fire offices, the insured is himself made a co-insurer for any excess of value in the property covered, above the amount of insurance which he carries on the same, and must contribute in that proportion to the loss. In almost all countries, however, the fire policies contain in one form or another a so-called contribution clause for the purpose of apportioning the losses among the underwriters themselves. The application of these clauses greatly vary in the different countries, and in nearly all of them arbitrary rules have been adopted by the adjusters with little regard to consistency, such rules obtaining among the companies such sanction as usage may give. Nowhere, I believe, has the principle involved received so much judicial consideration as in America. In the absence of such a clause, the insured may recover his entire loss within its policy against any company he may elect, leaving it to exact from the remaining co-insurers their share of the liability. In no country has it been allowed to reduce the amount of indemnity which the insured would otherwise receive, and the courts, especially in America, have always insisted on such a flexible construction of the clause as shall secure to the insured the fullest indemnity. This is one of the chief reasons why no fixed legal principle, aside from this, has ever been reached regarding its construction, which has varied according to the special peculiarities of each particular case.

Such, in brief, is the history of the contribution clause, which

lies at the foundation of the principle of apportionment suggested by Professor Whitney. That clause in the American fire insurance policy now reads as follows:

> "In case of any other insurance upon the property hereby insured, whether made prior or subsequent to the date of this policy, the insured shall be entitled to recover of this company, no greater proportion of the loss sustained, than the sum hereby insured bears to the whole amount insured thereon."

As in Great Britain, various rules have been employed by American adjusters for the construction of this clause, each usually instigated by the special advantages which would be secured to the company represented by the ruling spirit. Of course, where all the policies specifically apply to the same classes of property, no difficulty exists. Each contributes in proportion to the amount insured. The trouble arises when some of the policies include, as well, property not contained in others, or apply generally to several items, each of which may be specifically insured under another policy. Various principles have been employed for dealing with such cases. According to one rule, the compound policy, covering several items, should be divided according to the value of each item and treated as specific insurance for such amounts on each. According to another rule, the compound policy should be divided according to the loss. According to still another, the specific policies should first be exhausted and then the compound should simply be called on to make good any deficiency. Another would compel the compound to first make good and allow the specific policies to supply the deficiency. Another would first apply the compound policy to make good the items not protected by specific insurance, and then contribute according to its unexpended balance with the specific policies. Various other rules have been suggested according to the complexity of the cases.

Turning to the numerous judicial decisions that have been rendered, the confusion and inconsistency have been nearly as great. The chief difference has been that the adjusters naturally view the problem from the standpoint of the insurer, while the courts, as naturally, regard it from that of the insured and adopt as their

starting point his most complete protection. This has perhaps led them to regard with most favor the principle last enunciated. But in all cases, such apportionments have been made on the assumption that the damage done by a single conflagration was instantaneous and occurred at the instant when the fire first attacked the property and the liability began. This theory, of course, is false. It is a legal fiction, due primarily to a broader and well recognized legal principle, that the law will not attempt to separate into distinct parts that which under a contract is treated as an entirety and which admits of no sharp lines of separation. The law will not ordinarily deal with fractions of a day, when the day has contractually been treated as an entirety. But when a specific hour of the day has been stipulated, the law will recognize the fraction thus delimited. The law will not recognize the intervention of minor causes, where the chain of causation leads sharply and directly from the efficient cause to the result. The accident underwriter is liable for a resultant death from accident, though a disease, for which he is not liable, may have intervened to produce the fatal ending, if the disease is but part of the chain of causation leading direct from the injury to the death. But, if disease intrudes as an independent factor, to which the cause of death is properly attributable, the case is altered and no liability exists.

How far is this principle applicable in the case before us? For the purpose of fixing liability, the loss must be deemed to relate back to the inception of the fire. But the real reason for this must be sought in the presumed intention of the parties as expressed in the insurance contract. A policy ends at the noon hour of a certain day. The fire breaks out a moment before its expiration, while the actual damage is practically done after the contract has terminated by its terms. The intention of the parties was protection against such damage as should result from the inception of a fire during its continuance, says the court. The links of casualty are too close for a judicial separation. Hence the legal fiction of an instantaneous loss. No such legal fiction, however, is essential to construe the intention of the parties in their adoption of the contribution clause for the apportionment among the companies themselves. This is an independent question of equity and may well be treated by another legal fiction of instantaneous contributions to a constantly progressive loss, if

thus the spirit of equity contemplated by the clause can be best preserved. This is the position assumed by Professor Whitney, on which he has based his theory. But the same result can be reached in another way. The loss may be regarded as instantaneous, while the contributions for subsequent settlement flow progressively from the companies, like streams from reservoirs, until the claim has been fully satisfied, or the reservoirs have been exhausted. I know of no legal objection to this conception. On the contrary, the principles of equity would require its adoption, if in this way the spirit of the contribution clause could be best conserved. That clause is, as I have said, feasible in its character and is governed in its interpretation and application by the principles of equity rather than of law. It contemplates a fair and just pro-rata apportionment of the liability according to the conditions of the case, rather than the application of fixed rules which do not in all cases do justice. This is the spirit of equity which adapts its methods to the attainment of the desired end. For this reason no fixed legal rules have been laid down by the courts for the application of this clause, and the decisions cannot be harmonized. But other things being equal, if a definite, practical principle can be established for its application that will secure uniform justice, such a principle will be entitled to judicial preference.

How far does the theory here presented comply with this requisite? The theory is that the losses should be apportioned between contributing insurers on the assumption that the conflagration burns up equal values of each of the subjects insured as it progresses, and that the companies during each instant contribute proportionately from such portion of their liability as may not be already exhausted; or if the loss be assumed as instantaneous, that the companies steadily contribute in like manner from their unexhausted balance until it is made good. The first statement is, as a matter of fact, the legal construction already placed on the contract in a succession of independent fires, no matter how closely they may follow each other. After each fire the companies contribute according to their unexhausted balances, while the assumption that equal portions are destroyed in equal times, is certainly less of a legal fiction than the assumption now made by the courts that the loss is instantaneous, and finds a

justification in the familiar legal custom of treating as equal, things whose relative magnitude cannot be determined. Or, if we take the second assumption above, and treat the loss as instantaneous as the courts have done, the familiar analogy of reservoirs of indemnity, whose relative flow is determined by their contents, is in its essence but an extension of the very principle on which the contribution is now made. The whole question is one concerning what practical method will produce the best results, and be most consistent with itself.

Theoretically, at least, the treatment of the problem as has been done by Prof. Whitney, as a question of fluent quantities, seems to me superior to any other in its close analogy to the actual facts and its applicability to the varied conditions which may arise. It assumes that a policy covering more than one item, as for instance a building and its contents, was primarily intended by the parties to apply equally to its full amount on each, and that when a second policy is obtained covering but one of these items, as the building, it was not intended to disturb that equality of the former contract, except in so far as by giving greater protection to the building it may release a greater portion of the unexpended balance for the security of the contents. Such construction is in harmony with that already adopted by underwriters in some of their rules of apportionment mentioned above. The only remaining question is as to the practicality of the method. Courts are reluctant to adopt complicated processes for determining moneyed interests, when familiar commercial methods will produce a satisfactory result. Well recognized business rules, which will give substantial equity, will be given preference to such as by their abstruse character may be difficult of understanding by the ordinary business man. How far this objection may apply to the proposed method of dealing with the apportionment of fire losses as a series of differential equations to be evaluated, I am not prepared to say. To those familiar with the calculus, the principle involved is a simple one. A series of differential equations are found, expressing the relative flows of the various indemnity funds to make good a loss at each instant of time, whose summation gives the total flow, or contribution, from each. Whether the results can be expressed in formulas sufficiently simple and general to admit of ready and intelligent application by practical underwriters, unfamiliar with the cal-

culus, and whether the process itself can be made sufficiently clear to them to command their acquiescence, or that of a court passing on the problem, I am not prepared to express an opinion. But from a purely theoretical standpoint, the solution of the contribution problem, offered by Prof. Whitney, seems to me superior to any other in the completeness with which it embodies both the mathematical and legal principles concerned.

One important feature of this subject, however, still remains to be considered. It often happens that whatever rule of contribution may be applied, one policy will be exhausted, leaving a portion of the loss still unpaid with an unexhausted balance in another policy applicable to the payment. Thus to take the most simple case, one policy may as before insure a building and its contents and another simply the building. The loss on the latter may be so great as to not only absorb the entire second policy, but to leave a deficiency which can only be made good by drawing from the first more than its pro-rata share of the loss. Where a number of non-concurrent policies are involved covering various items, very complicated cases of this kind frequently arise, which since the insured must if possible be made whole, require subsequent reapportionments on the companies liable, perhaps on a contribution basis entirely different from that first adopted. How does the method here suggested meet this difficulty? I conceive that in adherence to its principle, such reapportionment would be made by the company still having an unexhausted balance, making good the deficiency, and where more than one such company is concerned, each contributing on the same principle as before. In fact the proposed method of contribution is strictly analogous to the physical problem of the flow of water from various reservoirs of equal diameter through small tubes at their base to fill empty tanks below. The velocity of the flow and the consequent contribution of each reservoir will be determined by the relative height of the water in each, at each instant of time until either the tanks are filled, or the reservoirs are exhausted, and the mathematical determination of such contribution is determined by the solution of differential equations analogous to those here employed.

While the problem here discussed is not perhaps in its narrower sense of a strictly actuarial nature, as remarked by Prof. Whitney, in a broader sense it is directly related to the work of the actuary,

who is called on to marshal the various surplus funds of his company and the interests of its members and determine the equitable principles, on which the former should contribute to the dividends on the latter. It is a life insurance problem, too, in the sense that, as I have attempted to show, a similar practical question may arise where either life, or accident policies may chance to be impressed with conditions which render them contracts of indemnity.

Actuarial Notes.

BY R. HENDERSON.

I.

On the Determination of the Rate of Interest Yielded by an Investment.

As the following formulas for determining the rate of interest are partially empirical, I do not propose to enter into any elaborate theoretical investigation of their accuracy. It will be sufficient to indicate certain theoretical considerations which had a controlling influence in the selection of the type of formula to be used. By way of preface it may be remarked that they are all based on the observed fact that the second differential coefficient of $\frac{1}{a_{\overline{n}|}}$ with respect to the rate of interest is small in proportion to the first, so that the ratio of $\frac{1}{a_{\overline{n}|}} - \frac{1}{n}$ to i is a very slowly varying quantity.

(1). *Annuities Certain*

Denote $\frac{1}{a^{(m)}_{\overline{n}|}} - \frac{1}{n}$ by h.

Then, where j is the nominal annual rate of yield convertible m times a year, we see that when j vanishes the limit of the ratio of h to j is $\frac{n + \frac{1}{m}}{2n}$, also that for $n = \frac{1}{m}$ or $n = \infty$ the ratio of h to j is unity. Keeping these conditions in mind the following empirical formula for j in terms of h was constructed

$$j = \frac{h}{\frac{n + \frac{1}{m}}{2n} + \frac{\left(n - \frac{1}{m}\right)h}{6 + 2\left(n - \frac{1}{m}\right)h}} \qquad \text{(a)}$$

After it had been constructed it was found to be theoretically correct to the second power of h and practically so to the third. The following table shows the results of applying the formula to the values of annual annuities for 10, 20 and 40 years at 4 and 5 per cent.

n	$i = .04$	$i = .05$
10	.03997	.04995
20	.03997	.04996
40	.04006	.05016

It is to be noted that no prepared tables of annuity values are used in this formula which still gives results with sufficient accuracy for most practical purposes.

The principle of using $\frac{1}{a_{\overline{n}|}}$ instead of $a_{\overline{n}|}$ may also be applied to the determination of the rate by interpolation between known values, the gain in accuracy being very great. The formula in this case becomes

$$\frac{i - i_1}{i_2 - i_1} = \frac{\frac{1}{a_{\overline{n}|}} - \frac{1}{{}^1a_{\overline{n}|}}}{\frac{1}{{}^2a_{\overline{n}|}} - \frac{1}{{}^1a_{\overline{n}|}}} = \frac{{}^2a_{\overline{n}|}}{a_{\overline{n}|}} \cdot \frac{{}^1a_{\overline{n}|} - a_{\overline{n}|}}{{}^1a_{\overline{n}|} - {}^2a_{\overline{n}|}} \qquad \text{(b)}$$

For the benefit of those who wish to compare these formulas with those given in the Text-book it may be stated that applied to the example there taken, where $n = 30$ and $a_{\overline{n}|} = 20$, formula (a) gives $j = .0284392$ while formula (b), interpolating between $2\frac{3}{4}$ per cent. and $2\frac{7}{8}$ per cent., gives $j = .0284453$, the true value being .0284464.

(2). *Bonds*

We have, where g is the rate of dividend on a bond, k is the premium at which it is purchased and j the rate realized. $k = (g - j)a^{(m)}_{\overline{n}|}$ where $a^{(m)}_{\overline{n}|}$ is computed at rate j. Whence $j = g - \frac{k}{a^{(m)}_{\overline{n}|}} = g - \frac{k}{n}$

$$- k\left(\frac{1}{a^{(m)}_{\overline{n}|}} - \frac{1}{n}\right) \text{ or } j\left(1 + k\frac{1}{j}\left(\frac{1}{a^{(m)}_{\overline{n}|}} - \frac{1}{n}\right)\right) = g - \frac{k}{n}$$

Here again we have the slowly varying function $\frac{1}{j}\left(\frac{1}{a^{(m)}_{\overline{n}|}} - \frac{1}{n}\right)$ for which, as k is generally not large, a constant may be substituted. A good average value to use unless n is very large is .6 giving the formula

$$j = \frac{g - \frac{k}{n}}{1 + .6k} \qquad \text{(c)}$$

Todhunter uses $\frac{n+1}{2n}$ instead of .6 but where n exceeds 5 the latter is as a rule more accurate. In this connection it is to be remembered that the n in Todhunter's formula is the number of periods not the number of years.

If the rate of interest is approximately known the following method may be adopted. Let j_1 be the approximate rate and ${}^1a^{(m)}_{\overline{n}|}$ the annuity value at that rate. Then

$$j(1 + lk) = g - k\left(\frac{1}{a^{(m)}_{\overline{n}|}} - lj\right).$$

If then l be so determined that the first differential coefficient of $\frac{1}{a^{(m)}_{\overline{n}|}} - lj$, with respect to j, vanishes, we may substitute $\frac{1}{{}^1a^{(m)}_{\overline{n}|}} - lj_1$ for $\frac{1}{a^{(m)}_{\overline{n}|}} - lj$ and we have

$$j(1 + lk) = g - \frac{k}{{}^1a^{(m)}_{\overline{n}|}} + lj_1k$$

or
$$(j - j_1)(1 + lk) = g - \frac{k}{{}^1a^{(m)}_{\overline{n}|}} - j_1$$

or
$$j = j_1 + \frac{g - \frac{k}{{}^1a^{(m)}_{\overline{n}|}} - j_1}{1 + lk} \qquad \text{(d)}$$

Ordinarily .7 will be found to be a sufficiently accurate value of l, but if greater refinement is required we may put

$$l = \frac{n + \frac{1}{m}}{2n} \cdot \frac{1 + .8\left(n - \frac{1}{m}\right)j_1}{1 + .4\left(n - \frac{1}{m}\right)j_1}$$

Applied to the Text-Book example, where $n = 50$, $g = .02$ and $k = \frac{1}{15}$, m being unity, formula (c) gives $j = .017949$ while, using .0175 as the trial rate and using $l = .7$, formula (d) gives $j = .017967$, the true value being .017968.

II.

The Present Value of an Annuity-Certain at Simple Interest.

A concise and easily computed expression for the present value at simple interest of an Annuity-certain is of great assistance on the rare occasions where such a computation is necessary.

Since $\log_e \frac{1+ni+\frac{1}{2}i}{1+ni-\frac{1}{2}i} = \frac{i}{1+ni} + \frac{1}{12}\frac{i^3}{(1+ni)^3} + \&c.,$

it is evident that $$a_{\overline{n}|} = \frac{1}{i}\log_e \frac{1+\frac{1}{2}i+ni}{1+\frac{1}{2}i}$$

gives a fairly close approximation but that it errs a little in excess. It has been found empirically that a good working adjustment to correct this error is to add $\frac{i^2}{20}$ to both numerator and denominator, thus we have

$$a_{\overline{n}|} = \frac{1}{i}\log_e \frac{1+\frac{1}{2}i+\frac{1}{20}i^2+ni}{1+\frac{1}{2}i+\frac{1}{20}i^2}$$

For $i = .06$ and $n = 30$ this formula gives $a_{\overline{n}|} = 16.8434$, the true value determined by discounting each payment being 16.8433.

If greater accuracy is desired a more exact correction may be applied by using the formula

$$a_{\overline{n}|} = \frac{1}{i}\log_e \frac{1+(n+\frac{1}{2})i}{1+\frac{1}{2}i} - \frac{i}{24}\left\{\frac{1}{(1+\frac{1}{2}i)^2} - \frac{1}{(1+n+\frac{1}{2}i)^2}\right\}$$

This formula is correct to the third power of i and applied to the case above cited gives $a_{\overline{n}|} = 16.843253$, the true value to six places being 16.843256.

III.

Parallel Proofs of Woolhouse's and Lubbock's Formulas.

The following proofs of Woolhouse's and Lubbock's formulas of approximate summation may be of interest as they are exactly parallel and avoid some of the difficulties which occur in other proofs.

Woolhouse's formula.

By Taylor's theorem we have

$$W_t - W_o = t\frac{dW_o}{dx} + \frac{t^2}{\lfloor 2} \cdot \frac{d^2W_o}{dx^2} + \frac{t^3}{\lfloor 3} \cdot \frac{d^3W_o}{dx^3} + \frac{t^4}{\lfloor 4} \cdot \frac{d^4W_o}{dx^4} + \&c.$$

Suppose now that $\frac{dW_x}{dx} = u_x$, then

$$W_t - W_o = tu_o + \frac{t^2}{\lfloor 2}\frac{du_o}{dx} + \frac{t^3}{\lfloor 3} \cdot \frac{d^2u_o}{dx^2} + \frac{t^4}{\lfloor 4}\frac{d^3u_o}{dx^3} + \&c.$$

Whence (where A_t, B_t, &c., remain to be determined)

$$tu_o = (W_t - W_o) + A_t(u_t - u_o) + B_t\left(\frac{du_t}{dx} - \frac{du_o}{dx}\right) + \&c.$$

$$tu_t = (W_{2t} - W_t) + A_t(u_{2t} - u_t) + B_t\left(\frac{du_{2t}}{dx} - \frac{du_t}{dx}\right) + \&c.$$

$$\vdots$$

$$tu_{(mn-1)t} = (W_{mnt} - W_{(mn-1)t}) + A_t(u_{mnt} - u_{(mn-1)t}) + B_t\left(\frac{du_{mnt}}{dx} - \frac{du_{(mn-1)t}}{dx}\right) + \&c.$$

Adding we get

$$t(u_o + u_t + \ldots u_{(mn-1)t}) = (W_{mnt} - W_o) + A_t(u_{mnt} - u_o) + B_t\left(\frac{du_{mnt}}{dx} - \frac{du_o}{dx}\right) + \&c.$$

Putting $t = \frac{1}{n}$ we get

$$\frac{1}{n}(u_o + u_{\frac{1}{n}} + \ldots u_{m-\frac{1}{n}}) = (W_m - W_o) + A_{\frac{1}{n}}(u_m - u_o) + B_{\frac{1}{n}}\left(\frac{du_m}{dx} - \frac{du_o}{dx}\right) + \&c.$$

Let $u = 1$ then

$$(u_o + u_1 + \ldots u_{n-1}) = (W_m - W_o) + A_1(u_m - u_o) + B_1\left(\frac{du_m}{dx} - \frac{du_o}{dx}\right) + \&c.$$

$$\frac{1}{n}(u_o + u_{\frac{1}{n}} + \ldots u_{m-\frac{1}{n}}) = (u_o + u_1 + \ldots u_{m-1}) + (A_{\frac{1}{n}} - A_1)(u_m - u_o) + (B_{\frac{1}{n}} - B_1)\left(\frac{du_m}{dx} - \frac{du_o}{dx}\right) + \&c.$$

The equations for determining the coefficients A_t, B_t, &c., are easily seen to be the same as those for determining the coefficients of the successive powers of y in the expansion of $\frac{ty}{e^{ty} - 1}$. So that

$(A_{\frac{1}{n}} - A_1)$, $(B_{\frac{1}{n}} - B_1)$ &c., are the coefficients of the powers of y in the expansion of $\dfrac{y}{n(e^{\frac{y}{n}} - 1)} - \dfrac{y}{e^y - 1}$. This expansion may be made by direct division and gives the coefficients of Woolhouse's Formula.

Lubbock's Formula.

We have

$$V_t - V_o = t\Delta V_o + \frac{t^{(2)}}{\lfloor 2} \Delta^2 V_o + \frac{t^{(3)}}{\lfloor 3} \Delta^3 V_o + \frac{t^{(4)}}{\lfloor 4} \Delta^4 V_o + \&c.$$

where $t^{(r)} = t(t-1) \ldots (t-r+1)$, and Δ applies to unit intervals. Suppose now that $\Delta V_x = u_x$, then

$$V_t - V_o = tu_o + \frac{t^{(2)}}{\lfloor 2} \Delta u_o + \frac{t^{(3)}}{\lfloor 3} \Delta^2 u_o + \frac{t^{(4)}}{\lfloor 4} \Delta^3 u_o + \&c.$$

Whence (where $A_{(t)}$, $B_{(t)}$ &c., remain to be determined)

$$\begin{aligned}
tu_o &= (V_t - V_o) + A_{(t)}(u_t - u_o) + B_{(t)}(\Delta u_t - \Delta u_o) + \&c. \\
tu_t &= (V_{2t} - V_t) + A_{(t)}(u_{2t} - u_t) + B_{(t)}(\Delta u_{2t} - \Delta u_t) + \&c. \\
&\vdots \\
tu_{(mn-1)t} &= (V_{mnt} - V_{(mn-1)t}) + A_{(t)}(u_{mnt} - u_{(mn-1)t}) \\
&\qquad + B_{(t)}(\Delta u_{mnt} - \Delta u_{(mn-1)t}) + \&c.
\end{aligned}$$

Adding we get

$$t(u_o + u_t + \ldots u_{(mn-1)t}) = (V_{mnt} - V_o) + A_{(t)}(u_{mnt} - u_o) + B_{(t)}(\Delta u_{mnt} - \Delta u_o) + \&c.$$

Putting $t = \dfrac{1}{n}$ we get

$$\frac{1}{n}(u_o + u_{\frac{1}{n}} + u_{\frac{2}{n}} + \ldots u_{m-\frac{1}{n}}) = (V_m - V_o) + A_{(\frac{1}{n})}(u_m - u_o) + B_{(\frac{1}{n})}(\Delta u_m - \Delta u_o) + \&c.$$

Let $n = 1$, then

$$(u_o + u_1 + \ldots u_{m-1}) = (V_m - V_o) + A_{(1)}(u_m - u_o) + B_{(1)}(\Delta u_m - \Delta u_o) + \&c.$$

$$\frac{1}{n}(u_o + u_{\frac{1}{n}} + \ldots u_{m-\frac{1}{n}}) = (u_o + u_1 + \ldots u_{m-1}) + (A_{(\frac{1}{n})} - A_{(1)})(u_m - u_o) + (B_{(\frac{1}{n})} - B_{(1)})(\Delta u_m - \Delta u_o) + \&c.$$

The equations for determining the coefficients $A_{(t)}$, $B_{(t)}$ &c., are easily seen to be the same as those for determining the coefficients of

the successive powers of y in the expansion of $\frac{ty}{(1+y)^t-1}$. So that $(A_{(\frac{1}{n})}-A_{(1)})$, $(B_{(\frac{1}{n})}-B_{(1)})$ &c., are the coefficients of the powers of y in the expansion of $\frac{y}{n(1+y)^{\frac{1}{n}}-1}-1$. This expansion may be made by direct division and gives the coefficients of Lubbock's Formula.

IV.

Application of Makeham's Law to Survivorship Assurance Problems.

Formula 38 of Chapter XIII of the Institute Text Book may be easily modified to apply to the case where the law of mortality is different for the two lives, provided the value of c remains the same.

We have

$$\mu_{x+t} = -\log_e s_1 - \log_e g_1 \log_e c\, c^{x+t}$$
$$\mu_{y+t} = -\log_e s_2 - \log_e g_2 \log_e c\, c^{y+t}$$

So that

$$\frac{\mu_{x+t}+\log_e s_1}{\log_e g_1\, c^x} = \frac{\mu_{y+t}+\log_e s_2}{\log_e g_2\, c^y}$$

or $\quad h(\mu_{x+t}+\log_e s_1) = \mu_{y+t}+\log_e s_2$ where $h = \frac{\log_e g_2}{\log_e g_1}\, c^{y-x}$

Multiplying both side by $v^t {}_tp_{xy}dt$ and integrating

we get $\quad h(\bar{A}^1_{xy}+\log_e s_1\, \bar{a}_{xy}) = \bar{A}^1_{xy}+\log_e s_2\, \bar{a}_{xy}$

Also $\quad \bar{A}^1_{xy}+\bar{A}^1_{xy} = \bar{A}_{xy}$ or $\bar{A}^1_{xy} = \bar{A}_{xy}-\bar{A}^1_{xy}$

So that substituting and collecting like terms we have

$$\bar{A}^1_{xy}(1+h) = \bar{A}_{xy}+\bar{a}_{xy}(\log_e s_2 - h\log_e s_1)$$

But $\quad \bar{a}_{xy} = \frac{1}{\delta}(1-\bar{A}_{xy})$

So that if we put

$$\frac{1}{\delta}\cdot\frac{\log_e s_2 - h\log_e s_1}{1+h} = L$$

and

$$\frac{1}{1+h}-L = K$$

we have

$$\bar{A}^1_{xy} = K\bar{A}_{xy}+L$$

Whence approximately

$$A^{1}_{xy} = KA_{xy} + \frac{d}{\delta} L$$

$$M^{1}_{xy} = KM_{xy} + \frac{d}{\delta} LD_{xy}$$

$$R^{1}_{xy} = KR_{xy} + \frac{d}{\delta} LN_{xy}$$

$$(IA)^{1}_{xy} = K(IA)_{xy} + \frac{d}{\delta} L(1 + a_{xy})$$

$$P(IA)^{1}_{xy} = KP(IA)_{xy} + \frac{d}{\delta} L$$

This formula will be found useful in connection with the return premium element of Survivorship Annuities when valued by mortality tables following Makeham's law.

Abstract of the Discussion of Papers Read at the Previous Meeting.

THE RATE OF SICKNESS—HIRAM J. MESSENGER.

Vol. X, Page 371.

Written Discussion.

Mr. Young: When Mr. Gore asked me to make some remarks on Mr. Messenger's paper I think he had in mind that I might be able to obtain the sickness experience of some British company to compare with the experience of the Travelers. Many British companies have recently added a department of sickness insurance to their business, but few have had any considerable experience. There is, however, one company that has had an extensive experience, namely, "The Century Insurance Company." This company started twenty-three years ago with the special purpose of insuring against sickness, and has been ably and progressively managed. A comparison of its sickness experience with that of the Travelers would be interesting and instructive, but, unfortunately, it is not available at the present time. It is especially to be regretted that there is no considerable experience of a similar character with which to compare that of the Travelers, as many of the results of that experience are unexpected, or, rather, contrary to presupposed notions. As Mr. Messenger points out, comparisons with the extensive experiences of the British Friendly Societies are misleading—it is as if we compared the mortality of the Travelers with that of an industrial company. Besides the right to cancel the contract (a right, I may remark, that the "Century" Company does not reserve), the limitation of the benefit to twenty-six weeks, and the larger effect of feigned sickness on the experience of the insurance company, there is the difference in the occupations of the insured in the Travelers and those in the Manchester Unity, which would alone vitiate any conclusions we might arrive at from a comparison of their sickness experiences.

The subject of Mr. Messenger's paper is one with which few of us have had an opportunity to acquire practical experience. The paper is doubly valuable to us on this account, and the Society is appreciative of the spirit shown in publishing this experience and is deeply indebted to Mr. Messenger for the trouble he has taken in preparing the figures. As Mr. Messenger is a pioneer on this subject, there is little that one can say with reference to the tables or the inferences that are drawn from them. I think it is to be regretted that Mr. Messenger does not give us, in

addition to the number of weeks' sickness for each year of exposure, the actual number of years of exposure for each age group; also, it would have been interesting if he had given us the number of cases of sickness in each of the various age groups, or the average duration of the sickness. The latter would have been helpful in making comparisons such as of the relative healthfulness of the North and South. A knowledge of the actual number of years of exposure for each age group is very necessary in weighing the evidence which the figures contain, that the rate of sickness does not, like the rate of mortality, increase with the age. This evidence is to me the most interesting feature of the paper. As Mr. Messenger points out, it had previously been supposed that the rate of sickness gradually increased with increasing age. This supposition is a very natural one, and I should suppose a very convenient one in the practice of the business of sickness insurance. It is so natural as to appeal to the mind almost as an axiom, and the extensive statistics of the Friendly Societies have confirmed it. The fact that the Travelers' experience is mainly in the first few years after selection does not in itself explain this peculiar feature, for the selection would show most favorably at the earliest ages where there was no experience except that immediately following selection. I do not think either that the fact that the Special Health Policy is limited to certain diseases would explain the decreasing rate of sickness, in the experience under that Policy. Considering the diseases that make up the majority of the claims (viz., typhoid fever, pneumonia, and pleurisy, as stated in Mr. Mr. Messenger's paper read at the 4th Congress), it may be true that typhoid fever is most common during adolescence and the first decade of adult life; but, on the other hand, pleurisy and pneumonia probably reach their maximum frequency at about 35 to 45. Had tuberculosis been included in the list of diseases on account of which benefits would be paid, its effect would undoubtedly have been to emphasize this feature of the statistics.

Let us consider a group of lives in early manhood, of similar occupation and living in the same locality, and suppose (as is very probable) that the rate of sickness among them would be found to increase with advancing age. If we add to this group other groups of similar occupation and living in similar localities, but entering at later ages, and if we eliminate the experience immediately following selection, so that the total group remains fairly homogeneous, we should expect still to find that the rate of sickness increased with advancing age. It is interesting to inquire to what degree the Travelers' data would fulfill the condition of this hypothetical group. To what extent is it likely that the lives in that experience make a homogeneous group? The average duration is so short that the experience between, say, 30 and 35, is probably almost confined to lives entering at those ages; in fact, it will not be far from the truth to treat the rate of sickness for the various age groups as relating solely to the groups of lives selected at the ages in the group, and the question arises, whether the lives who apply

for sickness insurance between, say, 25 and 30 are a similar group to those applying between, say, 45 and 50; and, further, whether the selection would be equally effective for these two groups. The first question could not be answered without a further analysis of the statistics, but it seems probable that the group entering at 45 to 50 would contain the larger percentage of employers, and the group entering at 25 to 30 the larger percentage of employees; and, as pointed out by Mr. Brown of the "Century" Company, sickness among employers has less pressure on the "financial" rate of sickness than that among employees. Further, the older age groups would probably contain a larger percentage of lives engaged in professional occupations (in which the rate of sickness is admittedly very light) than the groups entering at the younger ages. Again, in selecting lives for sickness insurance, the principles to be followed no doubt differ somewhat from those which guide us in accepting risks for life insurance. Such ailments as the applicant may have had in the past will, I should suppose, have considerable weight in determining acceptance or rejection for sickness insurance, and this history of past sickness will have much greater weight in the case of entrants in middle life than among those in early manhood. It seems to me not unlikely that a skilful selection of lives for sickness insurance would be more effective in the later age groups than in the earlier.

My conclusion is that the evidence that the rate of sickness does not increase with advancing age will not be convincing without a further analysis of the statistics; and, further, that the statistics will not likely afford conclusive evidence until the average duration of the policies is much longer than at present.

That the rate of sickness is likely to follow the rate of mortality finds confirmation in the statistics of sickness by geographical divisions, and I think the inference that there is less sickness in the North than in the South will be generally acceptable. Mr. Messenger infers from the lower rate of sickness in the industrial indoor group as compared with the industrial outdoor group that the evidence of his statistics is contrary to the often expressed opinion that it is much healthier to work outdoors in the pure fresh air than to follow an indoor occupation. If we admit that the rate of sickness in the South is greater than that in the North among similar classes, the inference referred to would only be warranted if the industrial indoor group contained about the same percentage of Southern lives as the industrial outdoor group, and if the age distribution of the two groups were similar. It seems to me likely that the industrial outdoor group will contain the larger percentage of Southern lives. Further, certain kinds of sickness which would disable in the case of an outdoor occupation would not disable at all, or only for a short time, if the occupation were indoors. In other words, the same amount of sickness might lead to larger claims in the outdoor class than in the indoor class.

Turning to the statistics by density of population, Mr. Messenger infers from the figures that the result is contrary to the general im-

pression that the large cities are not as healthy as the small cities and country towns. This inference would only be warranted if the percentage of Northern lives and the percentage of persons engaged in professional and clerical occupations in the larger cities were similar to those percentages for the small towns. It seems to me very probable that the lower rate of sickness in the larger cities may be due to the group of lives insured there containing the larger percentage of persons engaged in professional occupations and also a great majority of Northern lives.

The favorable experience under the Special Health Policy among men engaged in the liquor business is very different from the usual mortality experience among such lives, but I think it admits of explanation. The diseases insured against under that Policy were not such as would be caused by indulgence in alcoholic liquor. Under the Special Health Policy the largest percentage of claims was due to typhoid fever, but the most prolific cause of that disease is water drinking.

In obtaining from the sickness statistics of an insurance company information as to the relative sickness of different localities and occupations, or evidence of a law of sickness, the effect of the enormous proportion of feigned sickness cannot be ignored, and it is doubtful if the effect of this element could be eliminated in a study of the statistics.

Mr. Hann: The results of Mr. Messenger's investigation are so unlike anything of the kind we have been used to see, that one is lead to think there are causes at work which tend to conceal the real facts.

The number of weeks' sickness for each year's exposure shows the rate of sickness to *decrease* as the age *advances*. This is a reversal of any law of sickness we have seen derived from an extended experience. There surely must be some connection between the law of sickness and that of mortality.

Column K, gives the experience of the Manchester Unity, with its membership of about one million. It is shown here in a marked degree, that sickness materially increases with age, and this is a strong characteristic of all such investigations in Great Britain. It should be kept in mind that the Manchester Unity covers disability from accident in addition to sick benefit. If the former were eliminated, it would not greatly affect the relative increase with the age, since the young are as liable to many classes of accidents as the old.

It is stated by the author that the insured under accident policies in the Travelers' are largely drawn from professional men and clerks. We know that those in the British companies are largely drawn from the working classes. This is an important difference. The class of insured in the former would scarcely go on the fund for ailments of short duration, nor would there be likely to be much 'malingering' among them.

Again these are much better able to follow their occupations under comparatively slight ailments than those engaged in heavy labor. Hence, in the aggregate, there is considerable sickness that does not get on record.

The rate of sickness at the respective ages, in extensive experiences, always gravitates towards a constant ratio to the annual deaths. It is well known that the total expectation of sickness for a large portion of life, varies but little—it is equally well-known that with advancing age attacks of sickness become more frequent and of longer duration which finally overcome the power to resist destruction.

The author's valuable paper records the features of a short and somewhat limited experience under the most favorable conditions, but as it becomes older and comprises a larger proportionate number of the working classes, the results will doubtless reflect those characteristics that prevail among similar risks. The right of the insurer to cancel contracts has also an important bearing on the results. This might be exercised more largely among those of middle and advanced age.

Mr. J. M. Craig: I notice that one organization engaged in the business of health insurance was not mentioned by Mr. Messenger. This is not surprising because its reports were never published. The health insurance feature although carried on separately by a Society known as the Hildese Bund, was, so far as the individuals were concerned, inseparably connected with life insurance, and this latter feature of the business was placed with the company with which I am connected.

This Society had about 15,000 members in 1873, which was probably a larger membership than any similar society at that time. The sick benefits guaranteed were not limited to any specific forms of disease, but embraced all forms of sickness. The amount of weekly benefits ranged from $7.00 to $35.00 depending on the amount of insurance.

A person carrying $5,000 of insurance or over could have a weekly sick benefit of $35.00 or less as he chose to pay for. Full benefit was limited to 26 weeks and then dropped to one-half benefit for 26 weeks, and then dropped again to one-quarter benefit for 52 weeks, when the operation would be repeated—that is to say, if a person were permanently sick, he would get full benefit for 26 weeks, one-half benefit for the following 26 weeks and one-quarter benefit for the following 52 weeks, when he would begin again to receive full benefit for 26 weeks, etc.

In 1874 the maximum weekly benefit was reduced from $35.00 to $24.50 and in 1879 the minimum was reduced from $7.00 to $6.00 and the maximum from $24.50 to $9.00. This radical reduction in benefits was made at the reorganization of the Society and was thought necessary because of the depletion of its funds.

In 1896 the period for the renewal of the benefits was changed from two years to three years by granting 26 weeks full benefit, 26 weeks one-half benefit and 104 weeks one-quarter benefit instead of 52 weeks one-quarter benefit under the previous rule.

The Society is still in existence under a different name but has not been actively engaged in securing new business for many years and has but few members left.

MR. MESSENGER: I find that my paper on "The Rate of Sickness" has been somewhat misunderstood. I was very careful to qualify many of my statements—particularly where conclusions were drawn—but I fear that many on reading the paper have paid particular attention to the general conclusions and disregarded the qualifications.

On page 378 I say, "The general conclusion drawn from these various experiences given in Columns A. to J., inclusive, is that there is no more and may be less sickness at ages thirty-five to fifty than at ages eighteen to twenty-five," but a little further on I call special attention to the fact that these experiences represent new business, that they represent mainly select classes, that they do not cover disability from accident and that the contract gives the company the right to cancel the policy at any time upon returning the unearned premium. I did not intend to say and I do not think that anything in my paper can fairly be interpreted as meaning that among the general population there is no more and may be less sickness at ages thirty-five to fifty than at ages eighteen to twenty-five. I do think, however, that the health experiences submitted in my paper furnish very strong evidence that in the case of health risks selected at ages thirty-five to fifty there is no more and may be less sickness during the first five years than among risks selected from the same classes and under practically the same conditions except at ages eighteen to twenty-five; but the rate of sickness twenty years later on the survivors of these risks selected at ages eighteen to twenty-five would be a very different thing.

It should be borne in mind that the Travelers' health experience is a comparatively new experience and the more I study it the more I doubt whether it will ever become what properly might be called an old experience. Since the preparation of this paper I have added another year's exposure—making a total exposure of 82,131 years for the special health policy in place of 72,776 years, 28,736 for the instalment special health policy instead of 26,552 years and 19,356 for the general health policy instead of 12,373 years. The result was no important changes in the various rates of sickness—and of the small resulting changes the great majority tended to make the experience slightly more favorable. The addition of another year's experience then tends to confirm the conclusions drawn in my paper; but I trust it will be borne in mind that most of these conclusions are in regard to the health experience of the Travelers' Insurance Company and should not be applied to experiences where conditions are fundamentally different except with the greatest caution.

AN INQUIRY INTO THE NATURE OF THE FUNDAMENTAL PRINCIPLES OF A CONTRACT OF INDEMNITY—ALBERT W. WHITNEY.

VOL. X, PAGE 383.

WRITTEN DISCUSSION.

MR. HENDERSON: This paper is an example of the application of higher mathematics to the solution of problems in connection with fire insurance. After some investigation of the mathematical problem involved, it appears to me, however, that if any solution is ever arrived at, on the lines laid down in the paper, of the general problem, it would be of so complicated a nature as to seriously militate against its practical usefulness.

I have not been able to make any material progress towards the solution of the general problem. There is, however, a general principle which may throw some light on the solution. From equations (7) on page 392 it will be seen that when any one of the funds become exhausted the corresponding value of Z is necessarily infinite. Referring then to equations (6) it will be seen that some one of the quantities Z_1, Z_2, &c., for which the existence coefficient is unity in the corresponding equation, must vanish. Referring then to equations (3), in order that this may be the case, every fund, contributing to the loss to which the Z refers, must be exhausted at the same time.

The basis, however, of all this mathematical reasoning does not appear to me to be entirely satisfactory, and in particular axiom 2, as interpreted in the subsequent formulas, appears to me as an "onlooker" to be unduly onerous to the blanket policies, as they are required to contribute to each separate item of loss as fully as if they had to bear the burden of no other loss. It would appear to me more equitable if a slight change were made in the wording so that it would read as follows:

> "When a loss is contributed to by more than one indemnity fund the total flow from each fund during a given instant is proportional to the size of the fund at that instant."

In applying this axiom it will be necessary to observe that it does not follow that an indemnity fund is contributing to a loss, because the property being destroyed is covered by the corresponding policy, as the total equitable contribution of the indemnity fund may be required to meet losses occurring on other property covered by the corresponding policy which is not otherwise protected. With this modification of the axiom it will be seen that the proportions of the funds contributing to any particular loss remain unaltered, so that no complicated application of the infinitesimal calculus is required; all that is necessary is to analyze the various

policies and losses, so as to see that a uniform assessment would not divert the contribution of any particular fund to cover a loss on property not included in the corresponding policy. This can probably be most conveniently done in one or other of two ways:

1*st Way.*

Consider every possible combination of one or more of the policies entering into the problem and, with respect to each combination, calculate the percentage which the total loss incurred on all the property, insured by the combination, bears to the total amount of insurance. If any of these ratios are less than the ratio for the combination of all policies together, the combination giving the smallest ratio should be treated as a separate proposition, all of the losses covered by the policies entering into the combination being assessed against those policies and those only, the remaining policies being applied to cover the remaining losses.

2*nd Way.*

Consider every possible combination of one or more of the individual losses, and for each combination compute the ratio of the total amount of insurance policies, covering those losses, to the total amount of the loss. If any of these ratios are smaller than the ratio for the total losses taken together, the combination, for which the ratio is the smallest, should be treated as a separate proposition, the corresponding indemnity funds being limited to that particular combination of losses, and the remaining losses being provided for entirely out of the remaining indemnity funds.

This method of apportionment would always give results consistent with the principal laid down by the courts that the adjustment should be so made as to give the greatest possible indemnity, which principle would not always be complied with by an adjustment based on Professor Whitney's formula.

The whole of the preceding argument, like the argument in Professor Whitney's paper, is based on the assumption that none of the policies contain a co-insurance clause. Where this clause occurs in any of the policies it would impose an additional restriction on the amount which could be assessed against the particular policy in which it occurs.

MORTALITY EXPERIENCE IN TROPICAL AND SEMI-TROPICAL COUNTRIES—
ARTHUR HUNTER.

VOL. X, PAGE 395.

WRITTEN DISCUSSION.

MR. VAN CISE: Mr. Hunter certainly deserves great credit and the hearty thanks of the Society for the large amount of work which was necessarily performed in the preparation of his paper, and for the very valuable information contained therein.

I regret that I cannot agree with Mr. Hunter in quite all of his preliminary observations, although, in the main, they are entirely correct. He speaks of the difference in the rate of mortality as shown by the "Specialized Mortality Investigation" between natives of Ireland and Scandinavia, and seems to account for the excess of the former over the latter, by their difference in environment in the United States and Canada. In my judgment, this will not account for the difference. Many Scandinavians live in the cities, and those who live in the country, probably work harder than the insured Irishmen who live in our metropolitan districts. I am inclined to believe that the difference in the rates of mortality in America is about the same as in Scandinavia and Ireland, and, therefore, the reasons for this variation will have to be sought for in some way other than by difference in environment.

Neither can I entirely agree with Mr. Hunter in his judgment as to the effect of wine drinking on the Latin races, both in Europe and tropical America. Although the consumption of alcohol may be very moderate, and cases of drunkenness unusual as compared with colder climates, yet, in my opinion, moderate drinking tends to increase the rate of mortality, and total abstinence in the tropics is even more beneficial than in Great Britain, where the experience of "The United Kingdom Temperance and General Provident Institution," for fifty consecutive years, has shown a much lower death rate among total abstainers than among moderate drinkers.

I fully agree with Mr. Hunter, however, as to the great difficulties which must be encountered in any effort made to determine the effects of climate and race on differences in rates of mortality. I believe, however, that it may be laid down, as a general rule, that the warmer the climate, the higher the rate of mortality among adult lives. Longer life seems to be granted to the residents of countries where the Winters are long and severe, as some compensation for the hardships which must be borne, and the greater effort which must be made in the struggle for existence.

The company with which I am connected—the Equitable Life Assurance Society of the United States—formerly did quite a large business in tropical and semi-tropical countries. A few years ago our mortality

experience, up to 1899, was completed under the supervision of our Associate Actuary, Mr. R. G. Hann, and without going into lengthy details, I will here mention some of the results of his computations.

Our premium rates have been practically the same as those of the New York Life in all countries south of the United States, but in order to be conservative, 125% of the American Table in semi-tropical countries, and 150% in tropical countries were assumed as the mortality for which the premium rates would safely provide, with due consideration for the expense strain.

In South America, considered as a whole, the expected loss on a total amount exposed of $650,595,936. was, on the basis just mentioned, $9,744,255., against an actual loss of $9,599,040., or about 98½% of the expected. According to Mr. Hunter's calculation that the premium rates would provide for 150% of the American Table in semi-tropical countries, and 200% in tropical countries, the deaths might have been about $12,000,000., without loss to the Society.

Compared with our assumption of a loading of 25% of the American Table for semi-tropical, and 50% for tropical, countries, the percentage of the actual to the expected mortality for the first year was 75¾; for the second year, over 86; for the first five years, 87⅓, and thereafter, 110¾. The heaviest mortality was upon those entering in middle life, being 109½%.

Chile was the country having the largest amount of exposures, being $293,000,000. This is, of course, considered as a semi-tropical country. The percentage of the actual to the expected was 110, but in this connection it should be stated that nearly all the business had been done on the Ordinary Life plan. Had a large portion been transacted on the Endowment plan, the death rate would undoubtedly have been lower.

It is somewhat surprising that the most favorable experience was in the United States of Colombia, where on $48,500,000. of exposures, the death rate was only 67% of the loaded table. (50% of the American, being a tropical country.) This is partly explained by the comparatively high character of the lives insured, and the care with which the agency for that country was managed.

The Argentine Republic also made quite a favorable showing, as on $98,000,000. of exposures, the actual rate was less than 77% of the American Table loaded 25%, this being taken as the expected rate in semi-tropical countries.

In Brazil, on $92,000,000. of exposures, the percentage was 103¾ of the expected; that is, the American Table loaded 50%; and a prominent feature in this business was the heavy mortality experienced during the period of selection.

In Uruguay, which we consider a semi-tropical country, the amount exposed was a little over $28,000,000., and the mortality was 90.01% of the expected by the loaded American Table.

The business done in other parts of South America, was not in sufficient volume to afford reliable data as to the mortality to be expected.

In Central America, on business issued since 1884, at full tropical rates of premium, the actual to the expected mortality, (American Table loaded 50%) was only 72½% on nearly $82,000,000. of exposures. As in the case of Colombia, however, I attribute this comparatively favorable mortality, in a great measure, to the high character of the general agent through whom the business was done.

In concluding these comments, I wish to unite with Mr. Hunter in cautioning all who examine figures giving tropical experience, from attributing too much importance to them. Special causes, such as the character of the agents, the particular localities where the larger part of the business was done, and the kinds of policies issued, must have had great influence on the rates of mortality experienced, and, therefore, while general conclusions can be drawn from the data now attainable, it will probably be many years before the expected rates of mortality in tropical and semi-tropical countries can be computed with the same degree of accuracy as similar tables for the United States and Canada.

Mr. Wood: Mr. Hunter's paper is but the second contribution to the *Transactions* of the Society on the important subject of tropical mortality and like the former article by Mr. Jones deals with the experience of the New York Life Insurance Company. To those of us who are connected with companies actively engaged in transacting business in the tropics, this subject has a particular interest, but as every actuary is at times called upon to quote rates for foreign residence, none of us can afford to allow Mr. Hunter's paper to pass without careful study.

While the mortality rates prevailing in temperate climates have been very accurately ascertained through the combined experiences of many companies, statistics relating to the mortality in most unhealthy climates are extremely scant, our knowledge of tropical mortality being practically restricted to a few published experiences of individual companies. As a result of this lack of knowledge we find wide variations in the rates of premiums charged by different companies, even in some of the most important sections of the tropics such as the West Indies and British India. Under these circumstances the importance of ascertaining as accurately as possible not merely the general nature of tropical mortality but also the rates prevailing in various sections is apparent, and any additional light which can be thrown upon the subject is heartily welcomed. I feel, therefore, that we are all under a deep obligation to Mr. Hunter and to the New York Life Insurance Company for the publication of these extremely valuable statistics. The whole subject of tropical mortality has been dealt with by the author in the most comprehensive manner and the experience itself is the most extensive that has ever been published on this subject.

The first part of the paper showing the rates of premium charged in various parts of the world will, as the author suggests, serve as a guide to those who are occasionally asked to quote premiums for foreign residence, and in such cases the actuary cannot go far astray in simply

adopting the practice of a company which has had such an extensive experience. This information is also of importance in enabling us to form some conception of the wide area from which the material used in the construction of the final tropical and semi-tropical tables has been drawn, and the many different classes of risks which have contributed their data to the experience. Mr. Hunter shows that the tropical and sub-tropical rates heretofore charged have been sufficient in the aggregate to provide for double the American and one and one-half times the American rates of mortality respectively. In fact as is pointed out by the author the margin between the provision for mortality and the actual experience of the Company has been just about the same proportion as exists in well-managed companies in the North between their actual experience and the American rates of mortality on which the premiums are based.

The experience has been investigated both on the basis of the amounts assured and by lives, the ratios of actual to expected deaths by three standard tables being given according to policy years. The most striking feature in connection with the experience by amounts is the high death rate during the early policy years. In the case of the tropical business the ratio of actual to expected losses by the Jones' table is 93% in the third year of assurance and 95% in the fourth, while for all years after the fifth it is only 87%. In the case of the semi-tropical experience the ratio is 71% in the second year and 73% in the third, but only 65% for the years 6-20 combined. This unusual feature appearing in both the tropical and semi-tropical sections is hardly due to ordinary fluctuations in amounts. It is probably best explained as illustrating the effect of moral hazard, which as the author mentions in another part of his paper, is a factor of the utmost importance in connection with tropical business. It would appear that in spite of the care exercised by the Company some doubtful lives have succeeded in obtaining insurance for policies of comparatively large amount, and their presence is revealed in the heavy death rate in the early years of assurance.

The investigation according to lives is the more important part of the paper, not only because the data are more extensive but because they form the basis on which Mr. Hunter has constructed his graduated tables. Here also our attention is drawn to the comparatively short period during which medical selection appears to be effective in tropical business, the ultimate rate being practically attained in the third policy year. The semi-tropical table, however, does not present the same characteristics as appear in the experience by amounts, for here selection is distinctly traceable during the first five years of assurance.

Medical selection is undoubtedly not so potent a factor in reducing the mortality in the early policy years in the tropics as in temperate climates, for the extra mortality is largely due to diseases which cannot be eliminated by selection. Nevertheless, even in tropical business, we would hardly expect to find the ultimate rate attained as early as the third

year, and previous investigations at least as far as West Indian mortality is concerned do not seem to support this conclusion. The duration of selection will undoubtedly vary not only in different sections of the tropics but also among different classes of lives. It will also depend largely upon the methods of doing business and the degree in which the risk due to moral hazard is involved. It is probable, for instance, that selection continues for a longer period among native-born inhabitants of a tropical country than among foreign residents from a temperate climate; and consequently the inclusion of unacclimated lives in any experience would cause an abnormally heavy death rate in the early policy years. Then again a heavy mortality would probably occur in the early years of assurance in any class of risks in connection with which the moral hazard is particularly great, such for example as Asiatic lives insuring at ages 45 and over. It is hard to believe, however, that under normal conditions selection wears out in two years even in the tropics.

In the following table are given the ratios of actual to expected deaths according to years of assurance in the Scottish Amicable, the Barbados Mutual and the Jones' Tropical (Lives) as well as the corresponding figures for the present tropical experience.

Ratios of Actual to Expected Deaths by the Standards mentioned below.

Years of Assurance.	Scottish Amicable. % of Northampton.	Barbados Mutual. % of H^{m}.	Jones' Tropical (Lives). % of Am. Trop.	Present Tropical (Lives). % of Am. Trop.
1)	70	.93	65	70
2)				83
3 - 5	100	1.22	88	91
6 - end	114	1.59	100	94
1 - end	103	1.47	85	88

It will be observed that in both the Scottish Amicable and the Barbadoes Mutual experiences, selection is distinctly traceable throughout the first five years of assurance. This is also the case with the Jones' Tropical (Lives) which comprises the New York's experience, 1873-1891, and the data for which are included in the present investigation. Here the mortality for the years three to five inclusive was 88% of the ultimate rate excluding the first five years of assurance. In the present tropical experience, on the other hand, the mortality during the years of assurance three to five was 91% of the expected by the American Tropical table and only 94% for the years six and over. In other words the new experience is more favorable than the Jones' Tropical by Lives for the years six and over, but the mortality is actually heavier during the years of assurance three to five.

There is probably no doubt that in many tropical countries of the

world an actual improvement in mortality has taken place in recent years both owing to improved conditions and to increased knowledge on the part of the companies as to the proper method of conducting business. If this is so it would seem reasonable to expect to find evidence of this improvement in the early policy years as well as later on, or in other words that in the more recent experience the effects of selection would be found to continue for at least as long a period as in the earlier experience. That the ultimate rate is attained in the third policy year is not confirmed by the Scottish Amicable, the Barbadoes Mutual or the Jones' Tropical (Lives). On the other hand in the case of the British Empire's Indian experience, 1872-1902, contained in a paper by Mr. A. Winter in the Transactions of the Fifth International Congress of Actuaries, a similar condition is observed. The ratio of actual to expected deaths by the $O^{M(5)}$ table for the total Indian business is 150% for the years of assurance 3 to 5 and 153% thereafter. Mr. Winter investigated this point further by sub-dividing the experience into three classes of lives, European, Eurasians and Asiatics, and each of these he further sub-divided into two groups according to ages at entry 16 to 44, and 45 and over. The following table, extracted from his paper, is both interesting and instructive.

British Empire, "Indian Experience"
Comparison of Actual with Expected Deaths by $O^{M(5)}$.

Ages at Entry.	Years of Assce.	Europeans. Deaths.	%	Eurasians. Deaths.	%	Asiatics. Deaths.	%	Total. Deaths.	%
16 - 44	1 - 2	35	1.20	6	.71	25	.97	66	1.04
	3 - 5	52	1.72	5	.57	22	.89	79	1.24
	6 & over	62	1.36	24	1.87	30	1.19	116	1.40
45 & over	1 - 2	7	.57	6	1.95	12	1.29	25	1.01
	3 - 5	35	1.93	4	.95	32	2.34	71	1.97
	6 & over	195	1.54	42	2.28	67	1.48	204	1.59
All Ages	1 - 2	42	1.01	12	1.04	37	1.06	91	1.03
	3 - 5	87	1.79	9	.69	54	1.41	150	1.50
	6 & over	257	1.50	66	2.10	97	1.37	420	1.53

Among the European lives the mortality in each of the age groups was actually heavier during the years of assurance 3 to 5 than thereafter. Mr. Winter explains this as being due to the fact that the European lives were largely unacclimated, the first five years of residence often being concurrent with the first five years of assurance. The Eurasian experience, though small, nevertheless indicates a lower mortality throughout the first five years of assurance than thereafter. In the case of the Asiatics the younger groups of entrants have experienced a very light mortality during the years 3 to 5, but when we come to those who entered at ages 45 and over we find the death rate exceptionally heavy in the same period of assurance. The mortality among both Eurasians and Asiatics is relatively much higher among those who entered at 45 and

over than among the younger entrants. Although the data when sub-divided in this manner are not very extensive, this experience nevertheless indicates that both the question of race and age at entry may have a decided effect upon the duration of selection. The statistics comprised in the Jones' Tropical experience by Lives were drawn from the American Tropics exclusively and it is quite possible that the shorter period during which medical selection appears to be operative in the new tropical experience may be due not to any change in the character of American tropical mortality but to the inclusion in the statistics of data relative to native Indians and other Asiatic races. It would have been interesting if the present experience had been sub-divided at least into two broad sections, the American tropics in one group and other tropical parts of the world in the other, and these two sections further sub-divided according to groups of ages at entry. This would have enabled us to judge whether selection is more marked in the American tropics and also whether the age at entry has any bearing upon the question. The evidence of previous investigations into West Indian mortality does not seem to support the conclusion that as far as this section of the tropics is concerned selection wears off in two years.

In the construction of the ultimate tropical table Mr. Hunter has eliminated the first two years of assurance only. The following table shows how the rates of mortality and the net Whole Life premiums compare with those of the American Tropical Table.

Comparison of New Tropical with American Tropical Table.

Age.	q_x		% of	P_x @ 3%		% of
	New Tropical.	American Tropical.	American Tropical.	New Tropical.	American Tropical.	American Tropical.
25	10.48	12.22	86%	20.44	21.50	95%
30	11.31	13.00	87	23.40	24.35	96
35	12.59	14.22	89	27.24	28.04	97
40	14.64	16.13	91	32.26	32.87	98
45	17.86	19.20	93	38.89	39.26	99
50	22.96	24.01	96	47.78	47.81	100
55	30.98	31.59	98	59.85	59.41	101
60	43.57	43.48	100	76.48	75.38	101
65	63.21	62.01	102	99.67	97.61	102
70	93.45	90.69	102	132.34	128.91	103

At age 25 the ratio of mortality is only 86% of the American Tropical. The ratio gradually increases to 100% at age 60, after which the new rates are about 2% higher. The net annual Whole Life premium is 95% of the American Tropical at age 25. The ratio increases to 100% at age 50 and thereafter the premiums by the new table are slightly higher.

A comparison of the actual deaths in the new experience with the expected by the Jones' table for the groups of attained ages is also given.

Comparison of Actual Deaths in Tropical Experience (excluding the first two years) with Expected by the Jones' Table.

Ages.	Exposed to Risk.	Actual Deaths.	Jones' Table. Expected Deaths.	Jones' Table. Ratio of Actual to expected.
17 - 34	43,532	470	572	82%
35 - 44	72,883	1050	1179	90%
17 - 44	116,415	1522	1471	87%
45 - 54	41,459	953	950	100%
55 - 84	10,815	441	456	97%
45 - 84	52,274	1394	1406	99%
All Ages	168,689	2916	3147	93%

For ages 17 to 34 combined, the actual deaths are 82% of the expected by the Jones' Table, 90% for ages 35 to 44, 100% for ages 45 to 54 and 97% thereafter. The very low mortality in the new experience during the earlier period of life may be partly due to an actual improvement in mortality, but it is hard to believe that there has been such a marked improvement in the mortality in the American Tropics. It is also possible that the effects of selection may not have been entirely removed, but the inclusion in the statistics of data relative to the Asiatic races has probably tended to lower the mortality rates in the early period of life.

The curve of American Tropical mortality is believed to be quite different in character from that which applies to Asiatic lives, higher in the early period of life and lower at the older ages. If there is a wide difference in the nature of the mortality curve in these two principal sections of the business, a table based upon the combined statistics will not correctly represent the mortality in either section, and as many companies restrict their business to certain parts of the tropics it is very important to satisfy ourselves that the table used as a guide represents with sufficient accuracy the mortality likely to prevail in such localities. In the table on page 406 Mr. Hunter shows the relative mortality in the principal countries from which the data have been drawn, by comparing the actual deaths with the expected by the American table. A better idea as to the sufficiency of the new tropical rates would have been obtained if the actual deaths had also been compared with the expected by the new table. It appears, however, that in such an important field as the West Indies the death rate has been considerably heavier than the average in each of the four separate sub-divisions, the ratios of actual to expected deaths ranging from 160% to 170% of the American table, while for the total experience by Lives the ratio was only 144%, and excluding the first two years of assurance 153%. The inference is that the new tropical rates of mortality would not have been sufficiently high for this section of the tropics.

In order to ascertain how the new tropical table compares with the experiences of the other companies mentioned in Mr. Hunter's paper,

the expected deaths by that standard have been calculated approximately for the Scottish Amicable, the Standard, and the Barbadoes Mutual in Barbadoes, all West Indian experiences, and also for the principal experiences in British India, those of the Standard, the Oriental, and the British Empire.

WEST INDIAN EXPERIENCES (ALL POLICY YEARS).
COMPARISON OF ACTUAL WITH EXPECTED DEATHS BY THE NEW YORK LIFE TROPICAL TABLE.

Ages.	Scottish Amicable 1846-1890. Actual Deaths.	Expected Deaths.	% Actual of Expected.	Standard 1846-1900. Actual Deaths.	Expected Deaths.	% Actual of Expected.	Barbados (in Barbados)* 1846-1882. Actual Deaths.	Expected Deaths.	% Actual of Expected.
Under 30	12	9.4	127.1	46	50.6	92.7	24	29.3	81.9
30 - 39	58	42.4	136.9	178	151.6	117.4	92	80.5	114.3
40 - 49	66	64.4	102.6	275	202.6	135.7	105	101.0	104.0
50 - 59	75	64.4	116.5	234	208.0	112.5	94	91.6	103.7
30 - 59	199	171.2	116.2	687	562.2	122.2	291	273.1	106.5
60 & over	59	77.9	75.7	260	270.3	96.3	70	79.3	88.3
Total	270	258.5	104.4	993	883.1	103.1	385	281.7	100.9

EXPERIENCES IN BRITISH INDIA (ALL POLICY YEARS).
COMPARISON OF ACTUAL WITH EXPECTED DEATHS BY THE NEW YORK LIFE TROPICAL TABLE.

Ages.	Standard 1870-1900. Actual Deaths.	Expected Deaths.	% Actual of Expected.	Oriental 1874-1891. Actual Deaths.	Expected Deaths..	% Actual of Expected.
Under 45	697	758.4	91.9	481	652.5	73.6
45 - 49	191	190.0	100.5	125	124.2	99.5
50 - 59	244	238.5	102.3	135	128.1	105.4
60 & over	—	—	—	38	29.4	129.2
All Ages	1132	1186.9	95.5	779	934.2	83.0

Ages.	British Empire 1872-1904. Actual Deaths.	Expected Deaths.	% Actual of Expected.	Total Three Companies. Actual Deaths.	Expected Deaths.	% Actual of Expected.
Under 45	260	315.9	82.3	1438	1726.8	83.2
45 - 49	120	113.3	106.0	436	427.5	101.6
50 - 59	202	181.1	111.5	581	547.7	106.1
60 & over	92	88.0	104.6	130	117.3	110.9
All Ages	674	698.3	97.7	2583	2818.5	91.6

The total for the three companies is given as the three experiences cover practically the same period of time.

It is impossible to base too definite conclusions upon a comparison of

*The experience of the Barbados Mutual in Barbados was considerably more favourable than in the other West Indian Islands combined.

this kind without knowing the relative amount of new business contained in the several experiences, and also because the West Indian experiences cover a longer period of time. Nevertheless the figures indicate that there has been a marked difference in the past between the mortality in the West Indies on the one hand and in British India on the other. In each of the West Indian experiences (all policy years included) the death rate has been much heavier than the new tropical table throughout the important period of life from 30 to 60. In British India, on the other hand, a favorable rate of mortality has been experienced below the age of 45, the ratio of actual to expected deaths for the three companies combined being only 83% of the new tropical table, while for ages 45 and over the ratio is 105%. These comparisons indicate that the mortality in British India has been considerably lighter under age 45 than in the West Indies, but somewhat heavier at the more advanced ages.

The following table contains the experiences of the Oriental and the British Empire in regard to Asiatic lives only.

ASIATIC LIVES.

RATIOS OF ACTUAL TO EXPECTED DEATHS BY TROPICAL TABLE.

Ages.	Oriental.			British Empire.			Total Two Companies.		
	Actual Deaths.	Expected Deaths.	%	Actual Deaths.	Expected Deaths.	%	Actual Deaths.	Expected Deaths.	%
Under 45	356	487.4	73.0	76	113.5	66.9	432	600.9	71.9
45 - 49	89	88.4	100.7	47	38.2	123.0	136	126.6	107.4
50 - 59	98	89.3	109.7	51	48.0	106.3	149	137.3	108.5
60 & over	23	18.7	123.0	17	18.9	90.0	40	37.6	106.4
All Ages	566	683.8	82.6	191	218.6	87.4	757	902.4	83.9

The mortality among Asiatic lives was considerably lower than the average Indian experience of the three companies combined under age 45 and somewhat higher than the average thereafter.

The experience of the Indian Postal Insurance Scheme 1884-1904 which consists almost entirely of native Indian lives, is as follows:

RATIO OF ACTUAL TO EXPECTED DEATHS BY TROPICAL TABLE.

Ages.	Actual Deaths.	Expected Deaths.	% Actual of Expected.
Under 45	428	512.2	83.5
45 - 49	78	87.8	88.8
50 & over	109	74.1	147.1
Total	615	674.1	91.2

Here also the mortality is seen to be very favorable up to age 50, but exceptionally heavy thereafter. It should be mentioned, however, that this experience includes an unduly large proportion of the recently selected lives and it is also worthy of note that the new business in recent years has been effected chiefly on the Endowment plan, whereas all of the

older policies were on the Whole Life plan. Mr. Hunter has expressed a doubt as to whether the selection exercised by the assured in his choice of form of policy has any appreciable effect upon the mortality in tropical countries. The experience of our Company has been that the choice of plan is an even more important factor in the tropics than in the north in producing a favorable mortality. Not only is the endowment plan the means of attracting a much better class of lives generally, but it is also effective in eliminating to a large extent the moral hazard.

These comparisons indicate that the mortality rates of the new tropical table are probably too low during the earlier period of life to correctly represent the mortality likely to be experienced by most companies in the American tropics. On the other hand it is very probable that for native Indian and other Asiatic lives the rates will not be sufficiently high at the older ages. These are points to be carefully considered in framing a table of tropical premiums.

Mr. Hunter has called particular attention to the many obstacles in the way of doing a successful business in the tropics. Many of us can testify from personal knowledge that these difficulties have not been magnified. The experience of the New York Life Insurance Company, however, proves that success can be attained, but it also impresses upon us the fact that the first essential to success is a substantial rate of premium. Wide variations in the mortality will occur in different sections of the tropics and among different classes of lives. The mortality experience of the company cannot be controlled within the same narrow limits as in temperate climates. It must also be remembered that policyholders in the tropics expect at least as high a rate of bonus as in the north which, of course, means larger cash profits. A considerable margin, should, therefore, be provided in the construction of the premium rates not merely to cover all possible fluctuations which a regard for safety demands, but to enable the company to make as large profits from mortality proportionately as are realized in connection with northern business. In a well-managed progressive company in the north whose premium rates are based upon the American table, the actual mortality would be about 75% of the tabular rate. A company doing a similar business in the tropics basing its rates on the new tropical table and experiencing New York Life tropical mortality, would realize a corresponding profit from mortality of about 6% of the tabular rate.

Many years must elapse before the rates of mortality applying to different tropical countries and to different races can be ascertained with anything like the degree of accuracy which exists in northern climates. In the meantime the only wise course to follow is to err, if at all, on the side of safety.

One of the most interesting points which arise in connection with the study of tropical mortality is the relationship of the tropical curve to the northern. Mr. Hunter has shown that tropical mortality cannot be represented by the addition of a uniform percentage to the American q_x but that the percentage increases to a maximum at about age 50 and

then decreases. He also refers to the Barbadoes Mutual experience where the relationship to the H^M q_x is similar. Messrs. Hardy and Rothery also pointed out how the nature of the incidence of the extra mortality in the tropics could be exhibited in a striking manner by means of Makeham's expression for the force of mortality $\mu_x = A + Bc^x$, by assuming a common value for log c and determining the constants A and B as accurately as possible. It was shown that the Barbadoes table can be closely represented by a constant addition of .005 to the H^M force of mortality combined with an addition of three years to the age.

In the graduation of the tropical table Mr. Hunter has taken the value of log c as .04 which is approximately the same as in the text book graduation of the H^M table. The value of the constants A and B in the two tables are as follows:

$$\text{Tropical} - \mu_x = .00913 + .0001347c^x \ (\log c = .04)$$
$$H^M - \mu_x = .00619 + .0000960c^x \ (\log c = .039657)$$

From a comparison of these values we observe that Mr. Hunter's tropical table can be very closely represented by a constant addition of .003 to the H^M force of mortality combined with an addition of four years to the age. The following table shows how closely this approximation corresponds with the actual value of μ_x.

	Force of Mortality.	
Age.	Tropical μ_x	H^M $\mu_{x+4} + .003$
25	.01047	.01055
35	.01251	.01257
45	.01762	.01762
55	.03047	.03019
65	.06274	.06153
75	.14379	.13962

It will also be interesting to mention that the expected deaths on the basis of $M^M q_{x+4} + .003$ are 2921 as compared with 2916 actual deaths, and there is a very close agreement for the separate age groups.

This illustrates in a forcible manner the character of the extra mortality in the tropics. It is of a two-fold nature, being due in the first place to zymotic diseases, the risk from which is practically uniform at all ages, and secondly to diseases of the brain and digestive organs the risk from which increases with age and long continued exposure to a tropical climate.

With regard to the question of the valuation of policies issued at tropical rates, there can be no doubt that a tropical table of mortality should be used. The reserves by the American or H^M table are quite sufficient in the aggregate as long as the proportion of endowment assurance business is large, but this proportion will vary in different companies and the Life and Limited Payment Life policies will remain upon the books long

after the endowments have matured Then again all bonus additions and paid-up policies should, of course, be valued by a tropical table.

I had intended to present some figures relating to our Company's business, but I found it impossible to prepare the statistics in the limited time at my disposal. Speaking generally, however, I may say that our experience tends to confirm the opinion expressed that the character of American tropical mortality differs essentially from that applying to Asiatic lives, being higher at the younger ages and lower at the older ages. Furthermore, as I have already mentioned, we have found the mortality under Endowment assurance policies much more favorable than under Life policies in the same sections of the tropics.

In conclusion I know that I am expressing the feelings of all when I say that our thanks are due to Mr. Hunter for one of the most valuable papers which has ever been presented to the Society.

Mr. McClintock: Mr. Hunter's article on Tropical Mortality is memorable. It will be remembered for its importance, its timeliness, its carefulness and its completeness. Others have written and will write on the mortality in the tropics, and in time other tables may be produced in the light of future experience to take the place of those lately given to us, but I do not think that any paper so comprehensive is likely to be seen again. The opportunities enjoyed by Mr. Hunter as Actuary of the company which has had the most varied and extended experience in the tropics are of course unrivalled, and we must all congratulate him on having used his opportunities so well.

The author has perhaps erred in crowding too much into the title chosen for the paper. The distinction between semi-tropical and tropical rates is so well known to those of us who use such rates that the terms employed are no doubt more or less familiar to actuaries in this country. The title of this paper, however, does not relate to semi-tropical and tropical rates, but "mortality in semi-tropical and tropical countries." Tropical countries are known, up to the present time, as countries lying, approximately at least, between the Tropic of Cancer and the Tropic of Capricorn. Of course this existing fact is recognized by the author, who explains, on page 407, that for the purposes of his paper the terms used depend on the "premium scale, according as the semi-tropical or the tropical rate is charged." The phrase "semi-tropical countries" is novel, but allowable, if duly defined as in the present case, but it may be doubted that the phrase will come into general use. For one thing, if the definition in this paper came into use, one company might call a country semi-tropical, because it uses the rates having that name, while another charging full rates would call it tropical. Even the great company which does so large a tropical business might suddenly decide that a certain tropical country should thereafter pay semi-tropical rates, and thus by the present definition become a semi-tropical country. In short, I consider the phrase "semi-tropical countries" not happily chosen; but the fact remains that an author has a perfect right in a technical treatise like

this to adopt any terminology, however new, so long as it is defined, leaving it to the test of time whether or not it shall meet with the general acceptance among actuaries because of its convenience.

Long as the paper is, no part of it could be shortened to advantage. It is fertile in suggestions, I might say crammed with them, but for that very reason actuaries who examine it critically may find points on which they would like Mr. Hunter's opinion to be given even more completely. For example, we find it stated on page 397, "In tropical countries, it is questionable whether the selection exercised by the insured in his choice of the form of policy has any appreciable effect on the mortality. The plan of insurance partly depends on the tradition in each community as to the best plan of insurance, and partly on the relative safety of investments in each country." The circumstances named may have great effect in this or that community, and effects as great may be produced by the commission paid to the agent. On this point I would merely say that, unless human nature in the tropics is radically different from the same article in the temperate zone, applicants who feel themselves to be long lived will proportionately be more likely than the rest to favor the endowment plan. It is conceivable that the agent and the doctor know that a doubtful risk will be accepted by a company more readily on the endowment, and that the effect of his knowledge may under certain circumstances and in certain companies produce a selection of bad risks to an extent as great as the selection exercised by the good risks in favor of endowments. Mr. Hunter's statement of the universal preference given to endowments by the Japanese and Chinese is most interesting, and may be due to some difference in the kind of self-sacrifice to which they are addicted—for example, the cause may be that the Japanese prefer to sacrifice themselves for the Emperor, the Chinese for their parents, and the western nations for their wives and children.

Of course Mr. Hunter takes the correct view that neither lives alone nor amounts of insurance alone should be made the basis of a mortality table like the present. Thus, on page 407, reference is made to the Jones' Tropical Table prepared in 1894, and it is said that "It is based upon amounts insured;" and the first figures now presented by Mr. Hunter are stated as being based in the same way, upon amounts insured. The difficulty of adjusting the results so found led the author (page 412) to prepare data based on lives. The results obtained by taking lives alone were not however entirely satisfactory. The final statement about the method of graduating the table is (page 417) as follows: "In obtaining this graduation, which is based on lives, we kept in mind the experience on amounts insured, as well as the necessity of having a practicable table."

Doubtless some of the younger members present may not be familiar, except perhaps in a general way, with the reason why it is necessary to take the amounts into consideration as well as the lives. Some actuaries have a decided prejudice in favor of basing a life table on lives alone. The very name "Life Table" seems in their eyes conclusive. It shows that there are so many persons living at each age, and that is all that

there is to be said about it. I do not know whether the idea of basing a table on amounts as well as on lives was acted upon, or even proposed, before the investigation made 35 years ago by the Committee of Actuaries which was appointed by the Chamber of Life Insurance. I know that at that time it seemed heterodox, but I am glad to say that my ideas carried conviction to the minds of the Committee, and the Thirty Offices' Table was the result. Since then, the idea has spread more or less, but it labors under the disadvantage that the opposing theory appears to occupy the ground. A life table is a life table, and if a life table appears not to be a life table, but a table based on the amounts of insurance, why, it is confessedly not a life table at all. It would seem better to take the bull by the horns and to style such a table a Death Loss Table, and the phrase Rate of Loss might thereupon be substituted for Rate of Mortality, and so on. The objection to this, of course, is that there seems to be no use for two sets of terms for the description of two things so exactly the same in all respects excepting their origin as the life table and the loss table. I suppose, on the whole, that it is better to retain the old phraseology, in spite of its disadvantages.

In the case of Mexico, for example, the insuring population is more homogeneous than in this country, and therefore a life table cannot differ so greatly from a loss table; while on the other hand there is likely to be a greater moral hazard, which again works in favor of a life table rather than the loss table. In both respects Mr. Hunter's judgment in laying greater stress on the life table is therefore sound. On the other hand, in an old settled country, there will be less moral hazard but greater variations in amount of insurance; the mercantile class, for example, taking on the average considerably larger policies than the farming class. The object being to provide rates of premium corresponding to probably rates of loss, the enlightened actuary will weigh the experience before him as well as measure it. Assuming, for example, that there are two classes of people to be considered, of equal numbers, one, the mercantile class, taking three times the amount of insurance as the other, the farming class, the former must have three times the weight of the latter. In any such case the loss table is of course much more useful than the life table.

I end as I began, in congratulating Mr. Hunter on his memorable paper.

Mr. Hutcheson: Before proceeding to transact business in any country in the tropics, or semi-tropics, it is very necessary that a thorough study of the local conditions be made. It will generally be found that the laws, customs, and standards of morality differ greatly from what they do in America. It may sometimes happen that justice to a foreign corporation cannot be relied on in the courts. Reliable agents may be few and far between. Properly qualified medical examiners and satisfactory inspectors of risks may be very hard to get.

When these and many other difficulties have been overcome, there still

remains the question of the rate of mortality to be expected. Meagre statistics may exist regarding the mortality of the general population. It may perhaps be known that the mortality of the population as a whole is, say, 150% of the American, but nothing may be known of the incidence of the extra mortality. Without a knowledge of the rate of mortality at each age, we can calculate neither the premiums to be charged, nor the reserves to be held. And without such knowledge, and without some knowledge of the effect of selection, we are unable to state whether the business is being conducted at a profit or at a loss.

In such circumstances, if it is decided to transact business in the country in question, the company will probably either decide upon a scale of premium rates based upon the domestic rates with some addition thereto, or will adopt the rates of some company already doing business there. An endeavor will be made, on the one hand, to have a rate which will be ample; and, on the other hand, competition, if there be any, will be an influencing factor in the scale of rates determined upon.

When sufficient time has elapsed, and when a sufficient amount of business has been transacted, it will evidently be very desirable that an investigation be made into the mortality experienced with a view to seeing, in the first place, whether or not the business has been transacted profitably, and, in the second place, whether it is not possible to deduce a table of mortality therefrom for future use.

Mr. Jones made such an investigation into the experience of the New York Life in the American tropics, and gave us the result fourteen years ago in his Tropical table. And now Mr. Hunter has made another investigation into the mortality in countries in which his company charges the tropical and the semi-tropical rates of premium, and he has given us a tropical mortality table deduced from the tropical experience, and a semi-tropical table based on his tropical table and on the American Table.

The data used excluded the first two years' experience, and seems to have been about 30,000 lives, amongst whom 2,900 deaths took place, and the experience extended over 31 years. The average duration does not appear to have been very long, but, as the effect of medical selection lasts for only two years, the data can be used properly to form a mortality table.

The data was not as homogeneous as might have been desired, part of it coming from Mexico, another part from South America, still another from Asia, still more from Africa, and so on. The proportions of the business which came from the various countries is a matter of importance if the mortality rates vary in the different countries. The table might be a poor guide to a company doing business only in one of the various countries if the mortality in such country varied greatly from the average.

On the other hand, the data was doubtless insufficient from which to deduce a separate mortality for each country. And even if such separate tables had been deduced, they would probably have borne such a resemblance to each other that it would have been quite unnecessary to have made use of more than one. Too great refinements are impracticable,

and it may be doubted whether the rates of mortality in any two of the various countries included in the tropical experience would differ more than do the rates in some of the Northern and some of the Southern States of the Union in which no distinctions are made by many companies in premium rates.

The fact that the data from all the various tropical countries was combined, will make the table more generally useful. Companies which have done business in tropical countries may compare their mortality experiences in each country with that expected by the Hunter tables, and they can be guided by these comparisons of "actual" and "expected," as to the applicability of the table for their use in any particular part of the world.

To return, however, to Mr. Hunter's data: he found, by a comparison of actual and expected deaths (1) by the Compound Progressive, (2) by Jones' Tropical, and (3) by the American Table, that a multiple of q_x from one of these standard tables could not be used to express the rate of mortality in the tropics, so he proceeded to construct a table from the data, omitting the first two years' experience, as he found that the effect of medical selection could be disregarded after that period. The graduation was made by Makeham's formula, and it is gratifying that he was able to do this, and still to produce a table which followed the facts so closely.

Compared with the Jones' Tropical table, we find that the Hunter rates of mortality are lower up to age 60, after which age they exceed the rates by the Jones' Table slightly. The mortality curves, however, follow each other closely for the greater part of life.

I have not been able to make any practical test of Mr. Hunter's tables, but I hope ere long to see whether or not they are suitable tables for Mexican business, as shown by the experience of The Mutual Life, Mexico being the only part of the globe in which The Mutual has transacted business to any extent at tropical and semi-tropical rates of premium.

There has been great improvement in mortality in the business coming from many tropical and semi-tropical countries, owing to better agency, medical and inspection work and increased knowledge at the home office of local conditions. Improved sanitary conditions in some places have reduced the death rates of the population at large, but this improvement has not been so marked amongst the insuring classes as amongst the lower classes.

There can be no doubt that the reserves held for tropical and for semi-tropical business should be based on tables of mortality to which such business is subject. If Mr. Hunter's tables comply with this requirement, then reserves by these tables certainly should be used.

In conclusion, I want to thank Mr. Hunter for the great amount of information which he has brought before the Society, and for the new mortality tables which he has given us.

Mr. Macaulay: Mr. Hunter's paper is one of the most valuable additions yet made to our knowledge of tropical mortality. The foundation

statistics are so extensive and the results have been so carefully and thoroughly worked out, that the paper reflects credit not merely upon its author, but upon the Actuarial Society of America, in whose records it will appear. I myself am so impressed with its value, and with the immense amount of labor which it represents, that I hesitate greatly to say anything that partakes of the nature of criticism. I hope Mr. Hunter will pardon me if I draw attention to one feature which appears to me to be a weakness in any tropical table prepared as his has been.

The point which impressed me most when reading Mr. Hunter's paper was the difference between his table and that of Mr. Jones, as set forth in his contribution to our Society a few years ago, based also on the experience of the New York Life, but in the American tropics only. That table is known as The American Tropical Table, and has hitherto been regarded as perhaps the most accurate exponent we have of tropical mortality. Our own company, for example, and at least one other company, have taken it as the basis upon which to calculate our premiums for use in the tropics. It has also been used extensively in the calculation of reserves. All the statistics which were made use of by Mr. Jones are included in those upon which Mr. Hunter's results are founded, and we should naturally, therefore, expect considerable similarity between the two mortality curves. A strange and interesting fact, however, is that these curves differ radically. How is this phenomenon to be explained? Is it due to improved sanitation in the tropics? Or if not, what is the cause? That portion of Mr. Hunter's statistics on which Mr. Jones based his table, would, of course, if it were again tabulated, show the mortality curve brought out by Mr. Jones, and not that brought out by Mr. Hunter's table based on the total statistics. The difference must therefore be sought in some very pronounced peculiarities of the additional statistics included by Mr. Hunter—peculiarities so pronounced as to actually obliterate some of the features of Mr. Jones' statistics. That additional experience consisted in part of additional lives located like those of Mr. Jones in the American tropics, but to a very much greater extent of lives resident in other sections of the tropics, especially in Asia. I cannot but think that Mr. Wood, in his remarks yesterday, which I appreciated very highly, put his finger upon the real explanation. The differences are I think due in a great measure, possibly even entirely, to the fact that the new experience included by Mr. Hunter consisted largely of native lives. Now, the mortality on native Asiatic lives follows an absolutely different curve from that on white lives in the tropics. In the case of native Indians, for example, a remarkably low mortality is shown at the younger ages, but, on the other hand, as a race they are short lived, they mature early, they age early, and they show a very heavy mortality later in life.

The Oriental Life, of Bombay, was founded in 1874, and its experience among native lives, to the end of 1891, 566 deaths, showed a mortality of but 121 per cent. of the expected by the H^M table. Among its European

lives the corresponding percentage was over 152. The course of the native mortality curve is instructive:

Ages.	Actual Mortality P. C. of Expected by H^M.
20–24	53.75
25–29	84.50
30–34	96.89
35–39	117.65
40–44	124.66
45–49	144.24
50–54	163.85
60–64	173.41
65–70	201.21

This uniform steadily progressive increase is most impressive. It shows that the favorable mortality occurred only at the younger ages, the lives under thirty-five actually showing a mortality under that of the H^M table. We have no reason to believe that these favorable percentages at young ages, prevail among European lives—the experience of the Oriental Company itself in fact disproving this.

The addition of a large body of native lives, especially if of comparatively young ages, would therefore naturally tend to reduce the rate of mortality for the entire experience in which they are included, if attention be directed only to Mr. Hunter's statement in his paper that his experience includes a "comparatively large number of policies issued in recent years on the lives of Japanese and East Indians." Most of these lives, therefore, would be not merely young in age, but recently selected, and the habit of issuing endowment policies only in such cases, would also tend to eliminate the higher mortality naturally expected on these lives.

The Oriental Company, in its official reports, made several references to the fact that the favorable mortality of its earlier years had not been maintained. The valuation report presented in 1899, for example, stated that "Tables of contributions framed on the company's experience, show that rather smaller contributions at the younger ages will meet the claims, but that as the ages advance, the contributions of the company require to be considerably augmented." In other words, native mortality is remarkably low at the younger ages and remarkably high at the older ages. It follows, therefore, that the addition of a large body of young native lives would tend to lower the mortality percentages of any tropical experience, as compared with an experience among Europeans alone, and moreover, would tend to change the entire mortality curve, lowering it at the younger ages, and raising it at the higher. May not the addition of native lives, therefore, have much to do with the difference between Mr. Hunter's table and Mr. Jones' table on American Tropical lives? If Mr. Hunter could but see his way to divide his statistics into two parts—separating natives from Europeans—he would add very greatly to the value of his paper, and confer a decided favor on all of us. I do hope that he will see his way to do this.

Mr. Hunter's figures at page 410 indicate that the loss by amounts, in his experience, was about 81 per cent. of the expected by Jones' table at ages 44 and under, and 99 per cent. at ages 45-54. This is in accordance with the suggestions which I have just made.

The percentages, however, show something of a falling off at ages 55 and over, and this on its face appears inconsistent with the idea that the peculiarities of the table are due to the inclusion of native lives, since such lives become progressively worse after 54. I think, however, that an explanation is to be found in the fact mentioned by Mr. Hunter, and to which I have already referred, that the policies on these native lives were issued in comparatively recent years, so that naturally this portion of the experience would not figure largely in the ages after 55.

There is, moreover, another fact to be noted. It is the custom of most of the companies, and I presume of the New York Life in most sections, to issue to native applicants policies on the endowment plan only. A large proportion of these endowment assurances would of course run off before 55, and thus still further reduce the proportion of native lives exposed after age 55. There is no inconsistency, therefore, in supposing that the favorable character of Mr. Hunter's experience, when all the ages are combined, may be due in no small degree to the low mortality at the younger ages among the native lives which were not included in Mr. Jones' material.

Another peculiarity which stands out prominently in this experience, is the shorter term during which the benefit of selection is shown as compared with Mr. Jones' experience, and in contrast the high mortality during the second and third policy years. The question naturally arises—Can this also be explained by the inclusion of native lives? If we turn to the latest East Indian experience, that of the British Empire Mutual, which comes down to 1902, we find this identical peculiarity of a heavy mortality in the second and third policy years shown by them also in their experience on native lives.

My own view is, that European and native lives are so utterly different in character that we should have separate tables of mortality for these lives. Their mortality curves are entirely different, and if they be combined into one experience, it would be possible to have an almost infinite number of tables, each differing from the other, and such differences depending chiefly upon the proportion which each of the two classes form of the total. The only satisfactory way of dealing with an Asiatic tropical experience, is, I think, to divide it rigidly into two parts, separating the European lives from the native. A table based upon European lives is not suitable for native lives, nor is one based on native lives suitable for use by a company doing business among Europeans, while the combined experience gives results which are apt to not agree with the mortality of either group taken separately. In practical business a company that assures natives should either calculate its rates by a mortality table based on native lives alone, or should eliminate the unfavorable mortality at the older ages by issuing endowment policies only.

If Mr. Hunter could but see his way at some future time to separate this splendid experience into its component parts, so that we may be able to look at the European and native groups separately, he would add immensely to the value of his table, and still further increase the obligation under which he has already placed us.

Another minor point in Mr. Hunter's paper interested me. Mr. Hunter expressed the opinion that the plan of insurance is not as important a factor in affecting mortality in the tropics as it is in the north, or, in other words, that there is less difference between the mortality under endowment assurances and that under life assurances than in the north. This suggestion came as a great surprise to me, for my own opinion has been that there is an even greater difference between the mortality of these various groups in the tropics than there is in the north. That view is not a mere matter of opinion either, but is based upon the actual experience of our own company in I think every tropical country for which we have analyzed our experience. It has been a common remark with us, what a tremendous influence the selection of the endowment plan apparently has in the tropics in the way of reducing mortality. There is nothing in our northern experience to at all equal it, and the uniformity with which this condition is found by us in the various tropical countries, has made a deep impression on us. The experience of the New York Life, however, is of course very much greater than ours. I may mention that so great has been our own faith in this, in the more favorable mortality under endowment assurances, that a few years ago we even tried the experiment of attempting to improve our Indian mortality by practically refusing to issue anything but endowment assurances, and behold, what we hoped for happened, for the mortality has steadily improved. How is this to be explained? It might be supposed that if the tropical applicants were restricted to endowment assurances, the inferior lives would insure on that plan almost as freely as under the whole life plan, but apparently this is not the case. It would seem as though when the agent is only able to offer applicants endowment assurances, he reaches a different and better class of life entirely, the probability being that speculative cases and bad lives feel but little tempted by the offer of a plan of assurance in which chief emphasis is laid upon the investment feature.

The interesting question has been asked—Why is it that Chinese and other native lives apparently prefer the endowment plan? The first point to remember is, that most companies give them but little option on this point, so that they have to a large extent to accept this plan or do without life assurance. It is nevertheless true that the Chinaman of his own free will prefers the endowment plan, and the cause is to be found in their own habit of thought. A Chinaman profoundly objects to having the question of death discussed at all, and in particular the question of his own death. If an agent were to approach a well-to-do Chinaman, and speak of the possibility of his dying, that Chinaman would, I understand, consider that he had been grossly insulted. According to their idea of courtesy and politeness, death is one of the subjects which must be abso-

lutely avoided. If an agent were to say to a Chinaman, "I would like to issue to you a policy which would protect your wife and children in the event of your death," he would, I am told, almost certainly fail to secure the business. To succeed he must present the investment feature, point out the amount which would be payable at the end of 15 or 20 years, and all that he dare say about death is to remark incidentally in an offhand manner—"Of course if you happen to die before the expiration of the period, your family will be paid the amount of your policy"—this in the most superficial way in order not to give offence. It is easily understood, therefore, why it is that Chinese business is almost always on the endowment plan. The Japanese do not have these ideas so far as I know, and the only reason I can assign for Japanese and native Indians taking chiefly endowment assurances, is that, as I have already said, most companies give them no alternative.

Mr. Hunter: In the first place, let me thank those who have discussed my paper for the kindly manner in which they have spoken of my work. I shall endeavor to reply to the principal suggestions and criticisms made by the various writers.

The first discussion presented was by Mr. Van Cise, who is of the opinion that moderate drinking among the Latin races tends to increase the rate of mortality, and that total abstinence would be even more beneficial than in a temperate climate like Great Britain. It would be practically impossible to determine whether or not Mr. Van Cise's view is correct, as wine drinking is so universal in Spanish-American countries that reliable statistics based upon abstainers alone would be exceedingly hard to find. I did not express any view on this point in my paper, but did state that the difficulty of dealing with the subject of alcohol in Latin America is less than in temperate zones, because the habit of wine drinking is practically universal, there is less drinking to excess, and there are no places like our saloons and public houses.

The interesting statistics given by Mr. Van Cise show that, on the whole, the Equitable experience is similar to that of the New York Life, although the incidence of the mortality may be somewhat different by policy years and in different countries. In South America, for example, Mr. Van Cise states that the actual deaths were 98½% of the expected, the latter being calculated on 125% of the American q for Semi-Tropical countries, and 150% for Tropical countries. An examination of the table given on page 406 shows that the New York Life's experience in the countries of South America in which the Semi-Tropical premium was charged was approximately 120%, and in countries in which the Tropical rate was charged, 150% of the American Table.

Mr. Van Cise gives it as his opinion that the hotter the climate the heavier the mortality. I can hardly agree with him, as I think that the sanitary conditions and the relative humidity have a great deal to do with the mortality.

With regard to Mr. McClintock's criticism of the title of the paper, I

should like to say that the title was chosen merely for convenience. I readily admit the anomaly in the expression "Semi-Tropical country," but, as defined in the paper, and as understood by actuaries, I do not think that its use as an abbreviation for "Country where Semi-Tropical rate is charged" is likely to lead to any error.

I do not disagree with Mr. McClintock that "applicants who feel themselves to be long-lived will proportionately be more likely than the rest to favor the Endowment plan." My statement that "in Tropical countries it is questionable whether the selection exercised by the insured in his choice of the form of policy has any appreciable effect on the mortality" was intended to cover the experience of the New York Life throughout the Tropics *as a whole*. I fully recognize that in certain communities the plan of insurance would undoubtedly have great weight, especially in countries where an intelligent selection of the plan was made by the applicant. In the majority of Tropical countries the insured have no knowledge of the different plans of insurance, so that conscious self-selection is not commonly exercised. In Mr. Macaulay's discussion of the same point he probably has in mind the British in India or the intelligent natives in that country, and I quite agree with him that there would be a lower mortality among those who took Endowment insurance than among those who took Ordinary Life policies and the same would probably be true among the educated natives of Mexico. In many other Tropical countries, however, there are very few or no native companies, and as insurance is very little understood by the inhabitants of these countries, the plan of insurance is more likely to be influenced by the agent than selected by the applicant. To put it in other words, an intelligent knowledge of insurance is necessary to select either in favor of or against the company so far as the plan of insurance is concerned, and there would be accordingly no such selection unless in countries where the benefits of insurance had been known for many years, and where some knowledge of it was part of the heritage of everyone. That would not apply, of course, to countries like Japan and China, where only Endowment policies are taken by the natives, and where an application for an Ordinary Life policy might reasonably be looked upon with suspicion.

Mr. Macaulay and Mr. Wood make the point that the difference between the Jones' Table and the new Tropical Table may be due to the inclusion in the latter of data derived from the Asiatic Tropics. I should like to say in this connection that the data of the Asiatic Tropics comprised less than 20% of the total data. Furthermore, in my judgment, there have not been kept clearly in mind the causes of the differences between Mr. Jones' and the new Tropical Table. The chief causes of the differences between the two are:—

1. That the new Tropical Table includes a much larger quantity of data relative to the American Tropics than Mr. Jones' Table—nearly four times as much.

2. That it includes a relatively small proportion of Asiatic data—not sufficient to materially affect the final table as an exhibit of American Tropical mortality.

3. That it does not include the experience in the Northern and Central States of Mexico, which was included by Mr. Jones, such experience being included in the new Semi-Tropical Table.

4. That the average duration of the policies in the new experience is longer than in the Jones' Table, the average duration in the latter being less than five years, the author himself stating that "the Tontine experience subsequent to the fifth year was too limited to serve as a trustworthy basis for a Table."

In my judgment the differentiation of the Tropical experience into two tables based on the mortality in countries classified according as the Semi-Tropical or the Tropical rate of premium was charged, is the chief advance which my paper makes in the presentation of the mortality in Tropical countries.

In order to determine whether Mr. Wood's statement is correct, "that the inclusion in the statistics of data relative to the Asiatic races has probably tended to lower the mortality rates in the early period of life," I have measured our expected experience on the natives of British India, which comprises two-thirds of the total Asiatic experience by the new graduated Tropical table, making proper provision for the mortality in the first and second years—*i. e.*, the Select Table was used. The following is the result:

NATIVES OF BRITISH INDIA.

Expected Mortality, measured by the Tropical rates of Mortality. (Select.)

Attained Ages.	Actual Deaths.	Expected Deaths.	Ratio.
15–34	242	259.4	93%
35–44	219	211.8	103%
45 and over	151	121.8	124%
Total, -	612	593.0	103%

The foregoing table shows that the result which Mr. Wood anticipated was realized, but the effect must have been small on the total Tropical experience, as the data on the natives of British India were about 15% of the data entering into the Tropical experience.

I may add, with special reference to Mr. Macaulay's remarks that the proportion of native to white lives insured in India is about 6 to 1, according to amounts exposed to risk, so that the effect of including Europeans in India in the Tropical experience is very slight.

Mr. Wood comments on the short period during which medical selection lasts in the Tropics, and suggests that it may be due to the inclusion of unacclimated lives. There was such a small proportion of unacclimated lives in the experience, however, that they would have very little influence on the mortality. It appears to me that the short period of effect of medical selection is due largely to the difficulty of obtaining a first-class medical examination, to the ignorance of many of the insured regarding hygiene and medicine, and to the suppression of information regarding

their physical condition. Besides these causes, climate may have a great deal to do with it. That the period of selection among Asiatic races is short may be judged from our experience among the natives of British India. The experience has been measured by the new graduated Tropical Table (Select).

NATIVES OF BRITISH INDIA, MEASURED BY GRADUATED TROPICAL RATES OF MORTALITY (Select).

Policy Year.	Actual Deaths.	Expected Deaths.	Ratio.
1st	128	113.6	113%
2nd	108	99.2	109
3rd	82	88.3	93
4th	74	71.9	103
5th	66	58.7	112
6th to 10th	123	133.9	92
11th to 20th	31	27.5	113

The above figures indicate that the period of medical selection among natives of British India was no longer than among the insured in the American Tropics in which the Tropical rate was charged. The ratio of actual to expected deaths by the ultimate table was approximately 88%, 102%, 93% and 103% respectively, for the first four policy years.

Mr. Macaulay draws attention to the apparently shorter period of medical selection in the new tables, as compared with the Jones' experience. As a matter of fact, Mr. Jones did not find under his experience by *amounts insured* any trace of medical selection after the end of the second year, his own statement being that "an examination showed no marked differences as between mortality ratios for policy years after the second." His paper needs to be read very carefully, otherwise there will be confusion in the mind of the reader between his tables on amounts insured and on lives. His Tropical table was based upon the combined exposures by amounts insured in the third and succeeding years. That table was tested by a comparison with the entire experience by lives instead of by amounts. Such a comparison showed that the effect of medical selection extended for five years when the data were based upon lives, but only for two years when the data were based upon amounts.

The fact which Mr. Wood points out, that the new Tropical Table can be closely represented by a constant addition of .003 to the H^M force of mortality combined with an addition of four years to age, was noticed by us in the preparation of the table. In fact, our first attempts at graduation were along these lines, but we finally decided to make a graduation without reference to any other table of mortality.

The principal criticism made by Mr. Wood is that certain comparisons "indicate that the mortality rates of the new Tropical table are probably too low during the early period of life to correctly represent the mortality likely to be experienced by most companies in the American Tropics." While such inferences may be drawn from his comparisons, I do not think

that it has been proved or will be proved until some future table is published on Tropical mortality. Granting the correctness of his idea, the slight inaccuracy should hardly be sufficient to prevent the use of the new Tropical tables, because the loading in the premiums should contain a sufficient margin for contingencies of such a nature. And, indeed, the rates of mortality in the new Semi-Tropical and Tropical tables are probably as close to the actual experience of companies now operating in the Tropics as the American Table is to the actual mortality experienced in the United States. The advantages of having a single Tropical table for valuation purposes which will practically cover all policies issued at the Tropical scale in the American and the Asiatic Tropics are great, and are sufficient to offset any slight differences which may exist in the incidence of the mortality in these two Continents.

In connection with the mortality among Eurasians, I should like to say that the final proof of my paper differs from the galley proof in regard to the mortality among Europeans, Eurasians and Asiatics in British India. Our Medical Officer for India told me that he would not place a great deal of reliance on any statistics which indicated that the mortality among Eurasians was distinctly better than among either natives or Europeans, because of the difficulty of differentiating between Eurasians and natives. For example, he told me of a man whose great-grandfather had been a Portuguese while all his other ancestors were Asiatics; that man called himself a European. On the other hand, there are a great many Eurasians who are brought up as Asiatics, and do not claim to be Eurasians.

In conclusion I should like to reiterate my warning regarding the dangers of business in Tropical countries. The mortality in Tropical countries is *not* the only factor to be considered. There is the important factor of currency. In many of the South American countries the paper currency fluctuates greatly in value, and if the premiums are converted into gold and forwarded to the United States or Canada and returned in later years to meet the maturing Endowments and the death claims, the Company will lose money in the event of an increase in the value of the paper currency and will make a gain if the reverse holds good. As an illustration of the difficulties in this regard I may mention one country where the value of the paper currency dropped, over a period of twenty years, to one-quarter of its value at the beginning of the period, and then returned in the course of three or four years to one-half of that value.

Another question to be seriously considered is the probable taxation for which proper provision should be made in the premium rates. It is possible, too, that laws regarding deposits may be put into effect, and that is a more serious question in countries with fluctuating currency than one which is on a gold basis. Mr. Wood's statement is therefore well worth repeating, that in tropical countries "the first essential to success is a substantial rate of premium."

MISSTATEMENTS THAT AVOID THE POLICY—J. M. LANGSTAFF.

VOL. X, PAGE 452.

WRITTEN DISCUSSION.

MR. IRELAND: Mr. Langstaff's paper is a careful and methodical resumé of decisions that have been rendered in American and Canadian courts, with some English citations, respecting misstatements that avoid the policy; it is the result of a good deal of painstaking study, covering hundreds of cases read and scanned, and is interesting, though very largely as history, and not so much as a guide to the future, for the reason that, as it was expressed by one of the hearers when the paper was first read, these matters are no longer of great practical importance in the United States. It is interesting to trace the change in the practice of the companies that has come about within the memory of the older members of our Society in the attitude of the companies toward claimants in cases where everything was not perfectly clear and satisfactory. It is to be borne in mind that the paper discusses only misstatements, which of course are mostly matters contained in the application for the insurance, and does not include the wide field of avoidance of policies by acts or omissions occurring after the time when they were issued. Fifty years ago, failure to pay premium on or before the day when due was considered ample cause for cancelling the policy and depriving the insured of all further rights to insurance under the policy, or to any return on account of what he had paid to the company. It was quite consistent with this attitude that a misstatement in the application should be held to work a forfeiture of the insurance, even though the misstatement had nothing to do with the loss. Statements in the application were covenanted to be warranties, and such decisions were rendered as some of those quoted in the early part of Mr. Langstaff's paper. He says, "A breach of warranty is fatal, even though the insured acted in perfect good faith." In an English case "the evidence showed that the answer to this question was untrue in fact, but not untrue to the knowledge of the Plaintiff." The court decided that the policy was void, and remarked that "whether untrue to the knowledge of the party proposing the life" was a matter of very little importance to the insurers. And again Mr. Langstaff says, "whether the statement made is *ipso facto* material to the risk or not is of no consequence." In a leading British case in a "suit on a policy after the death of the assured, it was proved that several misstatements had been made. The court held that the untruth of the answers avoided the policy, notwithstanding the jury found that no material information had been held from the insurers, and it was conceded that there was no intentional fraud;" these quotations relating to cases in which the statements in the application were made warranties. A little later we read,—"The apparently inequitable effect, in numerous

instances, of the strict interpretation of warranties is no doubt responsible for the repugnance evinced by many Courts to recognizing warranties when evasion is possible. It is to be feared, however, that, in at least a few instances, the laudable desire of the Judges to succor the oppressed (the insured) and rebuke the oppressor (*i. e.*, the insurer) has led them into decisions not absolutely consonant with good law. An examination of the cases on the subject reveals many arbitrary decisions which can hardly be reconciled with recognized legal principles, and these cases should not be relied upon as furnishing precedents of weight." And here we have found one of the causes that led to a change in the attitude of the companies on this subject. When a resisted claim resulted in paying the claim in full, as well as the considerable expenses of a suit, and gave the company a reputation for not settling legitimate losses freely and promptly, there were very evident practical reasons why suits should not be encouraged; and while, as our writer says, some of these decisions do not furnish legal precedents of weight, they do point out, in a practical way, the disadvantages of relying on strictly legal and logical processes. When the doubts and disadvantages attending what may be called the resistful attitude of the companies had become sufficiently apparent, one or two of the largest companies started off in a different direction, made their appeal to the public by paying claims promptly and fully, waiving most of the grounds on which policies had been contested, and ceasing to avail themselves of the usual sixty or ninety days after presentation of proofs, at the end of which time it had been customary to pay losses. Competition rapidly brought other companies, one after another, to something like the same attitude; it was not many years before agents used all their influence to have claims paid promptly, and where more than one company was on the same risk, it was quite a point with each agent to try to have his claim paid first. Now the pendulum has swung so far in the other direction that in the payment of losses, as in other ways, companies are doing, in some respects, more than justice and propriety require; but they have stopped the old cries and gibes about the readiness of companies to receive premiums, and their slowness in paying losses, and in the matter of resisting death claims they have probably gained more than they have lost by this course. The final influence in this direction has been the action of the lawmakers. As Mr. Langstaff points out, "the effect of common law principles, with which this paper has mainly dealt, is considerably modified in various respects by statutory enactments which override the common law." There are statutes providing exactly what adjustment must be made in case of a misstatement as to age, and there are provisions that statements in the application, in the absence of fraud, shall be regarded as representations and not warranties. The provision in the proposed new Canadian Insurance Act is quoted as a type. "The policy and the endorsement thereon shall constitute the entire contract between the parties, and all statements made by the insured shall, in the absence of fraud, be deemed representations and not warranties, and no such statement shall be used in defence to a claim under

the policy unless it is contained in a written application, and a copy of such application shall be endorsed upon or attached to the policy when issued." And the writer adds, in a brief and pointed sentence,—"The virtual effect of a clause like the above is to remove the subject of warranty out of the realm of practical insurance law."

The part of the paper that relates to concealment is an interesting exposition of what concealment consists in, and of the different views that have been held by courts as to whether the concealment must be intentional or not to avoid the policy, whether it can exist even if there is no inquiry in the application on the particular point concerned, and so on.

Under the head of "Exceptions to general rules," there are interesting points. The writer quotes Lord Mansfield, who says:—"There are many matters as to which the insured may be innocently silent. (1) He need not mention what the underwriter knows" what way soever it came to the underwriter's knowledge. (2) He "need not mention what the underwriter ought to know," "or what he waives being informed of." This brings up a question that the present writer has heard discussed, as to whether a company receiving an application from a person already insured by it is bound to compare the two applications and note any discrepancy between them before it can take advantage of any misstatement made in the second application. For instance, if a man took out a policy in 1870 and again made application in 1890, giving a different date of birth, or contradicting in some way a statement made in the first application, must the company compare the papers before the second policy is issued, or otherwise be restrained from taking advantage of any misstatement that the second application may contain?

It is not intended by the above to imply that all the concessions to the insured have been made under compulsion. Many insurance managers have seen and felt that a larger measure of liberality was the fair and manly thing to concede, and, following their lead, there has been introduced into the business a more fraternal feeling, a larger impulse in favor of making life insurance a true protection and a genuine blessing to those who are innocent of any wrongdoing in connection with the contract. When an endowment policy approaching maturity on the life of the late William M. Tweed was disputed on the ground that when he escaped from prison he did not apply for a permit to cross the ocean, it may be that the insuring company thought there were other and really stronger reasons for not paying the claim, but the grounds advanced savored a little of the ridiculous, and the withholding of payment savored something of cruelty. To-day such a policy ould probably be settled for a generous surrender value.

Mr. Langstaff's labors have had the seal of approval set upon them by the award to him of one of the Fackler prizes; and the paper will doubtless serve as an index and guide for those who may hereafter have occasion to study the legal history of the subject.

Mr. Macdonald: Mr. Langstaff has made such an exhaustive and careful compilation of the results of the decisions in the hosts of cases

dealing with the effect of Warranty, Misrepresentation and Concealment in relation to Policies of Life Insurance that his article is almost beyond criticism. The field occupied in the law of Contract by Warranty and Conditions (precedent and subsequent) is a wide one, and owing to the confusion between the different meanings given to the word "Warranty" is a difficult one. In life insurance the words "Warranty" and "Condition" are nearly always interchangeable. It has been stated so often as to become a commonplace that the law is simple and easily found, but that its application to a given set of facts not already exactly covered by existing decisions is not always easy. Mr. Langstaff has, by his article, collected so many decisions and opinions and has classified them with such care that he has rendered valuable assistance in determining whether a policy is, as apart from fraud, voidable for misrepresentation or concealment. We think, however, that he does not always realize the part played by the trend of decision in legal matters and the weight which courts give in their interpretation of contracts to the intention of the parties. He has admittedly viewed the subject from the old common law standpoint only. The same Courts, in Canada at least, now apply the principles of both the common law and the Equity Jurisprudence of England. One of England's great Authorities on Mercantile Law has said—"Parties may think some matter, apparently of very little importance, essential; and if they sufficiently express an intention to make the literal fulfilment of such a thing a condition precedent, it will be one; or they may think that the performance of some matter apparently of essential importance and *prima facie* a condition precedent is not really vital, and may be compensated for in damages, and if they sufficiently expressed such an intention, it will not be a condition precedent." If the court does not find in the contract such an expression of the intention of the parties the interpretation of the disputed terms will remain for the court, and unless the court holds that the matter was not a condition precedent its breach will not operate as a discharge.

That portion of Mr. Langstaff's article which deals with "What facts are material" gives some indication of this tendency of the courts to enquire into the intention of the parties before deciding whether "Warranty" is used as equivalent to a condition precedent in the sense of a promise vital to the contract. The question "Is such and such a promise vital to the Contract?" is one which always plays an important part. Mr. Langstaff's article is full of material and argument relating to this point and contains a most useful collection of cases. I may perhaps be pardoned for referring to a case not cited by him, decided in 1905, which illustrates my meaning. Ellinger *vs.* Mutual Life Insurance Co., New York (1905), 1. K. B., 31. The application contained the following clause: "I also warrant and agree that I will not commit suicide whether sane or insane during the period of one year from the date of the said contract." In rendering judgment Justice Collins said: "This last statement is in my opinion as well as all those statements which precede made the basis and part of the contract. The question is whether

the statement which purports to be a warranty has the effect of a 'Condition,' a sense in which the word 'Warranty' is often used in relation to contracts of insurance. I think that having regard to the contract that all the three statements or warranties which I have mentioned were intended to constitute conditions of the contract. If this promise is one which goes to the root of the whole engagement, then it becomes according to the ordinary principles of ordinary law a condition," etc.

Justice Bigham said: "Death within five years is the event upon the happening of which the defendants are to pay, but death by suicide within the first twelve months is excluded. Apt words have been used to express this intention, and it is, I think, impossible to read the clause as merely giving to the Company a cross claim against the estate for damages." In this case the Insured warranted and agreed that he would not commit suicide, sane or insane, during the year. The Court asked whether that statement which purported to be a "Warranty" had the effect of a "Condition" and held that it was so used. This is not, however, the primary meaning of the word "Warranty," but is the usual meaning of that term in Life Insurance Law.

Mr. Langstaff has done a service to the life insurance companies doing business in Canada by pointing out what a mass of legal learning will be rendered obsolete by the proposed Canada Life Insurance Act if Section 97, Sub-sections B and C are enacted. These sections are as follows:—

"That the policy shall be incontestable after two years from its date of issue except for non-payment of premiums and for engaging in military or naval service in time of war without the consent in writing of a responsible officer of the Company."

"That the policy and the endorsement thereon shall constitute the entire contract between the parties and that all statements made by the Insured shall, in the absence of fraud, be deemed representations and not warranties and that no such statement shall be used in defence to a claim under the policy unless it is contained in a written application and a copy of such application shall be endorsed upon or attached to the policy when issued."

If these Sections become law no statement will render a policy void or voidable unless contained in a written application for insurance a copy of which must be attached to or endorsed on the policy when issued in order to form part of the contract, and the policy is expressly made uncontestable after two years upon every ground except non-payment of premium or military service. The public through its representatives does not appear to realize that the ordinary life insurance company is really an agent to collect and manage a fund for the benefit of a multitude of persons, and that sums unfairly extorted from the company are really taken from the profits of its policyholders and not from the profits of its shareholders. The shareholders of the company are generally only remotely and indirectly affected by the results no matter which way the particular policy in dispute is disposed of, and the company is usually quite prepared

to deal justly and liberally in each particular case rather than resort to litigation.

There is no reason why the various legislatures should go farther then the companies have already gone by making it necessary to comply with certain technicalities such as endorsing or attaching the written application to the policy, because such technicalities are rather an aid to the occasional fraudulent litigant than a protection to the ordinary policyholder who as a rule is honest and fair minded.

Oral Discussion.

Mr. Sheppard: The wording in the proposed new Canadian Insurance Act given on page 485 as follows:—

> "Unless it is contained in a written application and a copy of such application shall be endorsed upon or attached to the policy when issued."

is identically the same as in the insurance laws of Minnesota and Massachusetts. The latter State has ruled that it is compulsory to attach a copy of the application to the policy, while the former has not. This it seems to me is due to it being uncertain as to whether the word "shall" is governed by "unless," or whether it stands by itself and is mandatory. The new Insurance Law of Ohio states that a copy of the application *must* be endorsed thereon.

PERMANENT DISABILITY BENEFITS—CHARLES W. JACKSON.

VOL. X, PAGE 490.

WRITTEN DISCUSSION.

MR. MORRIS: Mr. Jackson in his paper has written upon a subject of considerable interest, especially to those companies which have incorporated the so-called disability clause in their life contracts, and it is from such a standpoint that I shall attempt to review his paper.

One of the chief difficulties at the present time involved in the matter of permanent and total disability as brought out by Mr. Jackson is the dissimilarity of the different experiences involved, the data which has been derived from German societies and insurance companies and English friendly societies being radically different in character from that which would be expected to be an experience resulting from the disability clause in an American company. Mr. Jackson has clearly brought out the pension feature as manifested in the experience in Germany, giving an experience, in the older ages especially, which would be entirely foreign to what would be expected in America.

The obtaining of an experience on those totally and permanently disabled has been of importance to the fraternal orders of this country and Mr. Abb Landis, actuary for a considerable number of them, in his "Analysis of Fraternal Societies," 1906, has given the rates of permanent and total disability which he has obtained from the very large experience of the Knights of the Maccabees of the World. This has been supplemented by the experience of the Royal League, the experience being based on over 400,000 lives. Mr. Landis expresses himself as being confident of the experience up to age fifty-five. His experience above age sixty is taken from cases of chronic illness of the Ancient Order of Foresters and the Manchester Unity of England. He, however, graded this experience above age sixty in two ways—one to consider absolute disability at age seventy-six, which figures he obtained from the experience of the Manchester Unity, and the other showing absolute disability at age eighty, the latter experience being more to confirm the conditions that probably exist in this country. I include the rates of permanent and total disability as published by Mr. Landis.

MR. LANDIS' TABLE—PROBABILITY OF BECOMING TOTALLY AND PERMANENTLY DISABLED.

Age.	Absolute Disability at Age 76.	Absolute Disability at Age 80.
25	.00025	.00025
35	.00048	.00048
45	.00090	.00090
55	.00255	.00255
65	.03915	.02955
75	.91221	.43505
80	.....	1.00000

It should be remembered that in America, either in the fraternal orders or by the conditions of life contracts, in order that total and permanent disability benefits may be payable, the applicant must prove this condition if required—not only once but also the permanency of the disability from year to year. It is evident therefore that the conditions in America would be much more strict than in Germany especially, where in the older ages the pension feature would be important. Also, again, in America the contracts as published by some of the life companies do not provide for the total disability feature in the older ages where superannuation is known to be much heavier. Further, the composition of the experience from the standpoint of American life insurance companies would be of a very different nature from that published by Mr. Jackson in the foreign tables, these being based mainly upon workingmen and railroad employees. Note should also be made that the definition of total and permanent disability in American companies would be different from that already used in the foreign experiences, the inability of the insured to pursue any and all gainful occupations being generally included in the American definition.

From the tables published by Mr. Landis extra premiums have been figured providing for the cessation of premium payments in event of total and permanent disability, the formula used being similar to that given by Mr. Jackson (page 492); that is, for the annual premium of a whole life policy the numerator is the single premium for whole life insurance according to the American Experience, which already includes cases of permanent disability so that no modification of the numerator of the equation is necessary. The denominator of the equation, the life annuity immediate, however, has been modified from the American Experience insomuch as it has been necessary so that the annuity shall cease either at death or at the occurrence of total and permanent disability. The result of the equation is to obtain in addition to the annual premium for a life policy the extra premium that would be required to create an additional reserve sufficient to care for the liability of the company necessitated by the assumption to pay the policyholder's premiums in event of total and permanent disability.

Extra Premium per $1,000 Policy Providing for the Cessation of Premiums in Event of Permanent Disability, based upon the American Experience and Mr. Landis' Table of Total and Permanent Disability.

Age.	Whole Life.	20 Payment Life.
20	.43	.75
25	.61	.81
30	.87	.91
35	1.29	1.02
40	1.99	1.19
45	3.24	1.49
50	5.56	2.48
55	10.46	6.45
60	21.66	18.65

The above formula seems to me to be theoretically correct, but in practice it would have to be considerably modified for the reason that it is based upon the supposition that the insured will pay premiums until death or until total disability. It is clear that a modification will be necessary on account of policies which are continued as paid-up by the extended term feature and, to a less degree, on account of paid-up policies, for here we have cases where the premiums have ceased and the insured has neither died nor become disabled but where the insurance is still in force.

The lapse rate is also an important feature in the calculation by a company of extra premiums providing for the disability clause as the company's liability is removed at the cessation of premiums. While all tables show the rapid increase in total disability with the age, a company's liability, on the other hand, rapidly lessens as policies become paid-up or lapse.

The Travelers Insurance Company in 1905 made an investigation of the policies on its books that had attained the age of sixty and over, sixty-five and over and seventy and over. At age attained sixty and over there were in force 2,605 premium paying policies representing $8,293,585 of insurance (2,002 policies for $5,907,225 being Ordinary Life). At age attained sixty-five and over there were 1,001 premium paying policies representing $3,045,480 of insurance (874 policies for $2,455,320 being Ordinary Life). At age attained seventy and over there were found to be 359 premium paying policies representing $1,013,017 of insurance (347 policies for $878,117 being on the Ordinary Life form). The Travelers had then been writing a gradually increasing life business for over forty years, which fact should be considered in reviewing these results. It is evident, however, that the Company's liability from the disability clause would practically cease at age attained eighty.

From a practical standpoint also the above formula would be seriously modified by the treatment of one company at least of its disability clause, in practically avoiding all payments on account of total and permanent disability first manifested above age sixty. This modification decreases the liability of the company, at the same time making a corresponding reduction of the rates chargeable. Without modification considerable danger might accrue to a life company unless heavy rates were charged, which increased rates would, from a practical standpoint, remove the attractiveness of the feature.

It has been my aim in this discussion to emphasize the thought that it is not safe from an Ameri an standpoint to put great reliance in the disability tables given in Mr. Jackson's paper, principally on account of the different view of what is termed disability which we have taken in this country. This, however, is not intended as any slight on Mr. Jackson's work, for the Society is deeply indebted to him for bringing out this interesting subject and presenting the results of foreign experiences.

Undoubtedly it will be many years, for reasons which I have attempted to give, before we will be able to obtain a table of disability which will

come within the limits of the American conception. Total and permanent disability, except at the older ages, is in itself a remote but not improbable contingency. The present vagueness regarding investigations on the subject is well illustrated in the discrepancies in the extra premium rates for the cessation of premiums which have been given.

Mr. Cole: This paper of Mr. Jackson's deals with a subject which has recently been brought into some prominence in this country from the incorporation in the life insurance policies issued by certain companies of a total disability clause providing for cessation from payment of premiums in event of the insured's becoming totally disabled and unable to pursue any remunerative occupation. Mr. Jackson has computed premiums for this benefit on combinations of the American Experience Table of Mortality with rates of disability calculated by Karup and Zimmerman from observations upon the experience among German railway employees not engaged in train service and also according to a table derived from the British Government Reports on Friendly Societies, males, from 1861-1870. In all three cases, however, there appear to be certain fundamental objections to the use of these tables for the calculation of premiums for total disability in connection with a life insurance policy to which I will shortly refer.

It is practically impossible to make an intelligent comparison of these rates and of those actually in use owing to the great difficulty of placing a definite interpretation upon the term "permanent disability" or its synonyms on account of the variety of risks which may be involved. In the first place the phraseology of such disability clauses and the contingencies covered thereby in the way of loss of limbs, sight, and so forth will naturally vary widely and so accordingly will the risk assumed. Then, too, the different companies may construe the terms of the contract more or less strictly, although with the trend of the times it is altogether probable that a liberal interpretation will be allowed on such claims as may arise.

In making certain observations upon the statistical and mathematical basis of the German Law of Invalidity of May 23, 1889, Mr. T. E. Young in a paper presented before the Institute of Actuaries (*J. I. A.*, Vol. XXIX) points out that if the numbers involved in the calculation of sickness benefits, as derived from friendly society statistics, are of sufficient extent to admit of average results, then the size of the society will prevent an efficient check upon feigned sickness or invalidity, thus having a tendency to increase rates of disability based on the figures of any such society. This point is also emphasized by the statement of Mr. A. W. Watson in a paper on Mortality, Sickness and Secession Experience of Friendly Societies read before the Institute March 26, 1900 (*J. I. A.*, Vol. XXXV), who reaches the conclusion that the rates of sickness claim increase in proportion with the number of members in the lodges, which is clearly supported by the facts adduced in this connection. The charges for cessation of premiums computed on any Friendly Society tables will

consequently be higher than is necessary for the above reason, as it will hardly be contended that a person would pretend incapacity for work merely to obtain so immaterial a benefit, as relief from payment of premiums on his life insurance, when by so doing his source of income would be cut off, although we can conceive of a certain class feigning illness to secure without labor from one-third to one-half of their regular wages.

As a matter of fact these disability premiums, although based on British insured lives, as noted by Mr. Jackson, are nevertheless apparently somewhat more than sufficient to cover the benefits granted by the most liberally worded disability clauses, for undoubtedly the sickness and mortality among lives of the industrial grade insured in the British Friendly Societies would exceed that experienced among the carefully selected lives insured in the ordinary business transacted by the companies of this country, which obtain their risks from the better class of the population who, on account of their occupations and environment, will be less liable to sickness or injury by accident.

In case there is a provision to the effect that if disability occurs after age sixty the premiums will be charged as a lien against the policy, the company is relieved from a certain amount of risk and the benefit becomes of a temporary nature, so that adequate extra premiums would be lower than those given in Table XI of this paper. It would have been interesting had Mr. Jackson appended his commutation columns M' and ${}^{a}N$, so that the extra premiums on the above basis might have been readily computed and compared with the extra premiums which he has tabulated.

The extended insurance privilege now incorporated in practically all policies would also in some measure cover the risks under consideration, for when disablement preventing the pursuit of any gainful occupation occurs, it is probable that as the result of this decided impairment in the health of the insured death would in many instances ensue within the term for which the insurance would be extended under this option. When, however, there is an agreement to commence payment of the policy in instalments upon proof of permanent disability the extended insurance option will not to so great a degree protect the company on claims presented. On the whole there seems to be room for considerable difference of opinion as to the proper rates to be charged and these premiums must for the present be largely experimental, until the companies can obtain some definite experience upon which to base their computations. There is also some question as to how general the issue of policies containing such a provision will become, especially in view of the action of Massachusetts forbidding the issue in that State of a policy-form embodying in one contract a policy of life insurance and a provision for payment of premiums by the company in event of the insured's becoming totally disabled. I believe, however, that eventually the permanent disability clause providing for cessation of premiums in event of total disability will in some form be quite generally incorporated in policies of life insurance.

Mr. Flynn: A comparative study of the various rates of mortality and disability deduced by Mr. Jackson from the Experience of the British Friendly Societies, Males, 1861-1870, brings out some interesting points of discussion.

The value of q_x showing the rates of mortality among all members of the British Friendly Societies—sick and healthy—are smaller than those of the American Experience table up to age 35, are higher up to about age 78 and smaller again in the ages above 80. From ages 60 to 65 Mr. Jackson's table shows rates about 10 per cent. higher than the American Experience. In ages above 85 the rates are much smaller than one would expect—probably due to the inadequacy of data in these older ages.

Mr. Jackson does not give the table which would show the probability of becoming disabled at any age which could properly be compared with the disability tables of Zimmerman and Karup. The table which is given, however,—the rate of *being* disabled, *i. e.*, of having become totally and permanently disabled at age x or earlier and remaining disabled at age x—can be used with certain restrictions. The rates for the ages below 50 of this table are surprisingly large when compared with German tables, especially in view of Mr. Jackson's assumption that the mortality rate during the first two years of disability is the same as will be experienced after two years from date of disability. Experience has shown that the mortality among persons disabled in the younger ages is very high during the first year or two after disability; gradually approaching the rate of general mortality as time elapses. This is probably due to the fact that total disability in the early ages is most often the result of an accident which results in an early death from shock or serious injuries. In the older ages the rates of being disabled instead of going up rapidly to a value of 1 at or about age 80 or 85, as in most tables, advance slowly to age 84 and then decrease. It seems reasonable to assume that at very old ages, as age 90, the rate of being disabled should be nearly 1. The only explanation seems to be that the data at these older ages are inadequate. Mr. Jackson's table of being disabled therefore shows higher rates than the German tables in the earlier ages and lower rates in the older ages—in spite of the assumption which probably keeps down the rate in early ages and in spite of the probability that a table of being disabled would show a higher rate than the table of becoming disabled in the older ages where the mortality among disabled risks most nearly approaches the general mortality.

Mr. Jackson's table of ${}^{i}q_x$ (mortality among disabled) compared with that of Bentzien which is one of the most reliable of German tables and based upon the whole experience of German Railroad Employees, 1868-1889, covering all occupations, shows rates of mortality which are very high—almost double that of the German table in the earlier ages. Mr. Jackson's values of ${}^{i}q_x$ are heavier than those of *any* German table, even those of Caron, Kuttner and Kaan which are based upon observations of German and Prussian miners.

Mr. E. Hamsa, in a paper read before the Third International Congress (*Trans. Third Inter. Cong.*, page 154), has computed the extra premiums for the cessation of premium payments in case of disability using the disability table of Zimmerman based upon Office Employees of German Railroads, as the general mortality table that of "Twenty-three German Offices," and for the rate of mortality among invalids the table of Bentzien. As this table of Zimmerman shows light rates of disability and as in the opinion of Mr. Hamsa, Bentzien's table is the most reliable of the German experiences, the extra premiums which are obtained therefrom are considered by Mr. Hamsa to be the lowest which can safely be charged for this disability benefit. It is unfortunate that the extra premiums based upon Mr. Jackson's table are not given for the 20 Payment and 15 Payment Life forms which are the only plans for which rates are given by Mr. Hamsa. From the study of Mr. Hamsa's rates, however, it would seem safe to say that Mr Jackson's extra premiums while slightly lower do not differ greatly from those of Mr. Hamsa. This similarity of extra premiums seems remarkable in view of the comparatively large variance between Mr. Jackson's rates of disability and mortality among disabled and those of the German tables. The explanation is probably that Mr. Jackson's high rates of mortality among disabled give a low value for the annuity upon disabled lives and that when this is used with a high rate of disability the result is somewhat similar to the use of the high value of the annuity upon the disabled life and the low rate of disability of the German table. It appears therefore that although Mr. Jackson's tables differ materially from the German tables used by Mr. Hamsa, the extent and the manner in which the various rates of disability and mortality enter in the calculation of the extra premium for this benefit are such that the resulting premiums do not differ to any great extent.

Although the tables and extra premiums for disability benefits based upon the Experience of the British Friendly Societies made up by Mr. Jackson are undoubtedly a valuable addition to our knowledge on the subject of the proper charge for permanent disability benefits, the complete absence of any reliable figures based upon an American Company's experience compels one to continue to consider this question carefully and to view from all standpoints the problem of how far a company can afford to go in the face of such scant knowledge. The curve of the rate of total and permanent disability is such that the whole problem of what permanent disability benefits should be offered and what the proper rate for these benefits should be can, in my opinion, be best considered from two standpoints—that of the ages below 60 and that of the ages above 60. At the early ages the probability of becoming permanently disabled is very remote and no decided upward tendency is found in the curve denoting this probability until ages 50 or 60 are reached. Beginning at about age 60 the curve inclines sharply upward reaching the value of I in most tables at about age 80.

In the younger ages the question of the proper interpretation by the claim department of a company of the condition of "total and perma-

nent disability" is not a difficult one. In the contracts of some companies it is stated that loss by accident or disease of the "entire sight of both eyes, both feet, both hands or hand and foot" will constitute total and permanent disability. As disability at these ages is generally the result of acute diseases or accidents the correct interpretation of this condition—that which is given by a claim adjustor of an accident company—is probably not a difficult task. In addition to the definite nature of the condition in the earlier ages is the fact that all experiences show that the occurrence of this event is comparatively remote and that therefore the premium to be charged is small. Another point which has a large bearing upon the situation is that the Extended Term feature of the present day contract which has attained an age of a few years will generally cover the risk under this benefit because of the short duration of life after disability in these ages. It can be seen therefore that, as far as the early ages are concerned, *i. e.*, where the risk of disability over age 60 is not considered, the offer of disability benefits in a life contract is reasonably safe from an underwriting standpoint and the rates as brought out by foreign experiences although small are probably adequate.

When the matter is considered from the standpoint of the ages above 60, however, an entirely different situation is found. The question as to just what constitutes permanent disability in these older ages is a most difficult one to answer. Permanent disability at this time of life is due only in a small degree to accident but rather almost entirely to the infirmaties due to old age. Just what case a company shall accept as one of total and permanent disability is almost impossible to say in advance. Whether or not a policyholder 65 or 70 years old who has rheumatism or some other ailment of old age and who decides to give up work will be considered to be entitled to disability benefits depends entirely upon the particular company's business policy and liberality in adjustment of claims. Further, just what has constituted "total and permanent disability" in the older ages in the foreign experiences is not known. The fact that in various experiences this condition has been looked upon differently is shown by the comparatively large variance in the rates of disability of various tables at these older ages. It is probably true that most of the tables, particularly German statistics, have been made up with the idea of superannuation in mind rather than total disability. Again the Extended Term feature which in the earlier ages acts as a safeguard, is at the older ages entirely inadequate for the reason that the Extended Term period is shorter and the mortality among disabled risks in the older ages has been shown to differ but little from general mortality rates.

It is because of this unsatisfactory situation that several companies have confined the issue of contracts carrying full disability benefits to ages below 50. A policy, therefore, issued below age 50 will have acquired a large Extended Term value or will have passed a greater part of the premium paying period before the age of 60 is reached. Several other companies have provided a disability clause which gives full benefits only in case of disability before age 60 on any form of contract. In addition

to what has already been said there are several other good reasons for restricting disability benefits in ages above 60—the principal one being that the character of a disability benefit in the early ages differs greatly from that in older ages. In the productive ages of early life the disability benefit protects the policyholder from loss in case of total disability due to some unforeseen cause such as accident or acute disease. In the ages above 60 the heavy extra premium required and the high probability of a settlement under this benefit simply changes the character of the benefit contract as, for instance, under the benefit which provided for cessation of premium payments at disability, a 20 Payment Life is changed into a 15 Payment Life. In case of the benefit which provides for the payment of the face amount of the contract in instalments upon disability the heavy extra premium in older ages simply provides for the contract as an endowment payable in instalments—something in the nature of a superannuation allowance. In my opinion the true sphere for disability protection is in the ages below 60 and that the change of the character of the benefit and the lack of reliable information in regard to proper rates for American Companies makes the offer of the benefit at older ages undesirable.

It might be well to mention here that there are a great many companies, particularly the young, small companies, which are offering unrestricted disability benefits presumable without adequate charges for the risk at the older ages. If the interpretation of the condition of total and permanent disability is to be applied by these companies with anything approaching leniency there is but little doubt that a heavy expense will fall upon these companies because of the inadequacy of the charges for this feature.

Oral Discussion.

Mr. Moir: Mr. Chairman, I think we owe Mr. Jackson a debt of gratitude for bringing this subject before us, and especially as it has opened up an entirely new subject; one that has never appeared in our *Transactions*, and a subject on which we want information. There is undoubtedly a desire to extend this disability feature in life insurance, and a public demand for it, despite the antagonism of Massachusetts and other State laws.

We owe him our thanks, too, for having been the means of producing the three excellent discussions to which we have just listened. Mr. Morris covered a large part o' the ground, and submitted new figures obtained from another source which I have no doubt will be very useful. Mr. Cole took up quite another line of discussion, and Mr. Flynn's remarks were particularly *apropos*, especially in his division of this particular feature of our business into ages below and above sixty. I think he might possibly have taken the age a trifle higher for his division, but approve of the principle and regard it as very important.

This question is one of which we know so little that everything that

can be gathered is of value. I know that Mr. Jackson in preparing the paper was hunting for information, trying to get statistics that would be of practical value, and he got statistics which are, I think, reasonably safe on which to base a general opinion. Not that the rates are accurate, or that any one would think of calling them accurate, but they are a great advance on anything we have had heretofore, and they place us in a scientific attitude towards this subject, and when we reach this position we begin to get at the truth.

I arose only to say a few words of thanks and of appreciation of the paper and of the criticisms.

NOTE ON THE CALCULATION OF INSURANCE VALUES—CHRISTIAN JENSEN.

Vol. X, Page 503.

Written Discussion.

Mr. Sheppard: Mr. Jensen has done the Society good service by calling to its attention Dr. Gram's formulas, by means of which, a series of commutation columns at various rates of interest based on a standard mortality table being given, the table being graduated by Makeham's formula, various actuarial functions according to any other table may be calculated if the latter table also follows Makeham's law. For this purpose it is only necessary to know the constants on which the second table is based. As numerical examples help to fix one's ideas I have taken the $H^{(M)}$ Table as given at the end of the Text-book of the Institute of Actuaries as my standard table, the commutation columns being there calculated at various rates of interest, and the Tropical American Experience table given in Vol. III of the *Transactions* of this Society as the second table. Using the formulas given at the beginning of Mr. Jensen's paper Dr. Gram's constants work out as follows:

$$\rho = \alpha' - \alpha = .0047761 \qquad \lambda = \frac{\gamma'}{\gamma} = 1.00865 \qquad \epsilon = \frac{\log_e \beta' - \log_e \beta}{\gamma} = 3.081202$$

As an example, let us find the force of mortality at age 40 according to the second table. The age with which we must enter the first table is $40\,\lambda + \epsilon = 43.427$. Interpolating by first differences in the first table and adding ρ we get $\mu'_{40} = .01604$ which differs by only one in the fifth place of decimals from the correct value. This establishes the correctness of the values above given for ρ, λ and ϵ.

As net premiums and reserves for contracts where the sum assured remains constant can always be expressed in terms of temporary annuities, it seems to me that if this method is to be of practical use we ought, in the most general case where the three Makeham constants have different values in the two tables, to have a formula by means of which we can obtain the value of a temporary annuity according to the second table directly from values of temporary annuities and such other actuarial functions as are usually tabulated according to the standard table. Mr. Jensen states that we must interpolate on the continuous insurance values. In order to test the correctness of this statement the remainder of this discussion consists of an attempt to obtain an approximate formula which will give good numerical results, as I have not been able to evaluate accurately the double integral involved in Dr. Gram's formula. I have used immediate annuities throughout and accented letters to refer to the second table.

Dr. Gram's formula rewritten becomes

$$\mathrm{a}'_{x\,\overline{n}|} = \Sigma_{t=o}^{t=n-1} \mathrm{e}^{-\int_o^t (\delta + \mu'_{x+t})dt} \qquad (\mathrm{A})$$

But $\mu'_{x+t} = \rho + \mu_{\lambda x + \lambda t} + \epsilon$

Let $\lambda x + \epsilon = \lambda' x,\ \delta + \rho = \delta',\ \lambda t = t'$

Then $$\mathrm{a}'_{x\,\overline{n}|} = \Sigma_{t'=o}^{t'=\lambda(n-1)} \mathrm{e}^{-\frac{1}{\lambda}\int_o^{t'} (\delta' + \mu_{\lambda' x + t'})dt'}$$

where the interval for finite integration is λ. This is the expression which I hope some other member of the Society will be able to evaluate.

The first approximate formula is obtained as follows:

Starting out with the fundamental equation

$$d\,\log(v^t l_{x+t}) = -(\delta + \mu_{x+t})dt$$

Replace t by λt

$$\therefore\ d\,\log(v^{\lambda t} l_{x+\lambda t}) = -\lambda(\delta + \mu_{x+\lambda t})dt$$

Integrate between the limits 0 and $\frac{s}{\lambda}$

$$\therefore\ \log(v^s l_{x+s}) - \log l_x = -\lambda \int_o^{\frac{s}{\lambda}} (\delta + \mu_{x+\lambda t})dt$$

Replace s by λt, divide each side by λ and raise "e" to the power of each expression and we get

$$\left(\frac{D_{x+\lambda t}}{D_x}\right)^{\frac{1}{\lambda}} = \mathrm{e}^{-\int_o^t (\delta + \mu_{x+\lambda t})dt}$$

$\therefore$ using equation (A)

$$\mathrm{a}'_{x\,\overline{n}|} = \Sigma_{t=0}^{t=n-1} \mathrm{e}^{-\int_o^t (\delta' + \mu_{\lambda' x + \lambda t})dt}$$

$$= \Sigma_{t=0}^{t=n-1} \left(\frac{D_{\lambda' x + \lambda t}}{D_{\lambda' x}}\right)^{\frac{1}{\lambda}},$$ taken at the rate of interest corresponding to δ'.

By expanding the expression in the bracket an approximate formula was obtained, but the resulting numerical results were not satisfactory.

Another method is as follows:—

Starting again with our fundamental equation

$$d\,\log(v^t l_{x+t}) = -(\delta + \mu_{x+t})dt$$

Integrate between 0 and $\frac{t}{\lambda}$

$$\therefore\ \log v^{\frac{t}{\lambda}} l_{x+\frac{t}{\lambda}} - \log l_x = -\int_o^{\frac{t}{\lambda}} (\delta + \mu_{x+t})dt$$

Raise "e" to the power of each side of this equation

$$\therefore \quad \frac{v^{\frac{t}{\lambda}} l_{x+\frac{t}{\lambda}}}{l_x} = e^{-\int_0^t (\delta+\mu_{x+t})dt}$$

Sum for integral values of t from $t=0$ to $t=n\lambda-\lambda$ and we get

$$\sum_{t=0}^{t=(n-1)\lambda} e^{-\int_0^t (\delta+\mu_{x+t})dt}$$

$$= \frac{1}{l_x}\left[1 + v^{\frac{1}{\lambda}} l_{x+\frac{1}{\lambda}} + \ldots + v^{n-1-\frac{1}{\lambda}} l_{x+n-1-\frac{1}{\lambda}} + v^{n-1} l_{x+n-1}\right]$$

$$= \lambda a^{(\lambda)}_{x\,\overline{n-1|}} + \frac{D_{x+n-1}}{D_x}$$

which using the ordinary formula for an annuity payable λ times a year and stopping at the second term, becomes

$$= \lambda\left\{ a_{x\,\overline{n-1|}} - \frac{\lambda-1}{2\lambda}\left(1 - \frac{D_{x+n-1}}{D_x}\right)\right\} + \frac{D_{x+n-1}}{D_x}$$

$$= \lambda a_{x\,\overline{n-1|}} - \frac{\lambda-1}{2} + \frac{\lambda+1}{2}\cdot\frac{D_{x+n-1}}{D_x}$$

Now, as λ is nearly equal to unity we may, as a first approximation to the value of the integral on the right hand side of equation B, make $\lambda=1$ outside the integral sign and in the interval of finite integration and we have finally

$$a'_{x\,\overline{n|}} = a\lambda_{x+\epsilon\,\overline{n-1|}} - \frac{\lambda-1}{2} + \frac{\lambda+1}{2}\,\frac{D_{\lambda x+\epsilon+n-1}}{D_{\lambda x+\epsilon}},$$

the expression in the right hand side of the equation being taken at the rate of interest corresponding to δ'.

As a numerical example I have worked out the temporary annuities for 10, 20 and 30 years and the annuity for the whole life at ages 20, 40 and 60 according to the American Tropical Table of Mortality at 3 per cent. interest using the Commutation Columns at 3 and 3½ per cent. given in the Text-book of the Institute of Actuaries. The equivalent ages were 23.254, 43.427 and 63.600 respectively, and the rate of interest obtained by the equation

$$i = \delta + \frac{\delta^2}{2} + \frac{\delta^3}{6} + \&c., \text{ was } 3.493\ \%.$$

The following table shows the resulting values, also the true values and the error in each case.

Approximate, True Values and Error in Calculation of Annuities.

$n=10$			$n=20$			$n=30$			$n=\omega-x$		
Approx.	True	Error	Approx.	True	Error	Approx.	True	Error	Approx.	True	Er
8.348	8.352	—.004	13.812	13.810	.002	17.280	17.272	.008	20.723	20.658	.(
8.148	8.155	—.007	12.953	12.954	—.001	15.316	15.302	.014	16.169	16.131	.(
7.069	7.072	—.003	9.314	9.303	.011	9.585	9.566	.019	9.589	9.569	.(

These figures show that the above formula of approximation may safely be used for the calculation of net premiums and reserves in the case of the endowments usually issued in this country and also for life and limited payment life policies taken out at high ages.

In conclusion I should like to state that I believe these formulas might be found capable of further development and to express the hope that the other members of the Society will turn their attention to them.

Mr. Kilgour: The paper of Mr. Jensen dealing with the calculation of Insurance Values based on any Mortality that follows Makeham's Law and an arbitrary rate of interest brings once more to our notice the eminent services rendered the actuarial profession by Dr. Gram, and the high standard of the contributions to the *Actuarial Journal* of Scandinavia. The question of sub-standard and special risks is one of recurring interest to Actuaries, and we should therefore welcome any addition to our working equipment for treating with this extremely practical and important problem.

As pointed out by Mr. Jensen, we are already indebted to Mr. Hunter for extensive tables enabling the Actuary to calculate values based upon important modifications of the Makehamized American Table. The enunciation of a principle, however, and its generalization, apart from its practical value arouses interest in the problem, and is of the greatest possible service.

The paper itself scarcely lends itself to critical review, but one or two steps which are not self evident may perhaps call for elucidation.

(1) $$\bar{a}_{x\,.\,\overline{n}|} = \int_0^n \frac{D_{x+t}}{D_x}\,.\,dt$$

$$= \int_0^n v^t s^t g^{cx(c^t-1)}dt, \text{ where } l_x = ks^x g^{c^x}$$

and $\mu_x = \alpha + \beta c^x$

$$= -\log_e s - \log_e g \log_e c\, c^x.$$

$$= \int_0^n e^{(\log_e v + \log_e s)t + c^x(c^t-1)\log_e g}\,.\,dt$$

Now $$\mu_{x+t} = -\log_e s - \log_e g \log_e c\, c^{x+t}$$

$$= -\log_e s - \log_e g \frac{dc^{x+t}}{dt}$$

$$\therefore \int_0^t \mu_{x+t}\,dt = -t\log_e s - \log_e g\, c^x(c^t-1)$$

$$\therefore\quad \bar{a}_{x\overline{n}|} = \int_0^t e^{-\int_0^t(\delta+\mu_{x+t})\,.\,dt} \quad \text{where } \delta = -\log_e v.$$

(2) $$\bar{a}'_{x\,.\,\overline{n}|} = \frac{1}{\lambda}\bar{a}_{z\,.\,\overline{v}|}$$

Here $$\bar{a}'_{x\,.\,\overline{n}|} = \int_0^n e^{(\log_e v' + \log_e s')t + c'^x(c'^t-1)\log_e g'}\,.\,dt$$

$$= \int_0^n e^{-(\delta'+\alpha')t - \frac{\beta'}{\log_e c'}c'^x(c'^t-1)}\,.\,dt' \ldots . (1)$$

Since $\beta' = -\log_e g' \log_e c'$

Also $$\bar{a}_{z\overline{v}|} = \int_0^v e^{-(\delta+\alpha)t - \frac{\beta}{\log_e c}c^x(c^t-1)}\,.\,dt.$$

$$\therefore\quad \frac{1}{\lambda}\bar{a}_{z\overline{v}|} = \frac{1}{\lambda}\int_0^v e^{-(\delta+\alpha)t - \frac{\beta}{\log_e c}c^x(c^t-1)}\,.\,dt$$

$$= \int_0^v e^{-(\delta+\alpha)t - \frac{\beta}{\log_e c}c^x(c^t-1)}\,d\left(\frac{t}{\lambda}\right)$$

Making $\frac{t}{\lambda} = t'$ and adjusting the limits, we have

$$\frac{1}{\lambda}\bar{a}_{z\overline{v}|} = \int_0^{\frac{v}{\lambda}} e^{-(\delta+\alpha\,\lambda t - \frac{\beta}{\log_e c}c^z(c^{\lambda t}-1)}\,.\,dt$$

$$= \int_0^n e^{-(\delta+\alpha)\lambda t - \frac{\beta}{\log_e c}c^z(c^{\lambda t}-1)}\,.\,dt \ldots . (2)$$

where v is selected so that $v = \lambda n$.

Since functions (1) and (2) are equivalents, we have

$$(\delta'+\alpha')t + \frac{\beta'}{\log_e c'}c'^x(c'^t-1) = (\delta+\alpha)\lambda t + \frac{\beta}{\log_e c}c^z(c^{\lambda t}-1)$$

for all values of t.

Expanding and arranging in powers of t; and equating coefficients we get

$$\left\{(\delta'+\alpha') + \beta' c'^x\right\} = \left\{(\delta+\alpha)+\beta c^z\right\}\lambda \ldots . (3)$$

$$\beta' c'^x\,.\,\log_e c' = \beta c^z \lambda^2 \log_e c \qquad \ldots . (4)$$

$$\beta' c'^x (\log_e c')^2 = \beta c^z \lambda^3 (\log_e c)^2 \qquad \ldots . (5)$$

etc.

Dividing (5) by (4) we get $\log_e c' = \lambda \log_e c$

$$\text{or } \lambda = \frac{\log_e c'}{\log_e c}$$

Also $$\beta' c'^{x} = \beta c^{z} \lambda \qquad \ldots . (6)$$

or $$z = \frac{\log_e c'}{\log_e c} x + \frac{\log_e \beta' - \log_e \beta - \log \lambda}{\log_e c}$$

$$= \lambda x + \frac{\log_{10} \beta' - \log_{10} \beta - \log_{10} \lambda}{\log_{10} c}$$

From (3) and (6) we get

$$(\delta' + \alpha') = \lambda(\delta + \alpha).$$

As far as the practical problem is concerned, most Actuaries I think will agree that a modification of (c) or γ might usually be disregarded, and that changes in . . . α and β are the chief modifications in Makeham's Law to be considered.

Mr. Henderson: Mr. Jensen brings to our attention in this paper a property of Makeham's law of mortality, whereby any isolated annuity value on a table following that law may be readily computed provided we have available a set of annuity values at different rates of interest by any table following that law. It may also, however, be applied to determine the constants of a Makeham law, which will closely reproduce the monetary values given by an ungraduated table. This arises from the fact that, where $\bar{N}_x = \int_x^\infty D_x \, dx$ and $\bar{S}_x = \int_x^\infty \bar{N}_x \, dx$ the quantities $\frac{D_x}{D_{x+t}}$, $\frac{\bar{N}_x}{\bar{N}_{x+t}}$ and $\frac{\bar{S}_x}{\bar{S}_{x+t}}$ remain unaltered by transformation.

But $$\log \frac{D_x}{D_{x+t}} = \int_x^{x+t} (\mu_x + \delta) \, dx$$

$$\text{og } \frac{\bar{N}_x}{\bar{N}_{x+t}} = \int_x^{x+t} (\bar{\mathrm{P}}_x + \delta) \, dx$$

$$\text{and log } \frac{\bar{S}_x}{\bar{S}_{x+t}} = \int_x^{x+t} \left(\frac{(I\bar{A})_x}{(I\bar{a})_x} + \delta \right) dx.$$

By the method of trial and error a Makehamized mortality table can be determined which will reproduce any three integrals such as those on the right hand side of these equations. Thus the average values of $\bar{\mathrm{P}}_x$ over three different intervals may be reproduced, or its average over two intervals and the average value of μ_x over one interval, or any other combination of three averages.

Address of the President,

JOHN K. GORE,

at the opening of the Semi-Annual Meeting,

October 15th, 1908.

Gentlemen of the Actuarial Society:

Insurance as a factor in economics is given scant treatment in the text-books and in general works on political economy. If we exclude references to the three parts into which the profit of the capitalist or enterpriser resolves itself; namely, interest, wages of superintendence and compensation for risk or insurance, it may be said that the so-called classic economists, such as Adam Smith, Ricardo and John Stuart Mill, make no reference whatever to insurance. Modern writers on political economy devote but a page or two to the subject of life insurance. It does not seem inappropriate, therefore, to comment in the briefest manner possible upon some of the economic phases of life insurance.

The birth of the science of political economy, the institution of savings banks and the establishment of the first successful life insurance organizations—all three events occurring about the end of the third quarter of the eighteenth century—mark an epoch of the greatest economic significance. Then it was that the weighing of the future wants of mankind, as compared with its present desires, was developed into a science, and probably the most beneficent products of that development were savings banks and life insurance.

Life insurance and savings banks have many attributes in common. Savings banks are builders of capital. Savings banks tend to prevent violent changes in income. Savings banks influence greatly the distribution of wealth. Savings banks work to decrease the non-economic or dependent class in society. All of these functions belong also to life insurance.

Life insurance builds capital. Not only are the proceeds of a life insurance policy or of a matured endowment a capital fund producing its own small flow of income, but the collective funds held in trust by life insurance companies become available as capital in the purchase of the securities of productive enterprises and in loans made on productive property.

Life insurance is a steadier of incomes. The temporary stoppage of the flow of income that would otherwise occur in the case of a family whose breadwinner dies is prevented by the proceeds of the policy. However small the amount, it becomes for the time being the whole or a part of the capital on which the survivors live. Although usually reduced, the family income is continued by the policy until such time as adjustments can be made to meet new conditions. What would have been a violent and perhaps disastrous change has been made, through the insurance policy, gradual and bearable. Those beneficiaries who are protected by life insurance can look ahead with a feeling of certainty rather than uncertainty. Such a condition if general in any community affects powerfully the entire economic life of that community.

Like savings banks, life insurance is a most important agent in the distribution of wealth. The hundreds of millions of dollars annually paid to the life insurance companies in comparatively small amounts, to be distributed by the companies among those upon whom losses may fall, work an incalculble good by enabling the beneficiaries to maintain their economic independence.

Life insurance, therefore, tends to decrease the non-economic or dependent class of society. If not recruited from the class above, the weakest, non-self-supporting group would eventually become extinct. This class, however, is being continually added to from the class just above it because of the misfortunes, incapacity or indulgences of some of the members of that class. It has often been claimed that if all the energy of church and state could be concentrated not upon criminals but upon those in danger of becoming criminals, law breaking would cease in a generation or two. So it may be said that one of the greatest economic results achieved by life insurance is the rescuing of hundreds of thousands of those just above the non-economic class and preventing them from becoming burdens upon the state.

In this connection reference should be made to Industrial life insurance. The industrial classes are never so far removed from

the non-economic stratum but that unexpected or unusual expenditures, even though comparatively small, will render a large percentage of those subjected to such expenditures dependent upon the bounty of friends or upon public charity. Recipients of charity do not as a rule long retain their self-respect. The loss of family pride and of self-respect soon dulls the sense of *meum* and *tuum*, and recruits are added to the criminal classes.

Ordinary insurance, as distinguished from Industrial, is generally placed upon the life of one member of the family, and is chiefly for indemnity for the loss of the family income that ceases at his death. Industrial insurance, on the other hand, is placed upon the lives of all the members of the family, and is primarily for indemnity for the cost of medical attendance and of burial. In other words, it is primarily for indemnity for expense incurred, and, secondarily, for loss of income. Even if the father, the chief breadwinner, dies, and the proceeds of the policy pay the costs of the final illness and of the funeral, and leave only a small residue for temporary expenses, the family will soon adjust its affairs and retain an economic standing, for among the industrial classes all except young children and the aged as a rule contribute something toward the family support. The amount of insurance that will be written upon the lives of young children is less than the average cost of burial. The child produces no income. In this case, therefore, the insurance is a partial indemnity only for expense incurred and not at all for loss of income. In this respect the Industrial plan has introduced an economic principle new to life insurance.

Among the earlier economists those engaged in developing the science of life insurance would have been classed as unproductive laborers. Labor in order to be productive had to bring about material results, or as John Stuart Mill says, "produce utilities embodied in material objects." From the old definitions one would conclude that the investigations of an actuary or the arguments of a life insurance agent are unproductive, while the labor of the one who prints the policy forms is productive. Modern economists, however, look for the test of productivity in feeling, not in material things. As actuaries, you are helping develop the habit of saving; helping increase the peace of mind of millions who are living under the protection of life insurance; helping distribute lightly among the many the burden that would crush

the few. You are adding to the psychic income of mankind. If any labor is productive, yours is productive.

Since our last meeting we have lost another charter member of the Society, our former President, Israel C. Pierson. During the first ten years of our Society's life he was its efficient and popular Secretary. Having been successively Second and First Vice-President, he became President in 1903 and in the same year presided over the Fourth International Congress, held in New York City.

To no Fellow of the Society has its welfare been dearer than it was to Mr Pierson. Always active in its interests, always kindly considerate of the feelings of others, his loss is a great one to us all.

I have asked Mr. Nichols to prepare an appropriate minute to appear in the next number of the *Transactions*.

Abstract from the Minutes of the Semi-Annual Meeting of the Actuarial Society of America, held in Newark, N. J., on Thursday and Friday, October 15th and 16th, 1908.

Board Room,
Mutual Benefit Life Insurance Company,
Newark, N. J., *October 15th*, 1908.

The meeting was called to order by the President, Mr. J. K. Gore, at eleven o'clock, A. M.

FELLOWS PRESENT.

Alsop,
Bradshaw,
Carpenter,
Craig, J. D.,
Craig, J. M.,
Crawford,
Flynn,
Fackler, E. B.,
Fackler, D. P.,
Gaylord,
Gore, J. K.,
Grow,
Hann,
Henderson,
Hunter, A.,
Hunter, R. G.,
Hutcheson,
Ireland,
Jackson,
Joffe,
Lee,
Johnston,
Macaulay,
Marshall, E. P.,
McClintock,
Messenger,
Moir,
Morris, E. B.,
Nichols,
McKechnie,
Papps,
Plumley,
Rhodes,
Salter,
Sanderson,
Sheppard,
Strong,
Tatlock,
Thompson,
Van Cise,
Welch,
Weeks,
Wells,
Wood,
Woodward, G. B.,
Woodward, J. H.,
Wright, W. C.,
Young.

ASSOCIATES ATTENDING.

Blehl,
Breiby,
Brough,
Brown,
Bulkley,
Cathles,
Carrington,
Davenport, J. S., Jr.,
Dickenson,
Forster,
Gamwell,
Gibb,
Gould,
Hammond,
Morris, W. O.,
Ryan,
Smith, E. H.,
Somerville,
Washburne, A. C.

After calling the roll, a quorum of the Fellows being present, the minutes of the annual meeting held in New York on May 21st and 22nd, 1908, were approved as printed in the *Transactions*.

The President of the Society then delivered an address (see page 723).

The Secretary then read an abstract of the proceedings of the Council since the last meeting. This included:

(*a*) The adoption of a diploma, to be granted to all the Fellows of the Society who desire it, irrespective of the date of their entry into the Society.

(*b*) The consideration of an investigation into the mortality among Term Insurance. The Council suggested that the matter of the Term investigation be dropped for the present owing to the heterogeneous data and the different methods of treating Term Insurance in the past by the various companies, but that the companies be recommended to keep their data in such shape, beginning with the year 1907, as to permit of an investigation say ten years hence.

(*c*) The adoption of certain changes in the syllabus of examination, the principal changes being the extension of Sections A and B of the Associateship examination to cover two days each, and the addition of two subjects to the Fellowship examination, all changes to go into effect in the year 1910.

(*d*) The investment of the Society's funds in certain first-class bonds.

(*e*) The consideration of matters regarding the 6th International Congress of Actuaries, including the appointment of delegates by the United States Government.

(*f*) The opening of the pages of the *Transactions* to correspondence on similar lines to that permitted in the Institute Journal, such communications to have the unanimous approval of the Committee on Papers.

(*g*) The Council also voted that Mr. J. M. Langstaff and Mr. J. B. McKechnie be admitted as Fellows of the Society, both of them having passed the regular examinations, and having attained the age of 25.

On motion, the proceedings of the Council were duly approved.

The following amendment to the constitution was duly adopted by the Society on the recommendation of the Council: Add the

words, "unless the Council should by vote appoint a different date in the month of May." The first sentence of Article VII of the constitution would accordingly read: "The annual meeting of the Society shall be held on the first Thursday after the 14th day of May in each year, unless the Council should by vote appoint a different date in the month of May."

The Society then proceeded to the reading of papers prepared for this meeting (pages 597-666).

At 12.30 addresses of welcome were made by the President of the Mutual Benefit Life Insurance Company, Mr. Frederick Frelinghuysen, and by the ex-President of the same Company, Mr. Amzi Dodd. The Society then lunched with the officers of the Mutual Benefit Insurance Company.

AFTERNOON SESSION, *October 15th*, 1908.

After the Society had reassembled at 2.30 o'clock, the reading of the papers was continued. The Society then proceeded to a discussion of the papers presented at the annual meeting in May of 1908.

At five o'clock the Society adjourned, to meet together at dinner at the Essex Club in the evening, and to reassemble for business the following morning at 10 o'clock in the Board Room of the Prudential Insurance Company, Newark, N. J.

SECOND DAY.

BOARD ROOM, PRUDENTIAL INSURANCE COMPANY,
NEWARK, N. J., *October 16th*, 1907.

The meeting was called to order by the President at 10 o'clock, and the discussion of papers read at the meeting in May was then resumed and completed.

On motion of Mr. Nichols, seconded by Mr. Hann, the thanks of the Society were voted to the Officers of the Mutual Benefit Life Insurance Company and of the Prudential Insurance Company, for their generous provision for the comfort and convenience of the Society, and for the entertainment of its members during this meeting.

At 12 o'clock the Society finally adjourned, to reassemble informally for luncheon at the Essex County Country Club, as guests of the Officers of the Prudential Insurance Company.

ARTHUR HUNTER,
Secretary.

Correspondence.

To the Editor of the *Transactions:*

Re — Reinstatement of a Policy Running Under Term Extension where Insured is an Impaired Risk.

Dear Sir:

In these days when term extension or continued insurance is a feature of practically every policy issued, a problem often arises to which I have been unable to find a satisfactory solution. I refer to those cases where the insured has ceased to pay premiums, the policy being carried on term extension, and where he is found to be an impaired risk on applying for reinstatement. There are four extreme cases, as follows:

(*a*) Where the insured is in a very poor state of health, and the period of extension is short.

(*b*) Where the insured is in a very poor state of health, and there are a number of years of term extension.

(*c*) Where the insured is only a slightly impaired risk and the period of extension is for a few months.

(*d*) Where the insured is only a slightly impaired risk, and the period of extension covers a number of years.

In cases (*a*), (*c*) and (*d*) the practice would be to reinstate the policy; but in the case of (*b*) the policy would be reinstated only if the insured were nearly certain to die within the term extension period—otherwise the company would naturally prefer to refuse to reinstate the policy. Between these four extremes there are many cases which it would be advisable to reinstate, and some cases which should be refused reinstatement. The problem, therefore, is to determine the line dividing those who should be reinstated and those who should be refused reinstatement. So far as I can determine, the practice is to treat each case on its merits; but it would be of great interest and value to the profession if a formula could be prepared whereby the dividing line could be determined. While I have devoted considerable thought to this problem and have found several solutions, none of them have been entirely satisfactory. I am therefore bringing the problem to the attention of our Fellows and Associates in the hope that someone can furnish a satisfactory solution.

Yours very truly,

Arthur Hunter.

Dr. Israel C. Pierson.

Through the death of Dr. Israel C. Pierson on September 11th, 1908, this Society lost a charter member, whose earnest and devoted labor in its behalf, during the earlier years of its existence, was a large factor in securing for the Society the important position which it now occupies among the learned societies of the world. Dr. Pierson was elected its first secretary and continued for ten consecutive years to perform, almost unaided, the multitudinous duties devolving on that office. Repeated testimonials from the Society bore witness to the fidelity with which he cared for its interests during this formative period of its development. He was the chief medium through which its relations with sister organizations in other countries were established and was one of its chosen representatives at International Congresses of Actuaries abroad. He was selected as President of this Society to preside over the International Congress, when America was made its meeting place and was honored by election as a Corresponding Member in both the French Institute of Actuaries and the Belgian Association of Actuaries. Probably no one member has given more time and study and active labor to the Actuarial Society of America than did Dr. Pierson during those earlier years. His work was that of a director and organizer of its affairs, as well as a recorder of its transactions. He was sensitive in his nature and shrank from the publicity of the stage. He was but an occasional contributor to its papers or discussions. His kind heart made him a helpful friend to many who sought his assistance and the sincerity of his friendships is well known by his professional associates. Business anxieties cast their shadows over his later years and contributed doubtless to his death, but he will be long remembered by the surviving members of the Society as the efficient secretary during its earlier days.

Dr. Pierson was for many years a trustee of the New York University, from which he graduated in 1865, and received the degree of Ph. D. in 1890. He received his actuarial training in the New York Life and the Equitable Life, from which he entered the service of the Washington Life as Actuary. He resigned in 1905 and became a consulting actuary and insurance journalist. He was also a Fellow of the American Statistical Society, the New York Academy of Sciences and the New York Mathematical Society.